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THE VERTEX CONNECTION

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Teacher's Guide

Grade level: 7–12

Materials: One set of worksheets for each student and a set of transparencies for classroom discussion if desired.

Objectives: Students will (1) discover and apply the formula for the number of diagonals in a polygon and (2) play and analyze the game of SIM.

Directions: Distribute the worksheets one at a time.

Sheet 1: Most students should have little difficulty discovering that the number of diagonals in an n-gon is $n(n - 3)/2$. However, some may require additional guidance to detect the pattern in the table. Note that this formula was based on an analysis of convex polygons. Students might be encouraged to investigate if their formula also holds for polygons that are not convex.

Sheet 2: In this activity, students are placed in a problem-solving situation that can be modeled using polygons. If class members are represented by the vertices of an n-gon, then the number of handshakes involved is equivalent to the number of sides of the polygon formed plus the number of diagonals, $n + n(n - 3)/2 = n(n - 1)/2$.

Students who have had some work with mathematical induction may wish to try proving this generalization and the one regarding the number of diagonals of an n-gon. Others might explore related questions such as these: How many triangles result if all diagonals from one vertex of an n-gon are drawn? What is the maximum number of lines determined by n points in the same plane?

Sheet 3: The game of SIM (Simmons 1969) is a two-person game that is handled most effectively by declaring the one who wins two out of three games the winner and then pairing the winners against each other until a class champion emerges. The results of this tournament will suggest that the game always has a winner and that the second player has a better chance of winning. An analysis of a winning strategy may be found in Mead, Rosa, and Huang (1974). The fact that there is always formed a triangle of one of the two colors guarantees an affirmative answer to problem 8. This fact can be easily proved as in Harary (1972). When SIM is played on five vertices, there need not always be a winner.

REFERENCES

Harary, F. "The Two-Triangle Case of the Acquaintance Graph." *Mathematics Magazine* 45 (May 1972): 130–35.

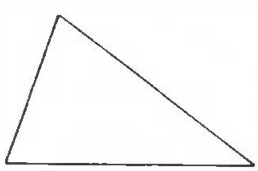
Mead, E., S. Rosa, and C. Huang. "The Game of SIM: A Winning Strategy for the Second Player." *Mathematics Magazine* 47 (November 1974): 243–47.

Simmons, G. "The Game of SIM." *Journal of Recreational Mathematics* 2 (April 1969): 66.

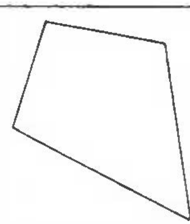
A diagonal of a polygon is a line segment connecting two vertices that are not endpoints of the same side.

1. Determine the number of diagonals in each of the polygons below and enter your answers in the following table:

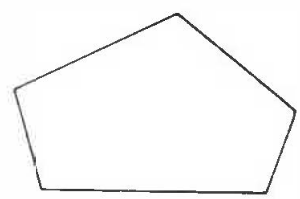
Polygon	Number of vertices	Number of diagonals from each vertex	Total number of diagonals
Triangle			
Quadrilateral			
Pentagon			
Hexagon			
Heptagon			
Octagon			



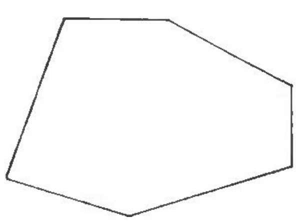
Triangle



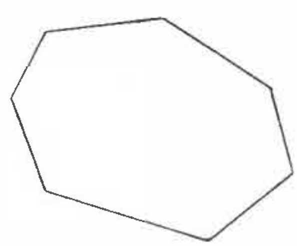
Quadrilateral



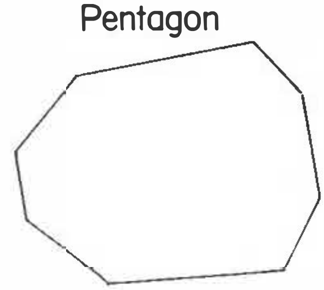
Pentagon



Hexagon



Heptagon



Octagon

2. a. How many diagonals do you think a polygon with ten sides would have? (Hint: Do you see any pattern in the table above?)

   b. Draw such a polygon and verify your answer.

3. How many diagonals has a polygon with n sides?

4. Suppose at the end of your class today, each student present shakes hands just once with every other student. How many handshakes would be involved?

   Hint: First consider some simpler problems:

   a. Suppose that there are only four members present.

      Represent each member by a vertex of a quadrilateral.

      Indicate all possible handshakes by connecting the appropriate vertices.

      How many handshakes are involved?

   b. Use the method above to determine the number of handshakes if seven students are present.

   c. Do you see any relationship between these solutions and your work on SHEET 1?

   d. Now try to answer the original problem which was posed.

5. How many handshakes would be involved if a class consisted of n members?

6. Here is an interesting game involving vertex connections which you can play with a friend.

   a. Place six points on a sheet of paper to mark the vertices of a regular hexagon as in Figure A below.

   b. Each player selects a color different from the other.

   c. Take turns connecting pairs of points with line segments. (See Figure B for a partially completed game.)

   d. The first player who is forced to form a triangle of his own color is the loser! (Only triangles whose vertices are among the six starting points count.)

Figure A

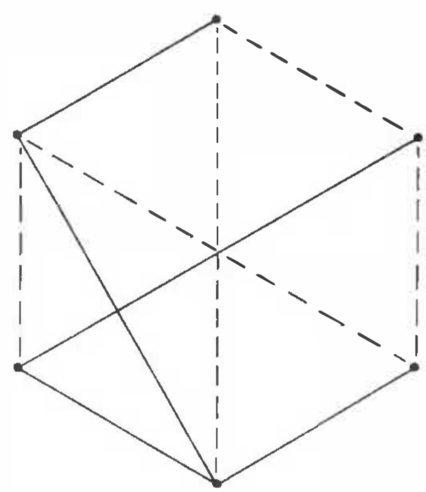


Figure B

7. Play this game several times and then answer the questions below.

   a. Is there always a winner?

   b. Which player has the better chance of winning?

8. Use the results of question 7 (a) to help you solve the following problem.

   Of any six students in a room, must there be at least three mutual acquaintances or at least three mutual strangers?

9. Play several more games, but in each case starting with five points that are the vertices of a regular pentagon, and then answer questions 7 (a) and 7 (b).