# An Experimental Screening Examination in Mathematics for Students at the University of Alberta

*Jack Macki*
Department of Mathematics
University of Alberta

In September, 1976, the examination described below was given to approximately 500 non-honors students enrolled in introductory calculus courses. The purpose of the exam was:

(a) to give the U. of A. Math Department some idea of the students' background;

(b) to see which parts of the school program had been retained by the students;

(c) to help us decide whether a remedial course is necessary for students with deficient backgrounds.

The exam was given to between 100 and 200 students in each of the following courses:

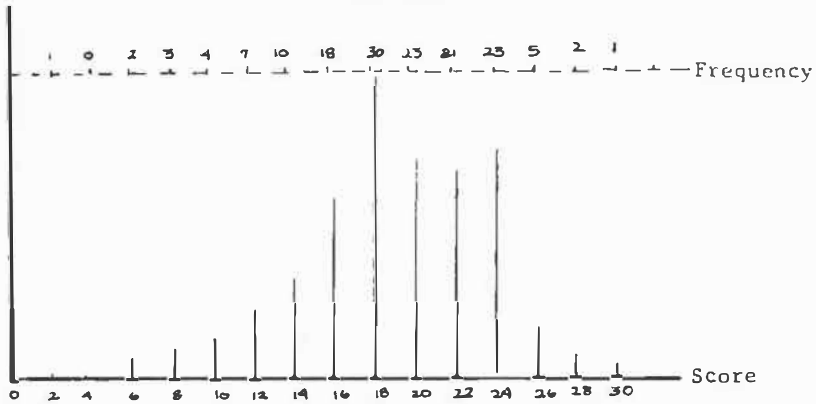
Math 202 (Math 203): The basic course in calculus for science majors and others. Math 203 requires Math 31 as a prerequisite, Math 202 requires only Math 30. (150 Math 202 students wrote the exam, 84 Math 203 students wrote.)

Math 212: The basic calculus course for engineering students. Most, but not all, have had Math 31. (116 students wrote the exam.)

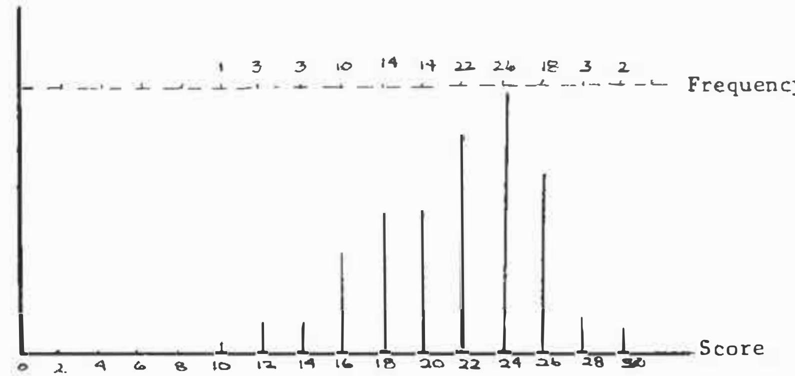
Math 240: A rudimentary calculus course designed originally for Commerce students. Taken by students with low aptitude and/or confidence in Mathematics. Math 30 is a prerequisite. (170 wrote.)

The exam, which is analyzed, question by question, below, consisted of 30 multiple-choice questions and a single question on induction (Question 31). On the multiple choice part, students were given +1 for a correct answer, 0 for a wrong answer, 0 for a blank, so a perfect paper would get 30 points. The total score was distributed as follows:

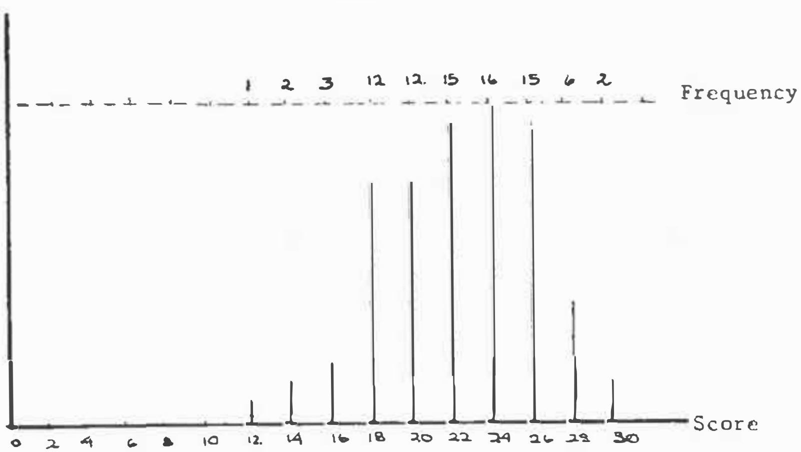
MATH 202



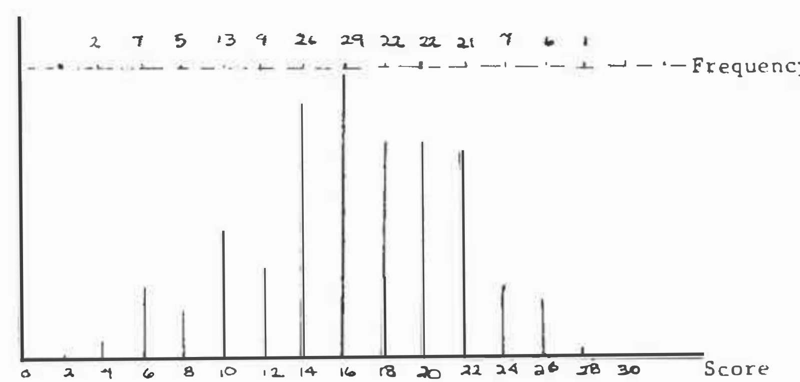

MATH 212




MATH 203




MATH 240



The performance on individual questions was as follows: (we give the percentage of students answering correctly)

1. The multiplicative inverse of 3 is

   (a) 1    <u>(c)</u> 1/3    (e) none of these

   (b) -3    (d) 4

   202: .91    203: .96    212: .95    240:.92

2. The number 1/2 is (a) irrational <u>(b)</u> rational (c) a natural number

   (d) an integer (e) all of these

   202: .84    203: .93    212: .83    240: .79

3. The number $1{\cdot}4999\cdots$ (9 is repeated forever) is (a) irrational

   <u>(b)</u> rational (c) a natural number (d) an integer

   (e) all of these

   202: .37    203: .26    212: .15    240: .18

4. The fact that $(3+2) + 4 = 3 + (2+4)$ is due to the

   (a) commutative law of addition (b) commutative law of multiplication

   (c) associative law of multiplication <u>(d)</u> associative law of addition

   (e) distributive law

   202: .55    203: .76    212: .59    240: .54

5. The multiplicative inverse (reciprocal) of 0 is

   (a) 1    (b) -0    (c) -1    (d) infinity

   <u>(e)</u> none of these

   202: .73    203: .75    212: .65    240: .73

6. $(2^3)^2$ is <u>(a)</u> 64    (b) 32    (c) 120    (d) 1

   (e) none of these

   202: .96    203: .99    212: .96    240: .95

7. $(2^{-3})^2$ is (a) 1/32    (b) -64    <u>(c)</u> 1/64    (d) -1/8

   (e) none of these

   202: .73    203: .88    212: .85    240: .64

8. $(2^3)^{-2}$ is (a) $-64$ (b) $1/32$ (c) $1/64$
(d) $-1/8$ (e) none of these

202: .73 203: .93 212: .85 240: .60

9. $(2^{-3})^{-2}$ is (a) $1/64$ (b) $64$ (c) $1/32$
(d) $32$ (e) $-64$

202: .77 203: .91 212: .84 240: .68

10. $a^x \cdot a^y$ is the same as

(a) $a^{xy}$ (b) $a^{x^y}$ (c) $a^{x+y}$ (d) $a^{2xy}$

(e) none of these

202: .72 203: .88 212: .88 240: .67

11. $4!$ is (a) 1 (b) 24 (c) 10 (d) 32 (e) none of these

202: .71 203: .93 212: 91 240: .69

Remark: Some exam copies had a confusing misprint on this problem.

12. $\log_{10}(10)$ is (a) 0 (b) 1 (c) $-1$ (d) 2
(e) none of these

202: .81 203: .85 212: .81 240: .64

13. $(x^2-4)$ factors into

(a) $(x-2)^2$ (b) $(x+2)^2$ (c) $(x+2)(x-2)$ (d) $x(x-4)$
(e) none of these

202: .91 203: 1.0 212: .97 240: .85

14. Given that $x = 1$ is a root of $x^3 - x^2 + x - 1$, we can factor this cubic into (a) $(x+1)$(something) (b) $(x-1)$(something)

(c) $x$ (something) (d) $x^2$ (something) (e) none of these

202: .55 203: .81 212: .79 240: .43

15. $(x^2-1)(x^3-x+1)$ is

(a) $x^5 + x^3 + x^2 - 1$ (b) $x^5 - 2x^3 + x^2 - x + 1$

(c) $x^5 - 2x^3 + x^2 + x + 1$ (d) $x^5 - 2x^3 + x^2 + x - 1$

(e) none of these

202: .73 203: .88 212: .88 240: .73

16. If $x^2 + 2bx + c = 0$ , then

(a) $x = -b \pm \sqrt{b^2 - c}$ (b) $x = -c \pm \sqrt{c^2 - b}$

(c) $x = b \pm \sqrt{b^2 - c}$ (d) $x = c \pm \sqrt{c^2 - b}$

(e) none of these

202: .33 203: .24 212: .29 240: .30

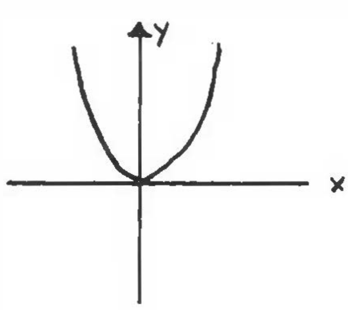
17. $x^5 - 2x^3 + x^2 + x + 4$ divided by $x + 1$ is $x^4 - x^3 - x^2 + 2x - 1$ with remainder

(a) 3 (b) 5 (c) -1 (d) 0 (e) none of these

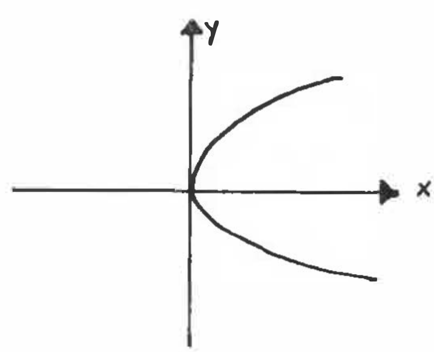
202: .44 203: .64 212: .62 240: .44

*THE NEXT FIVE QUESTIONS BELOW INVOLVE THE FOLLOWING FIVE POSSIBLE ANSWERS:*

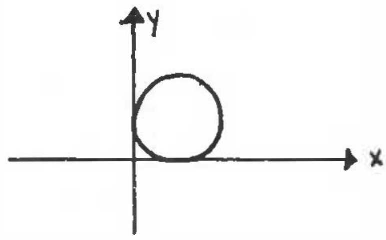
(a)



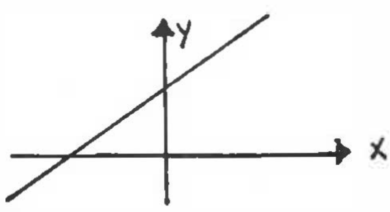

(b)



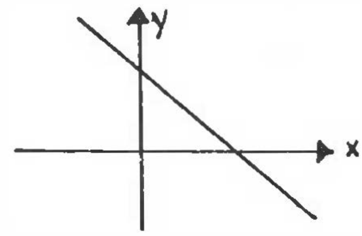

(c)




(d)




(e)



18. The graph of $y = 1 - x$ looks like: (e)

202: .67 203: .85 212: .87 240: .51

19. The graph of $(x-1)^2 + (y-1)^2 = 1$ looks like: (c)

202: .90 203: .95 212: .94 240: .81

20. The graph of $y = x^2$ looks like: (a)

202: .71 203: .87 212: .82 240: .59

21. The graph of $y = 1 + x$ looks like: (d)

202: .71 203: .83 2.2: .87 240: .55

22. The graph of $x = y^2$ looks like: (b)

202: .69 203: .87 212: .81 240: .61

23. The expression $\dfrac{\frac{1}{a} - \frac{1}{b}}{\frac{1}{a} + \frac{1}{b}}$ simplifies to give

(a) $-1$ <u>(b)</u> $\dfrac{b - a}{a + b}$ (c) $1$ (d) $\dfrac{b + a}{b - a}$

(e) none of these

202: .47 203: .58 212: .46 240: .27

24. The expression $\dfrac{1}{a - b} + \dfrac{1}{a + b}$ simplifies to give

(a) $\dfrac{2}{a}$ (b) $\dfrac{1}{a^2 - b^2}$ (c) $\dfrac{1}{2a}$ <u>(d)</u> $\dfrac{2a}{a^2 - b^2}$

(e) none of these

202: .53 203: .77 212: .72 240: .33

25. The solution set of $\dfrac{1}{x - 1} + \dfrac{1}{x - 2} = \dfrac{2}{x - 3}$ is

(a) $1$ (b) $\dfrac{1}{2}$ <u>(c)</u> $\dfrac{5}{3}$ (d) $-2$ (e) none of these

202: .31 203: .58 212: .49 240: .24

26. Written in the form $y = a(x-h)^2 + b$ , the equation $y = 2x^2 - 12x + 19$ becomes

(a) $y = 2(x-3)^2 + 19$ (b) $(x-2)^2 + 8$ (c) $y = 2(x-3)^2 + 1$

(d) $2(x-4)^2 - 1$ (e) none of these

202: .41 203: .57 212: .57 240: .41

27. The solution set for $\frac{x^2 - x}{x - 1} = x$ is

(a) {1} (b) {0} (c) $\{x : x \neq 0\}$ (d) $\{x : x \neq 1\}$

(e) none of these

202: .47 203: .55 212: .57 240: .34

28. $\log_{10}40$ is: (a) $2 \log_{10}20$ (b) $\log_{10}2 + \log_{10}20$

(c) $(\log_{10}20)^2$ (d) $\log_{10}20 + \log_{10}20$ (e) none of these

202: .21 203: .27 212: .41 240: .18

29. $\log_{10}(1/3)$ is: (a) $1/\log_{10}3$ (b) $-(\log_{10}3)/3$

(c) $-1 + \log_{10}3$ (d) $-\log_{10}3$ (e) none of these

202: .41 203: .32 212: .46 240: .34

30. Which one of the following is not an equivalent pair?

(a) $y = \log_{10}x$, $x = 10^y$ (b) $|a|$ , $|-a|$ (c) $x = 2$, $x^2 = 4$

(d) $|x| = \sqrt{4}$ , $x = 2$ (e) $|x| < 2$, $-2 < x < 2$

202: .09 203: .21 212: .19 240: .07

31. Give a proof, by mathematical induction, of the identity

$$1 + 2 + 3 + \ldots + n = \tfrac{1}{2}n(n+1):$$

Comments: There were only about 10 solutions to this problem. It is clear that the incoming students were almost universally incapable of carrying through a simple induction.

# Conclusions

*The Curriculum Review Committee of the Department of Mathematics at the University of Alberta met in October to discuss this examination. At this meeting it was agreed to continue giving a screening examination in certain first-year courses in order to compare the data from year to year. It was the general consensus of the committee that the performance on the examination by the students was reasonable. The committee took into account several facts, for example, the students had been away from their books for a summer, several items on the test related to material that had been taught to the students in the lower ranges of high school so they had not seen it for several years, and some of the material on the test does not appear in the high school syllabus. The students' performance on question 31 was rather disappointing, and we intend to do a little more experimenting in that area.*

*In the course of the committee's discussion of this experimental examination, we discussed both the syllabus for Mathematics 30, and the manner in which Mathematics 30 and Mathematics 31 are being timetabled in the high schools. The committee expressed almost unanimous dismay at the large number of topics in Mathematics 30. It seemed to us that there were far too many to be covered in any kind of depth. In particular, since mathematical induction was the tenth item in a list of 10 items, it seemed clear that the students were probably getting short-changed on this particular topic. It also came to our attention that many high schools are timetabling Mathematics 30 and 31 as one-semester courses offered simultaneously. We would like to hear from high school teachers as to their experiences in teaching mathematics in this rather rapid and condensed manner. Our own feeling was that these two courses deserve to be spread out over a little longer time interval, perhaps by being offered in different semesters, in order to give the students time to absorb and master the difficult concepts involved.*

*We would enjoy hearing from any high school teachers concerning this exam, whether their reactions are positive or negative. We wish to emphasize that, unlike the recent controversy over literacy, the exam was not formulated with the intention of criticizing the teaching of mathematics in the secondary schools. We are indeed interested in the quality of teaching in the mathematical schools as manifested in the performance of students at university, but we are well aware that the question of quality of teaching in the secondary schools is related to a great many factors that cannot be dealt with by an examination. The questions of discipline, how much homework can be assigned, support of teachers by administration, size of classes, teaching loads, and so on are probably far more relevant to the quality of teaching in secondary schools than whether or not certain topics are covered. We intend to continue giving the examination on an experimental basis for several reasons. One is to compare classes of incoming students with each other, and also to use the examination as a source of information for our teachers, to give them some idea of what they should expect from their students. In addition,*