THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

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Volume XVI, Number 2, December 1976

Thoughts from the Editor's Desk

Our MCATA conference in Red Deer was a surely a success from every method of measurement. The attendance was high, with many at both the Friday and Saturday sessions. The low point was the number of people leaving after the luncheon rather than taking advantage of the excellent afternoon speakers. The program was one of the most complete available with respect to variety and/or meeting the needs of the teachers in attendance. All the speakers were very well received and very highly spoken of by attendants at the sessions as they gathered together after each session.

The motel service was obviously very good as the committee received only compliments, rather than complaints. Our display group was at its best in giving good counseling service on how and where to use the materials available, rather than emphasizing the high pressure commercial sell often shown by publishers and distributors. Our dinner speaker was entertaining while getting across the point that we need more effective teaching of communications in Math and in everyday life. We have intentionally not commented on the content of Frank Ebos' excellent keynote speech. He has written an article on his theme which is enclosed, and only your presence at the meeting to see Frank in action could add to the impact of the message, as the printed page can never catch Frank's enthusiasm or the effectiveness of using what he has offered.

The questionnaire enclosed in the September issue will be reported on in February, as returned questionnaires will not arrive in time to meet our deadline for submission of material.

We wish to acknowledge the fact that delays in the printing room at Barnett House have made the September issue late, resulting in partially stale news, since our conference was the week before you received your letter. This delay has also left a *Monograph* still undelivered at the time of writing but hopefully not at the time of reading.

In This Issue

CROSS NUMBER PUZZLES

Grades IV - IX

These attractive, interesting puzzles were developed and field-tested in schools. Each one provides practice in basic computational skills. Puzzles are organized so you can give practice exercises tailored to specific needs of individuals. Every puzzle problem is keyed to a clearly-defined objective. Clever cartoon format makes these puzzles motivating, fun to do. Book I contains 27 puzzles for practice in whole number operations (Grades IV-VII). Book II contains 28 puzzles for practice in fraction, decimal and percent operations (Grades V-IX). Puzzle pages are designed to be reproduced by thermal process. Reproduction permission is granted for classroom use. Answers are printed on puzzles in nonreproducible blue ink to give you a handy correction key. Spirit Duplicating Masters are also available.

If interested, write to:

Western Educational Activities Limited
10577 - 97 Avenue
Edmonton, Alberta

METRIC SYSTEM GUIDE LIBRARY

J.J. Keeler & Associates are now offering their five-volume reference work, the *Metric System Guide Library*, at 50% off the original price. This looseleaf series has been designed to provide easy accessibility to learning and teaching the metric system.

The complete series is available for $147.50, and individual volumes can be purchased for $59.00 each. For further information, contact J.J. Keeler & Associates, 145 West Wisconsin Avenue, Neenah, Wisconsin 54956, or call, toll-free, 1-800-558-5011.

Some Reactions to ICME - Karlsruhe

Tom Kieren
University of Alberta

I, along with Doyal Nelson and Jack Bana from the University of Alberta and a fairly large contingent of Canadians, participated in the Third International Congress on Mathematics Education held this August in the lovely city of Karlsruhe in the Black Forest area of Germany. The meeting was attended by teachers at all levels, from elementary school to university, from over 80 countries, with the majority coming from various western European countries.

What comes from such a congress? As you might well imagine, there were many logistical difficulties. There were problems of translating into at least three languages, as well as a problem of the congress meetings being rather spread out. The latter problem prevented a lot of valuable incidental personal contact from occurring. It might be said that many of the presentations and discussions got stuck on a general level or on the level of platitudes due to the broad nature of the topics and audiences, and also partly due to various political considerations. Speakers and panelists were carefully chosen to reflect a spectrum geographically and politically. Many people felt a great need to get up and say something just so they could return and say they had represented their country.

These problems not withstanding, there were some observable trends and common concerns. I will only give a few brief highlights.

There seems to be a global concern for the manipulative skills (or lack thereof) of students entering post-secondary education. There is a universal tendency to blame "new mathematics" for this, but that term hardly has universal meaning.

Although we in Alberta tend to focus on very specific objectives and on the attainment of "basics" in our curriculum, this is not a universal trend. Indeed, there is a feeling in most European countries that this approach is entirely wrong and there is a need for broader, not more narrow, goals in mathematics instruction.

Three content themes struck me as worth mentioning. Many countries seem to be working toward including probability and statistics in upper elementary and junior high school curriculums in important ways. Second, there seems to be confusion in many countries, including Canada, as to what represents geometric instruction for all children and adolescents. Finally, there seems to be widespread attention given to the role of computing devices in mathematics instruction. Representatives from diverse countries mentioned the need to integrate the use of hand-held calculators into instruction and to develop an understanding of the workings of computers with *all* students before they finish school. It was felt that this latter topic was one important lead into the entire important area of attaining a broader focus on applications of mathematics.

One trend, of which we in Alberta are a part, is important in light of the above comments; that is the rapidly increasing role of teachers in mathematics curriculum development. This is held as a good thing by all. However, this responsibility demands many new skills of teachers. There were, in the ICME meetings, many calls for professional organizations (such as MCATA), teacher training institutions, and school and government agencies to act cooperatively in providing the means for this needed growth in mathematics teacher capabilities.

Oh Those Positive Whole Numbers!

Howard Larson
Retired School Superintendent
and Math Teacher
Red Deer

Generally speaking, there is little point in differentiating between whole numbers and positive integers. But somewhere the distinction should be made. Whole numbers are not elements of integers, even though we tend to use them as such. In teaching mathematics at any level, we should be constantly aware of the beauty of mathematics, and precision is one of the factors that makes it such a delightful subject.

In the early grades we go to some length in showing integers on the number line - and this is good. But do we go to the same trouble showing set "W" on a number "RAY?" Clearly, the geometric comparison (or contrast) in "point set" thinking is quite exciting. Here is where we can show that set "W" has no opposites as does set "I." Clearly, too, we can show why subtraction is a closed operation under set "I" but not under set "W."

As the student progresses, he will need to know that a whole number represents answers to "how many," while positive integers always indicate direction from some starting point, as well as answer how many. There are many places where this distinction is important in real life situations. For example:

> A farmer has 3 cows. He lost 6 cows. Impossible?
> Well, if 3 could mean +3, then of course the farmer
> now has -3! Can we imaginge that?

A farmer has 3 dollars. He lost 6 dollars. He now has -3 dollars. This is possible because we can think of 3 and 6 in terms of "I". But why can we do this with dollars and not cows? The answer, of course, is that dollars are abstractions and we can play around with abstractions in our mind! Note that if we think of 3 dollars as dollar bills, in no way can the farmer lose 6 dollar bills!

And this is the crux of the point I wish to make. Set "W" represents concrete THINGS while set "I" permits us to extend our thinking to whatever limits we are capable. If we teach youngsters that +4 and 4 are identical, just because it is too cumbersome to always identify the positive sign, we are setting the stage for confused thinking later on.

For example, let's take absolute values. The absolute value of -4 is 4. Now if 4 is in set I, then of course -4 = +4! Enough said.

Are We Shortchanging Our Students? or Back to Basics? New Basics? or Old Basics?

Frank Ebos
Faculty of Education
University of Toronto

Reprinted from *Ontario Mathematics Gazette*, Volume 15, Number 1, September 1976.

Probably each of us in our role as a teacher has been confronted by an anxious student inquiring about some of the math we teach. How many times have each of us heard:

> Why are we taking this stuff?
> What's it good for ... sir?

I would be the first to admit that we can't justify everything we do *all the time*. To explain how a specific topic in mathematics "fits" into the scheme of things is often difficult. The student must trust that what we are doing day by day in the math classroom is useful, is relevant, and is needed for today's activities as well as for tomorrow's. Unfortunately, for many students tomorrow never comes. The students then become parents, and the cycle of asking "Why are we taking this stuff" is continued. Here are certainly many indications that there is concern about the curricula we teach. Many studies have been and are being completed in both the United States and Canada to ascertain "What should be the math curricula?" Unfortunately, to predict the content, the skills, or even the methods needed at some future date is difficult. If you listen to the experts and read the journals, you soon would develop a complex about what we are not doing in the math classroom, but thank goodness for the so-called new math. We have in it at least a scape-goat. We have seen the headlines "New Math has failed! Back to basics!", and now we have a replacement for the new math, the BTB (Back to Basics).

When the "New Math" was in vogue, each person you talked to had a different "understanding" of what the "New Math" was: There were the *Set-new-math* followers, the *Base-new-math* followers, the *Structure-new-math* followers, and so forth. (I apologize to those new-math groups not identified at this time). The parents identified new math from another point of view. They only saw what their children brought home; sets, and words, commutative, associative, distributive, inverse, base 2, base 8, third base - to name a few - soon the scene is set for the BTB to be formed, and the parents are willing to join.

These days, I am hearing more and more of the BTB, but I think that before we change we need to decide *"what are the basics."* Parents whose experience with math was almost completely computation, evaluate a new program or curriculum according to their backgrounds. We must ask: are the basics solely related to computation? I think not. No one would disagree that "the old basics" are essential, but there are "new basics" that parents as adults use in their everyday living, and which need to be dealt with. The computation content, so familiar to many and part of the "Happy Days Syndrome" to return to basics, is not by itself suitable even for today's world, let alone tomorrow's. My main concern is not that the BTB wants change (improvement, or whatever), but that they appear to want the pendulum to swing back to a "shut-up: do-this" curriculum, based on computation.[1] We can't afford to have the pendulum swing back too far. Everyone will basically agree that computational skills are important and basic to the students' mathematical development, but to stop there would short-change our students. Before the BTB make the same errors as the New Math groups, they (there are probably more than one) should decide what the basics are!

I want to help the BTB by offering the following suggestions. (These suggestions could be added to the computational platform already advocated by the BTB. A little review is useful here:

BTB = a group of persons who want to return Back to Basics.)

1. Students want us to be accountable. We should have some reasonable explanation or justification for "Why are we taking this stuff?" If we were to provide examples of applications for the present curriculum, some students and parents would be partially satisfied. To tell a student he is taking a topic because he needs it for next year's math is a weak argument. Many topics have applications, not only to satisfy the present questions asked by students, but also to provide a foundation for their future study of the topic in next year's math class. There are at present at least five studies that are looking at the curriculum from this point of view. Hopefully, the results of these studies will be expressed in practical terms and eventually made available to practising teachers. *Applications should be basic to any curriculum.*

2. In our everyday living, we are never given "neat problems" solved in a "neat way" that result in a "neat solution." A real-life problem requires

[1]A description of the "shut-up: do this" curriculum was in a speech delivered by Eric MacPherson of the Faculty of Education, University of Manitoba at the Annual Meeting of the National Council of Teachers of Mathematics in Denver, Colorado, April, 1975.

us to sort out the useful (needed) information from the extraneous information. Do we ever give students problems that contain more information than is really required to solve the problem? Will our students ever be given a problem to solve when they enter the "world of work" for which the "boss" provides only the information that is necessary? I doubt it! Yet how much of this practice in solving do we provide our students in our math classrooms (if you do provide these types of problems, then consider this section a brief review)? *To decide on what information is necessary or needed to solve a problem, to me, is a basic skill.* How many times have you given a problem which has a missing piece of information that the student is to provide in order to solve the problem? (Try assigning 5 problems, each having an essential piece of information missing. Provide a second sheet that contains the 5 missing "pieces," as well as 25 other pieces of extraneous but closely related information). This skill will probably be used more often than the skills advocated by the BTB. Without this skill the BTB skills are often confusingly applied by students to solve problems. (I think a mathematician might say the BTB skills are sufficient, but not necessary, or is that ... necessary but not sufficient ... ?)

However, "let me make it perfectly clear" that we need the BTB basic skills once the information is properly interpreted and the essential computational operations decided upon. Unfortunately, "good old" Euclidean geometry provided an opportunity for a student to "sort out" the needed information, but an EGID movement (Euclidean Geometry Is Dying) seems to have sprung up. (E GAD!)

3. Every day, as adults, we read about the Gallup Poll, the latest statistics on "why we are paid more and more but are eating less and less" and this poll and that poll. However, do we provide any basics for students to tackle the world of statistics? We give a brief look at the topic in Grade VIII (if at all in some classrooms), completely ignore it in Grades IX and X, but give some hope to those who stay on and finish high school and study the topic again. (The key question here is - how many will finish high school?) Could these students not find the skill of working with statistics useful just for everyday living, and thus be given the opportunity to study the topic in Grades IX and X? *Working with, interpreting and reading statistics is a basic skill.* Statistics permeate too much of our everyday living to be ignored as they have been in our mathematics curriculum. We are shortchanging our students by not teaching this skill. Continuously through Grades IX and X we are especially shortchanging those students who do not reach Grade XI.

4. How many students have asked you "Is this right?". How many are completely lost if they cannot find the answer at the back of the book? Students should be taught (and taught and taught ...) to know how to check that their answers are reasonable. *To know the answer is reasonable is a basic skill.* A student must by Grade XII "feel" whether the answer is reasonable. The "boss" does not assign problems with the answer at the back of the book. Teaching this skill partially can be accomplished by providing students with skills of approximating, estimating, as well as decision-making. Too often students are given problems that have "neat" answers. The following problem about a corn roast seems trivial at first, but when it was assigned

to Grade VIII students, it introduced the need to make decisions.

"How much would it cost our class to go on a corn roast?"

The students list the assumptions, and make decisions to arrive at the cost of going on the corn roast. I have assigned this problem to teacher groups and have had to referee arguments, as well as to impose a maximum time limit of 15 minutes because the problem, although simple in appearance, can be complex in finding a "reasonable" solution. The solution will eventually involve answers to these questions:

Where are we going? How do we get there? Do we take drinks? Do we need salt, butter, napkins, pepper, and so on, and so on?

These are but a few of the problems hidden in arriving at a "reasonable" price for a "reasonable" corn roast. What's your answer?

Problems suitable for different grade levels should be assigned to develop and strengthen this basic skill "Is my answer reasonable?"

5. I am sure many teachers already provide students with a variety of strategies for "starting" to solve a problem. I will never forget when I posed a problem to a class and got the immediate reply, "We haven't taken that yet ... sir!" We all remember too well the complete blanks left for some problems tackled by students writing exams. Some students freeze as soon as we say "word problem," and will sit and look, but really do nothing to "start" the problem. *Teaching students to "sketch" the problem or make a diagram to help them solve the problem is a basic skill that needs to be emphasized continually.* Many problems have been solved by "doodlers." Often a problem is solved by "doing" something rather than "waiting" for an inspiration. A blank page provides very little inspiration (but there are exceptions, of course).

My list of recommendations to the BTP is not exhaustive or complete. Each of us has probably many other recommendations to add, but *we do* have to make recommendations.

There is much going on in curriculum development as well as studying how students learn. The process is painstakingly slow. Studies are being conducted on developing the problem-solving abilities of our students. All you need to do is name a problem in education today, and there is probably a study going on somewhere investigating the problem and formulating a solution.

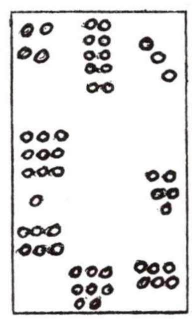
Curriculum development is progressing, but to a new teacher the educational scene must appear confusing. I once heard a description of curriculum development that seems to describe the present scene. The scene opens in the cockpit of a 747 (big bird). One pilot remarks to the other "We seem to be in a fog." The other pilot remarked, "Yes, but *we are* making headway." The evidence, as valid or invalid as we wish to make it, seems to indicate that there are problems in our curriculum, and different pressure groups are making it known that our present curriculum is not making headway. Students do not want to be shortchanged, nor do we want to shortchange them. They want to be able to understand "why we are taking this stuff;" and perhaps their reoccurring question might indicate a weakness in our curriculum. We are so over-preparing for the future that we are neglecting the present. We do have to have informed consumers. We do want them to use Math in their everyday living (accurately too!). We do want our students to be mathematically literate when they read the newspaper, pay that charge account or calculate the percentage increase of their raise (a very controversial issue). At the same time, we do not want to shortchange the students by providing them with a curriculum that accommodates the present but ignores the future. They must have some "computer sense," some appreciation and working knowledge of calculators, as well as an appreciation of mathematics, "as a science." At the same time, we need to prepare them to be mathematically sound, and for this we would need to provide a foundation involving relations and functions, transformations, vectors - the list could go on and on.

To keep everybody happy is impossible, but we all have to participate in lifting the fog.

Ideas for the Primary Class

NUMBER CLOTHESPIN

Norma Grace Scott
Leaf Rapids, Manitoba



Print number words on clothespins - one to ten. Prepare a cardboard sheet as shown in the diagram. The children can pin the correct clothespin to the dots on the cardboard sheet.

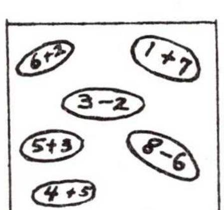
BEAN BAG TOSS

Cathy Dearden
Elwick Community School

Prepare a large chart to be placed on the floor. Put numbers or number facts on it. A sheet with numbers on it for each player.

2	9
5	4
3	6
0	7

Number chart



Number facts chart

2	8	5	3
6	4	0	9

Player's score sheet

The children throw the bean bag, then put an X on the appropriate square on their own sheet. The first person to complete the sheet is the winner. The sheets should be prepared so that all the answers are not on every sheet.

SPILL THE BEANS

Russ Erickson
Minneapolis, Minnesota

This is a similar game, which could be played with the number chart prepared for the Bean Bag Toss. The players have two lima beans in a shaker. They take turns "Spilling the Beans" onto the chart. To score a point, they must be able to give the total of the numbers on which the beans landed.

MUSICAL SETS

This game appears in the teacher's guidebook to *Mathways*, published by Copp-Clark. It's active and noisy, so the children like it. Play it in the gym or the other teachers won't like you! It's like musical chairs, except that at the pause in the music, the teacher holds up a number. The children have to form themselves into sets of that number. Any children not in a set are 'out.'

ACTIVITIES USING A GEOBOARD

Olof Hardy
Department of Education
Government of Manitoba

Materials needed:

Graph paper or paper that is dotted into one cm squares
Selection of elastics
Geoboard or pegboard and pegs

A. 1. Make shapes on the geoboard: triangles, squares, rectangles, houses, animals, anything you wish.
2. Make a design with the elastics on the geoboard
3. Make a shape like this on the geoboard.

Name this shape. Make other shapes with only three sides.

4. Make as many different sized triangles as you can on the geoboard using a different color elastic for each one. The triangles can be the same kind or different. It does not matter in this particular exercise.

B. 1. a. Make a triangle on the geoboard. Now try to make another triangle the very same size in a different place on the geoboard.
b. Make a triangle that is bigger than your first one.
c. Make a triangle that is smaller than your first one.

2. a. Make a small square on the geoboard. Now try to make another square the very same size in a different place on the geoboard.
b. Make a square that is bigger than your first one.
c. Make a square that is smaller than your first one.

3. a. Make a small rectangle on the geoboard. Now try to make another rectangle the very same size in a different place on the geoboard.
b. Make a rectangle that is bigger than your first one.
c. Make a rectangle that is smaller than your first one.

C. 1. a. Make one triangle. Now using different color elastics, make bigger triangles on top of this one.
b. On another place on the geoboard make a different kind of triangle. Then using different color elastics make bigger triangles on top of this one.

2. a. Make the smallest square you can on the geoboard using one elastic.
b. Make the largest square you can using one elastic.
c. Now make as many other squares of different sizes that you can.

3. a. Make a square on the geoboard. Then make it into two triangles using another elastic.
b. Make another square the same size in a different place on the geoboard. Can you make this square into two triangles in a different way from the way you did with the first square?

D. 1. Make a square on the geoboard. Now make another one with its sides twice as big.

2. a. Make two triangles on the geoboard so that they form a triangle.
b. Make two triangles on the geoboard so that they form a square.

3. Make a triangle with no square corners on the geoboard.

4. Make any shape on the geoboard. What shape is it? Using a different color elastic make another of the same shape inside the one you just made. Is your second shape bigger, smaller, or the same size as your first one? Does your second shape have the same name as your first one?

Ideas for the Intermediate Class

Games to Reinforce Metrics

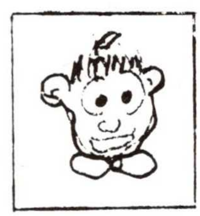
OLD MAN METRIC

A card game played like Old Maid

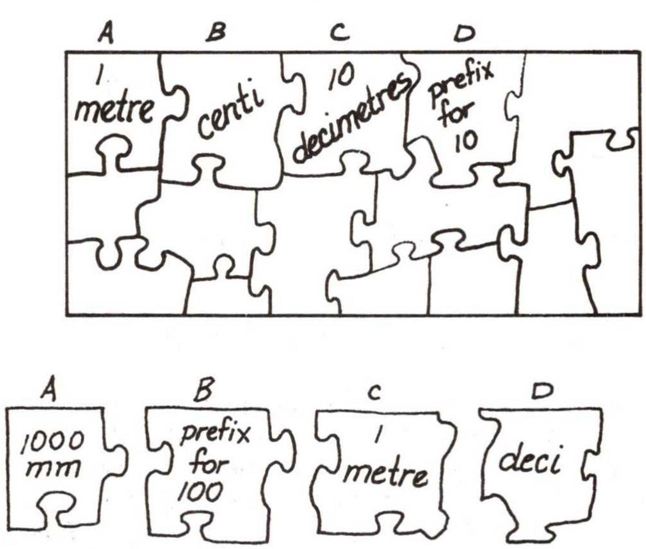
1. Shuffle all of the cards
2. Deal them to players
3. Lay down all pairs
4. Always take a card from the player on your left.
5. The player left with "Old Man Metric" is the loser.

Some sample cards:

100 cm	1,000 grams	$0\ ^{o}$ C
= ↓	= ↓	= ↓
1 metre	1 kilogram	water freezes



METRIC PUZZLE



Use any commercial puzzle as long as it has a frame. Trace around the empty space in the frame (after you remove one puzzle piece at a time). On the back of the puzzle piece write the metric term. In the frame put the equalling notation (see sample).

METRE MAID

Materials

Game board

Spinner with numbers 1-12 or dice

Color	mm section	orange	lima beans also
"	cm "	blue	sprayed orange,
"	dm "	green	blue, green, and
"	m "	red	red.

m	dm	cm	mm

Directions

1. Give each child a game board. (Four children play at once and another child acts as metre maid or banker. She keeps the beans and makes sure exchanges are right.)
2. Child who goes first spins spinner. If he rolls a 7, he gets 7 orange beans to keep on his board.
3. Other children do the same - roll and take beans.
4. When first player gets his turn again if he rolls an 8, he can take 8 orange beans or he can trade 10 orange for 1 blue, leaving 1 blue and 5 orange on the board.
5. The winner is the one who gets 1 red first.

Rate of Exchange of Beans:			
	10 orange	=	1 blue
	10 blue	=	1 green
	10 green	=	1 red

METRIC CONCENTRATION

Materials

1. Gameboard and about 36 cards
2. Cards with matching metric terms and some wild cards
3. Answer card to check answers

Directions

1. Turn all cards face down.
2. First player turns up two cards. If they match, he keeps them. If no match is made, then he turns them face down again.
3. Next player takes his turn.
4. Play continues until no more matches can be made.
5. Player with most cards wins.

Ideas for the Junior High Class

Make Algebra Enjoyable

G. Strempler

It is important that the interest of students be kept alive. If you are looking for commercially prepared help, here is an idea. *Accent on Algebra* is an enrichment book designed for use with any level algebra student or in any course where algebraic topics are introduced and used. It contains 128 pages of crossnumber and crossword puzzles, word games and much more. It is available at Creative Publications.

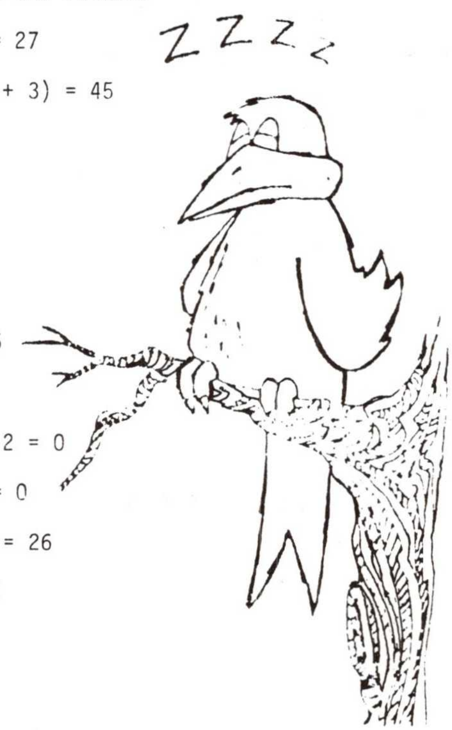
Try this sample when you teach equation solving:

FAMILIAR PHRASE

6 11 20   14 15 2 25 14 6 20 30 6   8 20 9 6 11 20 15 20 10

13 0 14 20 10   30 20 0 24 20 30   6 11 20   9 5 5 20 12 0 10

SOLVE THE FOLLOWING EQUATIONS FOR SOME HELPFUL CLUES TO THE PUZZLE ABOVE.
USE THESE CLUES TO COMPLETE THE PHRASE.



1.	$(a - 1) + a + (2 + 1) = 27$	a = ____
2.	$(b + 1) + (b + 2) + (b + 3) = 45$	b = ____
3.	$103 - 4d - 6d = 3$	d = ____
4.	$-85 - e + 4e = -25$	e = ____
5.	$2f - (f - 1) = 9$	f = ____
6.	$7(h - 9) - 3h + 19 = 0$	h = ____
7.	$12.5 - 2(i + 2i) = 12.5$	i = ____
8.	$4L + \frac{1}{2}(-6 - 2L) - 33 = 0$	L = ____
9.	$5(m + 17) - 8(m + 1) - 2 = 0$	m = ____
10.	$3(8n - 44) - 3(1 - n) = 0$	n = ____
11.	$15o - (3o + 4) - (-3o) = 26$	o = ____
12.	$2(2 - 3p) = 9(5 - p) + 1$	p = ____
13.	$1 = \frac{2}{3}(9 - r) + 5$	r = ____
14.	$1 - \frac{3}{5}(20 - s) = 7$	s = ____
15.	$6t^2 - 3(t + 2t^2) + 18 = 0$	t = ____
16.	$5z - [z - 2(z - 49) - 3 - z^2] = -(-25 - z - z^2)$	z = ____

NOW THAT YOU HAVE DONE THE EASY PART, SEE IF YOU CAN SHORTEN THE PHRASE INTO A WELL-KNOWN PHRASE.

Practice and Discovery: the Alternating Diagonal Hundred Square

Bonnie Litwiller and *David Duncan*
Associate Professors of Mathematics
University of Northern Iowa
Cedar Falls, Iowa

Figure I: Alternating Diagonal Hundred Square

1	2	6	7	15	16	28	29	45	46
3	5	8	14	17	27	30	44	47	64
4	9	13	18	26	31	43	48	63	65
10	12	19	25	32	42	49	62	66	79
11	20	24	33	41	50	61	67	78	80
21	23	34	40	51	60	68	77	81	90
22	35	39	52	59	69	76	82	89	91
36	38	53	58	70	75	83	88	92	97
37	54	57	71	74	84	87	93	96	98
55	56	72	73	85	86	94	95	99	100

The Hundred Square is frequently used in middle school and junior high classrooms for discovering and reinforcing computational patterns. We wish to consider the "Alternating Diagonal Hundred Square" as shown in Figure I.

The arrows denote the order in which entries of the Square were listed. Several interesting patterns may be observed on the Square.

195		
	64	
259		-3
	61	
320		15
	76	
396		-7
	69	
465		11
	80	
545		-11
	69	
614		7
	76	
690		-15
	61	
751		3
	64	
815		

1. The sums of the entries of the rows (horizontal) and their consecutive differences are given to the right. Here and henceforth, the differences are found by subtracting the first number from the second.

   Note the second differences form two interlocking symmetric sequences, one the negative of the other.

2. The sums of the entries of the columns (vertical) and their consecutive differences are:

200		
	54	
254		17
	71	
325		-5
	66	
391		13
	79	
470		-9
	70	
540		9
	79	
619		-13
	66	
685		5
	71	
756		-17
	54	
810		

Observe a similar pattern to that in I.

3. Figure II shows the sums of the entries of the indicated diagonals

Figure II

1	1	2	6	7	15	16	28	29	45	46
5	3	5	8	14	17	27	30	44	47	64
15	4	9	13	18	26	31	43	48	63	65
34	10	12	19	25	32	42	49	62	66	79
65	11	20	24	33	41	50	61	67	78	80
111	21	23	34	40	51	60	68	77	81	90
175	22	35	39	52	59	69	76	82	89	91
260	36	38	53	58	70	75	83	88	92	97
369	37	54	57	71	74	84	87	93	96	98
505	55	56	72	73	85	86	94	95	99	100
		540	548	532	495	440	370	288	197	100

Consider the consecutive differences of the sums as shown.

Again the second differences from two symmetric sequences, one the negative of the other.

1		
	4	
5		6
	10	
15		9
	19	
34		12
	31	
65		15
	46	
111		18
	64	
175		21
	85	
260		24
	109	
369		27
	136	
505		
	35	
540		-27
	8	
548		-24
	-16	
532		-21
	-37	
495		-18
	-55	
440		-15
	-70	
370		-12
	-82	
288		-9
	-91	
197		-6
	-97	
100		

4. Figure III shows the sums of the entries of another set of indicated diagonals.

Figure III

1	2	6	7	15	16	28	29	45	46	46
3	5	8	14	17	27	30	44	47	64	109
4	9	13	18	26	31	43	48	63	65	141
10	12	19	25	32	42	49	62	66	79	214
11	20	24	33	41	50	61	67	78	80	240
21	23	34	40	51	60	68	77	81	90	315
22	35	39	52	59	69	76	82	89	91	343
36	38	53	58	70	75	83	88	92	97	412
37	54	57	71	74	84	87	93	96	98	450
55	56	72	73	85	86	94	95	99	100	505
55	93	162	190	265	291	364	396	459		

```
46
      63
109         -31
      32
141          41
      73
214         -47
      26
240          49
      75
315         -47
      28
343          41
      69
412         -31
      38
450          17
      55
505
     -46
459         -17
     -63
396          31
     -32
364         -41
     -73
291          47
     -26
265         -49
     -75
190          47
     -28
162         -41
     -69
93           31
     -38
55
```

Again note the pattern of two symmetric sequences, one the negative of the other.

The teacher and her students are invited to seek other patterns on the Alternating Diagonal Hundred Square. These activities may be used for maintenance of skills, discovery of patterns, or even practice with the hand calculator.

Ideas for the Senior High Class

Try Another Sequence

Cecil Grant

How have your students been performing on the units dealing with Factoring and Rational Expressions? If you have been following the sequence in which these topics are treated in any of the approved texts for Mathematics 100, you no doubt present these topics as two separate units: Special Products and Factoring as one unit, and Rational Expressions as the other.

The relationship between these two units is quite clear, and the logical necessity of teaching the factoring first is also quite obvious. Moreover, factoring is such an integral part of the work on algebraic fractions that it seems advisable to combine these two topics into one unit, entitled "Factoring and Algebraic Fractions." If this is done, some interesting consequences result.

The first of these is the increased meaningfulness to the students of the task of factoring. The purpose of factoring as a tool in working with algebraic fractions becomes much clearer to them. Secondly, certain types of factoring, like grouping to factor, and possibly, the sum and difference of two cubes, become questionable items to be included in this mathematical diet. Thirdly, and what I consider a most interesting consequence, is the possibility now offered to present the material in a sequence other than the one followed by so many mathematics texts which include these topics within their covers. I am referring to that traditional arrangement in which all the types of factoring are presented first, followed by the reduction of fractions, multiplication, division, addition and subtraction of fractions. A section on complex fractions may then follow. The point to note is that *all* the types of factoring are treated first.

Here is another sequence which I have found very effective. After completing the work on common factors, rather than going on to the next type of factoring, proceed to the reduction of fractions which involve common factoring only. Then move on to the multiplication and division of fractions which utilize common factoring only, and finally to addition and subtraction of fractions in which only common factoring is involved. The next type of factoring can now be taken up and the same procedure followed through from reduction to subtraction of fractions in which only the particular type of factoring is involved.

You may discover, as I did, that this arrangement can be very effective.●

Metric Ideas

Let's Be Reasonable

Activity: Ask students to place decimal points so that the following statements are reasonable.

1. Jim is 1545 centimetres tall.
2. Mary's new baby brother weighs 350 kilograms.
3. The fire truck sped by at 10000 kilometres per hour.
4. The car's gas tank was nearly empty. Dad filled it with 7800 litres.
5. The pop bottle holds 320 millilitres.
6. Mary drinks lots of milk every day, nearly 100 litres.
7. The school room is 320 metres high.
8.. The distance from Montreal to Vancouver is about 35000 kilometres.
9. In the high jump at school, Don broke the class record by jumping 1200 centimetres.
10. Mom's favorite cake recipe calls for 5000 millilitres of vanilla.
11. A two-page letter from Grandma weighs about 2000 grams.
12. Sam ran all the way home, averaging 100 kilometres per hour.
13. Carol's favorite hamburger stand serves big hamburgers. The meat alone weighs 2500 grams.
14. Beth's favorite candy bar weighs 1200 grams.
15. The thermometer dropped to 200°C last night. Fruit growers were worried about their orchards.
16. Peter Piper picked 10000 litres of pickled peppers. 10000 cm^3 of pickled peppers Peter Piper picked. If Peter Piper picked 10000 ml of pickled peppers, where's the 10000 dm^3 of pickled peppers Peter Piper picked?

Metric Recipes

BRAN MUFFINS

Preheat oven to 220°C. Thoroughly grease 16 medium-sized muffin cups.

Sift together	300 ml	cake and pastry flour
	7 ml	baking powder
	2 ml	baking soda
	2 ml	salt
Stir in	375 ml	natural bran
	125 ml	raisins
Cream together	50 ml	shortening
	100 ml	lightly packed brown sugar
Beat in	50 ml	molasses
	2	eggs
	250 ml	milk

Add liquids to dry ingredients and stir only until combined.
Fill prepared muffin cups 2/3 full.
Bake in preheated 220°C oven for 15 to 20 minutes.
Makes 16 muffins.

BROWNIES

Preheat oven to 160°C. Grease 20cm square cake pan.

Blend or sift together		
	175 ml	cake and pastry flour
	100 ml	cocoa
	1 ml	salt
Cream together	125 ml	shortening
	225 ml	granulated sugar
Add	2	eggs
	5 ml	vanilla

Beat until light and fluffy. Stir dry ingredients into creamed mixture alternately with water and nuts.
Turn into prepared pan.
Bake in preheated 160°C oven for 25 to 30 minutes.
While warm cut into bars. Makes 24 bars.

"Ideas for the Primary Class," "Ideas for the Intermediate Class," "Ideas for the Junior High Class," "Ideas for the Senior High Class," and "Metric Ideas" were all reprinted from *the Manitoba Mathematics Teacher*. Unfortunately, we are unable to supply the volume and date of the issue from which this material was drawn. Our sincere apologies to the *Manitoba Mathematics Teacher*.

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MATHEMATICS TALKS
FOR HIGH SCHOOLS

Members of the Mathematics Department at the University of Alberta have prepared a list of talks which are suitable for High School students, Mathematics clubs and/or teachers. Anyone interested in scheduling any of these talks for their school is asked to contact Ivan Baggs, Department of Mathematics, University of Alberta. Edmonton, or phone 432-3385. A partial list of these talks together with an abstract and names of the speakers is given below. If you would like to have a talk on a mathematical topic not listed below, let us know and we can probably accommodate you.

1. CANONICAL CALCULATION by Professor H.I. Freedman

   Summary: History of calculation and calculating instruments from ancient through medieval up to the present time. Topics include finger reckoning, ancient Roman hand calculators, bank accounts in the middle ages, the first mechanical adding machine. Slides and models utilized.

2. TRANSCENDENTAL Π-DITATION by Professor H.I. Freedman

   Summary: A chronology of Π from ancient times to the present. Topics include methods of calculating Π , Π in the Bible, irrationality of Π , squaring the circle, anecdotes on Π . Charts and slides utilized.

3. MATHEMATICAL BIOLOGY by Professor H.I. Freedman

   Summary: How mathematics can be used as an aid in explaining biological phenomena. Why is a louse not as big as a house? How do genes distribute from one generation to the next? This talk is intended for an audience of "better" students.

4. LARGE NUMBERS by Professor H.I. Freedman

   Summary: How does one write a large number? Can you ever own a complete set of bingo cards? What is a googol? Charts utilized.

5. THE ART OF PROBLEM SOLVING by Professor M.S. Klamkin

   Summary: The art of problem solving is related to that of problem proposing. For by considering other problems related to the one to be solved, we are often led to a key idea which unlocks its solution. This is illustrated by an elementary geometric minimization problem which is rather difficult to solve. To obtain its solution, we will consider the general patterns of "level lines," "relaxation," "homotheticity," and "symmetry." Each pattern in turn will be illustrated by several other problems.

6. PROBLEM SOLVING VIA TRANSFORMS by Professor M.S. Klamkin

   Summary: Here we take a broad viewpoint in solving problems via transformations. We illustrate (with slides) the philosophy and applications of transform theory by a series of problems starting off with some simple ones in arithmetic and geometry. We then consider some other problems in algebra, probability, number theory, combinatorics and physics. The choice of problems illustrated will depend on the background of the audience attending.

7. MATHEMATICAL CREATIVITY by Professor M.S. Klamkin

Summary: Although the psychological aspects of creativity in mathematics are important, we shall dwell mainly on the mathematical aspects. We will show how one can start with some rather elementary mathematical results and often end up with some rather sophisticated results.

8. ON THE TEACHING OF MATHEMATICS SO AS TO BE USEFUL by Professor M.S. Klamkin.

9. VECTOR PROOFS IN SOLID GEOMETRY by Professor M.S. Klamkin

Summary: In solving problems, one usually has the choice of using analytical geometric, synthetic geometric or vectorial methods. We discuss, with many illustrations, the advantages and disadvantages of these three general approaches. For many problems, especially higher dimensional ones, it seems that the vectorial approach is a good compromise insofar as the case of setting up the problem is concerned as well as its subsequent solution.

10. OPTIMIZATION BY MEANS OF LEVEL LINES AND INEQUALITIES by Professor M.S. Klamkin

Summary: We show, with many illustrations, how one can solve maximum and minimum problems with dispatch using level lines and/or inequalities. Also one can obtain the condition for the maximum or minimum of a rational function of one variable, in a given interval, without calculus.

11. MATHEMATICAL COMPETITIONS by Professor M.S. Klamkin

12. THE SANDY FOUNDATIONS OF MATHEMATICS by Professor J.W. Macki

Summary: (Math 31 or Math Club Level). The concept of numbers is usually introduced via the notion of set. But intuitive set theory contains paradoxes. When one tries to make set theory precise, problems arise. I discuss recent work in this area which seems to indicate that there are no obvious firm foundations for mathematics.

13. FINDING ROOTS OF POLYNOMIALS AND OTHER SORDID DRAMAS by Professor J.W. Macki

Summary: (Math 30 level). A discussion of techniques for finding roots of polynomials from ancient times through the escapades of Cardano, Tartaglia and Ferraro. Concludes with a discussion of Galois' and Abel's solution of the quintic problem.

14. SOME PROBLEMS ON PAIRINGS by Professor J.W. Moon

Summary: Suppose a class consists of an equal number of boys and girls and that some of the boys and girls like each other and some don't. When is it possible to pair off the boys and girls in such a way that each person is paired with someone he or she likes? This and some related problems and applications will be discussed.