And this is the crux of the point I wish to make. Set "W" represents concrete THINGS while set "I" permits us to extend our thinking to whatever limits we are capable. If we teach youngsters that +4 and 4 are identical, just because it is too cumbersome to always identify the positive sign, we are setting the stage for confused thinking later on.

For example, let's take absolute values. The absolute value of -4 is 4. Now if 4 is in set I, then of course -4 = +4! Enough said.

Are We Shortchanging Our Students? or Back to Basics? New Basics? or Old Basics?

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Probably each of us in our role as a teacher has been confronted by an anxious student inquiring about some of the math we teach. How many times have each of us heard:

> Why are we taking this stuff?
> What's it good for ... sir?

I would be the first to admit that we can't justify everything we do *all the time*. To explain how a specific topic in mathematics "fits" into the scheme of things is often difficult. The student must trust that what we are doing day by day in the math classroom is useful, is relevant, and is needed for today's activities as well as for tomorrow's. Unfortunately, for many students tomorrow never comes. The students then become parents, and the cycle of asking "Why are we taking this stuff" is continued. Here are certainly many indications that there is concern about the curricula we teach. Many studies have been and are being completed in both the United States and Canada to ascertain "What should be the math curricula?" Unfortunately, to predict the content, the skills, or even the methods needed at some future date is difficult. If you listen to the experts and read the journals, you soon would develop a complex about what we are not doing in the math classroom, but thank goodness for the so-called new math. We have in it at least a scape-goat. We have seen the headlines "New Math has failed! Back to basics!", and now we have a replacement for the new math, the BTB (Back to Basics).

When the "New Math" was in vogue, each person you talked to had a different "understanding" of what the "New Math" was: There were the *Set-new-math* followers, the *Base-new-math* followers, the *Structure-new-math* followers, and so forth. (I apologize to those new-math groups not identified at this time). The parents identified new math from another point of view. They only saw what their children brought home; sets, and words, commutative, associative, distributive, inverse, base 2, base 8, third base - to name a few - soon the scene is set for the BTB to be formed, and the parents are willing to join.

These days, I am hearing more and more of the BTB, but I think that before we change we need to decide *"what are the basics."* Parents whose experience with math was almost completely computation, evaluate a new program or curriculum according to their backgrounds. We must ask: are the basics solely related to computation? I think not. No one would disagree that "the old basics" are essential, but there are "new basics" that parents as adults use in their everyday living, and which need to be dealt with. The computation content, so familiar to many and part of the "Happy Days Syndrome" to return to basics, is not by itself suitable even for today's world, let alone tomorrow's. My main concern is not that the BTB wants change (improvement, or whatever), but that they appear to want the pendulum to swing back to a "shut-up: do-this" curriculum, based on computation.[1] We can't afford to have the pendulum swing back too far. Everyone will basically agree that computational skills are important and basic to the students' mathematical development, but to stop there would short-change our students. Before the BTB make the same errors as the New Math groups, they (there are probably more than one) should decide what the basics are!

I want to help the BTB by offering the following suggestions. (These suggestions could be added to the computational platform already advocated by the BTB. A little review is useful here:

BTB = a group of persons who want to return Back to Basics.)

1. Students want us to be accountable. We should have some reasonable explanation or justification for "Why are we taking this stuff?" If we were to provide examples of applications for the present curriculum, some students and parents would be partially satisfied. To tell a student he is taking a topic because he needs it for next year's math is a weak argument. Many topics have applications, not only to satisfy the present questions asked by students, but also to provide a foundation for their future study of the topic in next year's math class. There are at present at least five studies that are looking at the curriculum from this point of view. Hopefully, the results of these studies will be expressed in practical terms and eventually made available to practising teachers. *Applications should be basic to any curriculum.*

2. In our everyday living, we are never given "neat problems" solved in a "neat way" that result in a "neat solution." A real-life problem requires

[1]A description of the "shut-up: do this" curriculum was in a speech delivered by Eric MacPherson of the Faculty of Education, University of Manitoba at the Annual Meeting of the National Council of Teachers of Mathematics in Denver, Colorado, April, 1975.

us to sort out the useful (needed) information from the extraneous information. Do we ever give students problems that contain more information than is really required to solve the problem? Will our students ever be given a problem to solve when they enter the "world of work" for which the "boss" provides only the information that is necessary? I doubt it! Yet how much of this practice in solving do we provide our students in our math classrooms (if you do provide these types of problems, then consider this section a brief review)? *To decide on what information is necessary or needed to solve a problem, to me, is a basic skill.* How many times have you given a problem which has a missing piece of information that the student is to provide in order to solve the problem? (Try assigning 5 problems, each having an essential piece of information missing. Provide a second sheet that contains the 5 missing "pieces," as well as 25 other pieces of extraneous but closely related information). This skill will probably be used more often than the skills advocated by the BTB. Without this skill the BTB skills are often confusingly applied by students to solve problems. (I think a mathematician might say the BTB skills are sufficient, but not necessary, or is that ... necessary but not sufficient ... ?)

However, "let me make it perfectly clear" that we need the BTB basic skills once the information is properly interpreted and the essential computational operations decided upon. Unfortunately, "good old" Euclidean geometry provided an opportunity for a student to "sort out" the needed information, but an EGID movement (Euclidean Geometry Is Dying) seems to have sprung up. (E GAD!)

3. Every day, as adults, we read about the Gallup Poll, the latest statistics on "why we are paid more and more but are eating less and less" and this poll and that poll. However, do we provide any basics for students to tackle the world of statistics? We give a brief look at the topic in Grade VIII (if at all in some classrooms), completely ignore it in Grades IX and X, but give some hope to those who stay on and finish high school and study the topic again. (The key question here is - how many will finish high school?) Could these students not find the skill of working with statistics useful just for everyday living, and thus be given the opportunity to study the topic in Grades IX and X? *Working with, interpreting and reading statistics is a basic skill.* Statistics permeate too much of our everyday living to be ignored as they have been in our mathematics curriculum. We are shortchanging our students by not teaching this skill. Continuously through Grades IX and X we are especially shortchanging those students who do not reach Grade XI.

4. How many students have asked you "Is this right?". How many are completely lost if they cannot find the answer at the back of the book? Students should be taught (and taught and taught ...) to know how to check that their answers are reasonable. *To know the answer is reasonable is a basic skill.* A student must by Grade XII "feel" whether the answer is reasonable. The "boss" does not assign problems with the answer at the back of the book. Teaching this skill partially can be accomplished by providing students with skills of approximating, estimating, as well as decision-making. Too often students are given problems that have "neat" answers. The following problem about a corn roast seems trivial at first, but when it was assigned

to Grade VIII students, it introduced the need to make decisions.

"How much would it cost our class to go on a corn roast?"

The students list the assumptions, and make decisions to arrive at the cost of going on the corn roast. I have assigned this problem to teacher groups and have had to referee arguments, as well as to impose a maximum time limit of 15 minutes because the problem, although simple in appearance, can be complex in finding a "reasonable" solution. The solution will eventually involve answers to these questions:

Where are we going? How do we get there? Do we take drinks? Do we need salt, butter, napkins, pepper, and so on, and so on?

These are but a few of the problems hidden in arriving at a "reasonable" price for a "reasonable" corn roast. What's your answer?

Problems suitable for different grade levels should be assigned to develop and strengthen this basic skill "Is my answer reasonable?"

5. I am sure many teachers already provide students with a variety of strategies for "starting" to solve a problem. I will never forget when I posed a problem to a class and got the immediate reply, "We haven't taken that yet ... sir!" We all remember too well the complete blanks left for some problems tackled by students writing exams. Some students freeze as soon as we say "word problem," and will sit and look, but really do nothing to "start" the problem. *Teaching students to "sketch" the problem or make a diagram to help them solve the problem is a basic skill that needs to be emphasized continually.* Many problems have been solved by "doodlers." Often a problem is solved by "doing" something rather than "waiting" for an inspiration. A blank page provides very little inspiration (but there are exceptions, of course).

My list of recommendations to the BTR is not exhaustive or complete. Each of us has probably many other recommendations to add, but *we do* have to make recommendations.

There is much going on in curriculum development as well as studying how students learn. The process is painstakingly slow. Studies are being conducted on developing the problem-solving abilities of our students. All you need to do is name a problem in education today, and there is probably a study going on somewhere investigating the problem and formulating a solution.

Curriculum development is progressing, but to a new teacher the educational scene must appear confusing. I once heard a description of curriculum development that seems to describe the present scene. The scene opens in the cockpit of a 747 (big bird). One pilot remarks to the other "We seem to be in a fog." The other pilot remarked, "Yes, but *we are* making headway." The evidence, as valid or invalid as we wish to make it, seems to indicate that there are problems in our curriculum, and different pressure groups are making it known that our present curriculum is not making headway. Students do not want to be shortchanged, nor do we want to shortchange them. They want to be able to understand "why we are taking this stuff;" and perhaps their reoccurring question might indicate a weakness in our curriculum. We are so over-preparing for the future that we are neglecting the present. We do have to have informed consumers. We do want them to use Math in their everyday living (accurately too!). We do want our students to be mathematically literate