## Volume XVI, Number 4, May 1977



## MCATA Hosts Leadership Conference February 5, 1977

MCATA was the host group at an NCTM-sponsored CAG Western Canada leadership Conference at the U. of A. Education Building.

Joan Kirkpatrick, one of the NCTM directors and professor of elementary education at U. of A., represented President John Eggsgard, who is from Ontario and I believe the first Canadian to become president. Joan Routledge, CAG representative for Canada, acted as chairman for the meeting.

The Manitoba affiliate group was represented by Alan Wells, president, and Cecil Grant, publicity chairman and substitute for NCTM delegate.

The Saskatchewan affiliate group was represented by Darlene Kidd, vice-president, and Rev. Albert Ruetz, NCTM representative.

The British Columbia affiliate group was represented by John Epp, president; Tom Horwitz, NCTM representative; and Bill Dale, vicepresident.

MCATA was represented by A1 Neufeld, president, and Matt Pawluk, NCTM representative; others. from MCATA in attendance were Art Jorgensen, secretary; Bob Holt, annual meeting chairman; Earle Warnica, director; Lyle Pagnucco, director; and Edward Carriger, publicity chairman and newsletter editor.

From MCATA we have Dr. Joan Kirkpatrick on the NCTM board of directors, Dr. George Cathcart on the publications committee, and Bob Holt on the program committee for

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the annual NCTM meeting in San Diego next year. We have had other MCATA members in official NCTM positions in the past and have been well represented and well recognized as one of the NCTM affiliate groups.

At our conference the leaders of the groups were given an opportunity to learn more about what NCTM offers to us in the way of services which include assistance in obtaining speakers and/or material for our annual meetings and even giving financial insurance in the event unusual circumstances cause the meeting to have less than expected revenue. Other activities are name-of-site meeting sponsorship and publication of material that would otherwise be unavailable or excessively expensive. Each group represented exchanged information and ideas that will help us work better together to bring about exchange of information and program sources to reduce duplication of efforts in improving services and development of useful material to our teachers.


Art Jorgensen, Tom Howitz, John Epp, Bill Dale, Rev. Albert Ruetz, Cecil Grant, Darlene Kidd.

Alan Wells, Lyle Pagnucco and Earle Warnica



Bob Holt, Joan Kirkpatrick and Matt Pawluk


## THE VERTEX CONNECTION

Christian R. Hirsch<br>Western Michigan University<br>Kalamazoo

Reprinted from Mathematics Teacher, Vol. 23, Number 7, November 1976.
Teacher's Guide

Grade level: 7-12
Materials: One set of worksheets for each student and a set of transparencies for classroom discussion if desired.

Objectices: Students will (1) discover and apply the formula for the number of diagonals in a polygon and (2) play and analyze the game of SIM

Directions: Distribute the worksheets one at a time.

Sheet 1: Most students should have little difficulty discovering that the number of diagonals in an $n$-gon is $n(n-3) / 2$. However, some may require additional guidance to detect the pattern in the table. Note that this formula was based on an analysis of convex polygons. Students might be encouraged to investigate if their formula also holds for polygons that are not convex.
Sheet 2: In this activity, students are placed in a problem-solving situation that can be modeled using polygons. If class members are represented by the vertices of an $n$-gon, then the number of handshakes involved is equivalent to the number of sides of the polygon formed plus the number of diagonals, $n+n(n-3) / 2=$ $n(n-1) / 2$.

Students who have had some work with mathematical induction may wish to try proving this generalization and the one regarding the number of diagonals of an $n$ gon. Others might explore related questions
such as these: How many triangles result if all diagonals from one vertex of an $n$-gon are drawn? What is the maximum number of lines determined by $n$ points in the same plane?

Sheet 3: The game of SIM (Simmons 1969) is a two-person game that is handled most effectively by declaring the one who wins two out of three games the winner and then pairing the winners against each other until a class champion emerges. The results of this tournament will suggest that the game always has a winner and that the second player has a better chance of winning. An analysis of a winning strategy may be found in Mead, Rosa, and Huang (1974). The fact that there is always formed a triangle of one of the two colors guarantees an affirmative answer to problem 8 . This fact can be easily proved as in Harary (1972). When SIM is played on five vertices, there need not always be a winner.

## REFERENCES

Harary. F. "The Two-Triangle Case of the Acquaintance Graph." Mathematics Magazine 45 (May 1972): 130-35

Mead. E., S. Rosa, and C. Huang. "The Game of SIM: A Winning Strategy for the Second Player." Mathematics Magazine 47 (November 1974): 243-47.
Simmons. G. "The Game of SIM." Journal of Recreational Mathematics 2 (April 1969): 66.

A diagonal of a polygon is a line segment connecting two vertices that are not endpoints of the same side.

1. Determine the number of diagonals in each of the polygons below and enter your answers in the following table:

| Polygon | Number of <br> vertices | Number of diagonals <br> from each vertex | Total number of <br> diagonals |
| :--- | :--- | :--- | :--- |
| Triangle |  |  |  |
| Quadrilateral |  |  |  |
| Pentagon |  |  |  |
| Hexagon |  |  |  |
| Heptagon |  |  |  |
| Octagon |  |  |  |



Triangle


Hexagon


Quadrilateral


Heptagon


Pentagon


Octagon
2. a. How many diagonals do you think a polygon with ten sides would have? (Hint: Do you see any pattern in the table above?)
b. Draw such a polygon and verify your answer.
3. How many diagonals has a polygon with n sides?
4. Suppose at the end of your class today, each student present shakes hands just once with every other student. How many handshakes would be involved?

Hint: First consider some simpler problems:
a. Suppose that there are only four members present.

Represent each member by a vertex of a quadrilateral.

Indicate all possible handshakes by connecting the appropriate vertices.

How many handshakes are involved?
b. Use the method above to determine the number of handshakes if seven students are present.
c. Do you see any relationship between these solutions and your work on SHEET 1?
d. Now try to answer the original problem which was posed.
5. How many handshakes would be involved if a class consisted of $n$ members?
6. Here is an interesting game involving vertex connections which you can play with a friend.
a. Place six points on a sheet of paper to mark the vertices of a regular hexagon as in Figure A below.
b. Each player selects a color different from the other.
c. Take turns connecting pairs of points with line segments. (See Figure B for a partially completed game.)
d. The first player who is forced to form a triangle of his own color is the loser! (Only triangles whose vertices are among the six starting points count.)
-

。
-2 -2


Figure $A$


Figure B
7. Play this game several times and then answer the questions below.
a. Is there always a winner?
b. Which player has the better chance of winning?
8. Use the results of question 7 (a) to help you solve the following problem.

Of any six students in a room, must there be at least three mutual acquaintances or at least three mutual strangers?
9. Play several more games, but in each case starting with five points that are the vertices of a regular pentagon, and then answer questions 7 (a) and 7 (b).

# Do You Know Your Mathematics Terms? 

Originally printed in Summation, Newsletter of the Association of Teachers of Mathematics of New York City, Volume 22 Number 1, Fall 1976.

Can you match - the terms pertaining to mathematics on the left with the brief description or examples on the right? Five correct answers is passing; 6 is fair; 7 is good; 8 is excellent; 10 is perfect.
$\qquad$ 1. ACCELERATION
2. IRRATIONAL NUMBER
3. DISTRIBUTIVE LAW
4. FUNCTION
5. MATRIX
6. PYTHAGOREAN THEOREM
7. SLOPE
8. IMAGINARY NUMBER
$\qquad$ 9. UNDEFINED
10. VECTOR
A. $a^{2}+b^{2}=c^{2}$
B. rise/run
C. directed segment
D. $\sqrt{2}$
E. $\frac{d v}{d t}$
F. $\tan 90^{\circ}$
G. correspondence
H. $\sqrt{-1}$
I. rectangular array
J. $a(b+c)=a b+a c$

This quiz can be given to an eleventh or twelfth grade class. It provides a "lead in" for a discussion of advanced mathematics.

## ANSWERS:

|  | כ | $\ddagger$ | H | 8 | $\forall$ | 1 | 9 | $r$ | $\exists$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | OL | 6 | 8 | L | 9 | S | $\dagger$ | $\varepsilon$ |  |

Submitted by Sidney Penner, Department of Mathematics, Bronx Community College of the City University of New York.

# An Experimental Screening Examination in Mathematics for Students af the University of Alberta 

In September, 1976, the examination described below was given to approximately 500 non-honors students enrolled in introductory calculus courses. The purpose of the exam was:
(a) to give the U. of A. Math Department some idea of the students' background;
(b) to see which parts of the school program had been retained by the students;
(c) to help us decide whether a remedial course is necessary for students with deficient backgrounds.

The exam was given to between 100 and 200 students in each of the following courses:

Math 202 (Math 203): The basic course in calculus for science mafors and others. Math 203 requires Math 31 as a prerequisite, Math 202 requires only Math 30. (150 Math 202 students wrote the exam, 84 Math 203 students wrote.)

Math 2l2: The basic calculus course for engineering students. Most, but not all, have had Math 31. (116 students wrote the exam.)

Math 240: A rudimentary calculus course designed originally for Commerce students. Taken by students with low aptitude and/or confidence in Mathematics. Math 30 is a prerequisite. (170 wrote.)

The exam, which is analyzed, question by question, below, consisted of 30 multiple-choice questions and a single question on induction (Question 31). On the multiple choice part, students were given +1 for a correct answer, 0 for a wrong answer, 0 for a blank, so a perfect paper would get 30 points. The total score was distributed as follows:

MATH 202


MATH 203


MATH 212


MATH 240


The performance on individual questions was as follows: (we give the percentage of students answering correctly)

1. The multiplicative inverse of 3 is
(a) 1
(c) $1 / 3$
(e) none of these
(b) -3
(d) 4
202: .91 203: .96 212: .95 240:.92
2. The number $1 / 2$ is
(a) irrational
(b) rational
(c) a natural number
(d) an integer
(e) all of these 202: . 84

203: . 93 212: . 83
240: . 79
3. The number $1.4999 \ldots$ (9 is repeated forever) is (a) irrational
(b) rational
(c) a natural number
(d) an integer
(e) all of these
202:. 37 203: . 26 212: . 15 240:. 18
4. The fact that $(3+2)+4=3+(2+4)$ is due to the
(a) commutative law of addition (b) commutative law of multiplication
(c) associative law of multiplication (d) associative law of addition
(e) distributive law
202:. 55 203: . 76 212: . 59 240: . 54
5. The multiplicative inverse (reciprocal) of 0 is
(a) 1
(b) -0
(c) -1
(d) infinity
(e) none of these 202: . 73 203: . 75 212: . 65 240: . 73
6. $\left(2^{3}\right)^{2}$ is (a) 64
(b) 32
(c) 120
(d) 1
(e) none of these
202: . 96 203: . 99 212: . 96 240: . 95
7. $\left(2^{-3}\right)^{2}$ is (a) $1 / 32$
(b) -64
(c) $1 / 64$
(d) $-1 / 8$
(e) none of these 202: . 73 203: 88 212: . 85 240: . 64
8. $\left(2^{3}\right)^{-2}$ is
(a) -64
(b) $1 / 32$
(c) $1 / 64$
(d) $-1 / 8$
(e) none of these

202: . 73
203: . 93
212: . 85
240: . 60
9. $\left(2^{-3}\right)^{-2}$ is
(a) $1 / 64$
(b) 64
(c) $1 / 32$
(d) 32
(e) -64 202: . 77 203: . 91

212: . 84 240: . 68
10. $a^{x} \cdot a^{y}$ is the same as
(a) $a^{x y}$
(b) $a^{x^{y}}$
(c) $a^{x+y}$
(d) $a^{2 x y}$
(e) none of these 202: . 72 203: . 88 212: . 88 240: . 67
11. 4! is
(a) 1
(b) 24
(c) 10
(d) 32 (e) none of these

202: . 71
203: . 93
212: 91
240: . 69
Remark: Some exam copies had a confusing misprint on this problem.
12. $\log _{10}(10)$ is
(a) 0
(b) 1
(c) -1
(d) 2
(e) none of these 202: . 81 203: . 85 212: . 81 240:. 64
13. $\left(x^{2}-4\right)$ factors into
(a) $(x-2)^{2}$
(b) $(x+2)^{2}$
(c) $(x+2)(x-2)$
(d) $x(x-4)$
(e) none of these 202: . 91 203: 1.0 240: . 97 24
14. Given that $x=1$ is a root of $x^{3}-x^{2}+x-1$, we can factor this cubic into (a) ( $x+1$ ) (something) (b) ( $x-1$ ) (something)
(c) $x$ (something)
(d) $\mathrm{x}^{2}$ (something)
(e) none of these 202: . 55
203: . 81
212: . 79 240: . 43
15. $\left(x^{2}-1\right)\left(x^{3}-x+1\right)$ is
(a) $x^{5}+x^{3}+x^{2}-1$
(b) $x^{5}-2 x^{3}+x^{2}-x+1$
(c) $x^{5}-2 x^{3}+x^{2}+x+1$
(d) $x^{5}-2 x^{3}+x^{2}+x-1$
(e) none of these

$$
\text { 202: . } 73 \text { 203: . } 88 \text { 212: . } 88 \text { 240: . } 73
$$

16. If $x^{2}+2 b x+c=0$, then
(a) $x=-b \pm \sqrt{b^{2}-c}$
(b) $\mathbf{x}=-\mathrm{c} \pm \sqrt{\mathrm{c}^{2}-\mathrm{b}}$
(c) $x=b \pm \sqrt{b^{2}-c}$
(d) $x=c \pm \sqrt{c^{2}-b}$
(e) none of these

$$
\text { 202: . } 33 \text { 203: . } 24 \text { 212: . } 29 \text { 240: . } 30
$$

17. $x^{5}-2 x^{3}+x^{2}+x+4$ divided by $x+1$ is $x^{4}-x^{3}-x^{2}+2 x-1$ with remainder
(a) 3
(b) 5
(c) -1
(d) 0
(e) none of these
202: . 44 203: . 64
212: . 62 240: . 44

THE NEXT FIVE QUESTIONS BELOW INVOLVE THE FOLLOWING FITE POSSIBLE ANSWERS:
(a)

(b)

(c)

(e)

18. The graph of $y=1-x$ looks like: (e)

$$
\text { 202:. } 67 \text { 203: . } 85 \quad \text { 212: . } 87 \quad \text { 240: . } 51
$$

19. The graph of $(x-1)^{2}+(y-1)^{2}=1$ looks like: (c)

$$
\text { 202: . } 90 \quad \text { 203: . } 95 \quad \text { 212: . } 94 \quad \text { 240: . } 81
$$

20. The graph of $y=x^{2}$ looks like: (a)
202: . 71 203: . 87 212: . 82 240: . 59
21. The graph of $y=1+x$ looks like: (d)
202: . 71 203: . 83 2.2: . 87 240: . 55
22. The graph of $x=y^{2}$ looks like: (b) 202: . 69 203: . 87 212: . 81 240: . 61
23. The expression $\frac{\frac{1}{a}-\frac{1}{b}}{\frac{1}{a}+\frac{1}{b}}$ simplifies to give
(a) -1
(b) $\frac{b-a}{a+b}$
(c) 1
(d) $\frac{b+a}{b-a}$
(e) none of these

$$
\text { 202: . } 47 \text { 203: . } 58 \text { 212: . } 46 \text { 240: . } 27
$$

24. The expression $\frac{1}{a-b}+\frac{1}{a+b}$ simplifies to give
(a) $\frac{2}{a}$
(b) $\frac{1}{a^{2}-b^{2}}$
(c) $\frac{1}{2 \mathrm{a}}$
(d) $\frac{2 a}{a^{2}-b^{2}}$
(e) none of these 202: . 53 203: . 77 212: . 72 240: . 33
25. The solution set of $\frac{1}{x-1}+\frac{1}{x-2}=\frac{2}{x-3}$ is
(a) 1
(b) $\frac{1}{2}$
(c) $\frac{5}{3}$
(d) -2
(e) none of these 202: . 31 203: . 58 212: . 49 240: . 24
26. Written in the form $y=a(x-h)^{2}+b$, the equation $y=2 x^{2}-12 x+19$ becomes
(a) $y=2(x-3)^{2}+19$
(b) $(x-2)^{2}+8$
(c) $y=2(x-3)^{2}+1$
(d) $2(x-4)^{2}-1$
(e) none of these 202: . 41 203: . 57 212: . 57 240: . 41
27. The solution set for $\frac{x^{2}-x}{x-1}=x$ is
(a) $\{1\}$
(b) $\{0\}$
(c) $\{x: x \neq 0\}$
(d) $\{x: x \neq 1\}$
(e) none of these
202:. 47 203:. 55 212:. 57 240:. 34
28. $\log _{10} 40$ is:
(a) $2 \log _{10} 20$
(b) $\log _{10} 2+\log _{10} 20$
(c) $\left(\log _{10} 20\right)^{2}$
(d) $\log _{10^{20}} 2+\log _{10} 20$
(e) none of these 202: . 21
203: . 27 212: . 41
240: . 18
29. $\log _{10}(1 / 3)$ is:
(a) $1 / \log _{10} 3$
(b) $-\left(\log _{10} 3\right) / 3$
(c) $-1+\log _{10} 3$
(d) $-\log _{10^{3}}$
(e) none of these 202:.41 203: . 32 212:. 46 240:. 34
30. Which one of the following is not an equivalent pair?
(a) $y=\log _{10} x, x=10^{y}$
(b) $|a|,|-a|$
(c) $x=2, x^{2}=4$
(d) $|x|=\sqrt{4}, x=2$
(e) $|x|<2,-2<x<2$
202: . 09 203: . 21 212: . 19 240: . 07
31. Give a proof, by mathematical induction, of the identity

$$
1+2+3+\ldots+n=\frac{1}{2} n(n+1):
$$

Comments: There were only about 10 solutions to this problem.
It is clear that the incoming students were almost universally incapable of carrying through a simple induction.

## Conclusions


#### Abstract

The Curriculum Review Committee of the Department of Mathematics at the University of Alberta met in October to discuss this examination. At this meeting it was agreed to continue giving a screening examination in certain first-year courses in order to compare the data from year to year. It was the general consensus of the committee that the performance on the examination by the students was reasonable. The committee took into account several facts, for example, the students had been away from their books for a summer, several items on the test related to material that had been taught to the students in the lower ranges of high school so they had not seen it for several years, and some of the material on the test does not appear in the high school syllabus. The students' performance on question 31 was rather disappointing, and we intend to do a little more experimenting in that area.


In the course of the committee's discussion of this experimental examination, we discussed both the syllabus for Mathematics 30, and the manner in which Mathematics 30 and Mathematics 31 are being timetabled in the high schools. The committee expressed almost unanimous dismay at the large number of topics in Mathematics 30. It seemed to us that there were far too many to be covered in any kind of depth. In particular, since mathematical induction was the tenth item in a list of 10 items, it seemed clear that the students were probably getting short-changed on this particular topic. It also came to our attention that many high schools are timetabling Mathematics 30 and 31 as one-semester courses offered simultaneously. We would like to hear from high school teachers as to their experiences in teaching mathematics in this rather rapid and condensed manner. Our own feeling was that these two courses deserve to be spread out over a little longer time interval, perhaps by being offered in different semesters, in order to give the students time to absorb and master the difficult concepts involved.

We would enjoy hearing from anu high school teachers concerning this exam, whether their reactions are positive or negative. We wish to emphasize that, unlike the recent controversy over literacy, the exam was not formulated with the intention of criticizing the teaching of mathematics in the secondary schools. We are indeed interested in the quality of teaching in the mathematical schools as manifested in the performance of students at university, but we are well aware that the question of quality of teaching in the secondary schools is related to a great many factors that cannot be dealt with by an examination. The questions of discipline, how much homework can be assigned, support of teachers by administration, size of classes, teaching loads, and so on are probably far more relevant to the quality of teaching in secondary sciools than whether or not certain topics are covered. We intend to continue giving the examination on an experimental basis for several reasons. One is to compare classes of incoming students with each other, and also to use the examination as a source of information for our teachers, to give them some idea of what they should expect from their students. In addition,
one important and somewhat unexpected aspect of the examination has been to indicate to us that our present policy of offering several different calculus courses at different levels is worthwhile. It is quite clear that the mathematical talent of the students in the various courses fits in a rather general way our conception of the purpose of these courses. In closing, please note that we did not test our incoming honor students with this examination; we will be testing them next year for comparison.

## ANNOUNCEMENT OF SUPPORT FOR SPECIAL PROJECTS

Reprinted from NCTM BULLETIN FOR LEADERS, Apri1 1977.

The Mathematics Education Trust has been established by the National Council of Teachers of Mathematics to provide funds for special projects that would enhance the teaching and learning of mathematics beyond the regular activities of NCTM. These projects may include, but are not restricted to, special conferences, the publication of certain monographs, consultants for teachers, lecturers for students, awards for scholars in residence, exchange teachers, special centers, and research. Projects that are unusual, creative, and adventuresome are encouraged. Selecting appropriate projects and monitoring the progress of funded projects are among the responsibilities of a five-member Mathematics Education Trust Committee appointed by the NCTM President with the approval of the Board of Directors.

The first deadline for the submission of proposals for projects to begin during the period 1 September 1977 through 31 August 1978 is 1 July 1977 . Dead1 ines for projects to begin at a later date are 1 September 1977 and 1 March 1978.

Further information and applications/guidelines for project proposals (not to exceed $\$ 3,000$ ) may be secured from Mathematics Education Trust, National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 22091.

## NEWFOUNDLAND "NAME-OF-SITE" CONFERENCE

The Mathematics Council of Newfoundland is holding a "name-of-site" conference July 20 to 23, 1977, with NCTM as co-host.

Further information can be obtained by contacting:
Mr. Ed Carriger
R.R. \#1, Site 2, Box 4

Bluffton, Alberta TOC OMO

Activities that contribute to the student's personal understanding of key concepts in mathematics
IDEAS

## Prepared by George W. Bright, Northern Illinois University, DeKalb, Illinois

Each IDEAS presents açivities that are appropriate for use with students at various levels in the elementary school. The activity sheets are so arranged that they can be easily removed and reproduced for classroom use. Permission to reproduce these worksheets for classroom use is not necessary.

The activities for November provide practice in using two units of lengthcentimetre and metre. Be sure each student knows how to use a ruler before beginning these activities. The last worksheet (IDEAS) should be attempted only after students have had some experience measuring objects to the nearest centimetre and metre. Other measurement activities are contained in the 1976 NCTM Yearbook, Measurement in School Mathematics.

One "thank you" goes to Millie Lewis and Candy Slitor (Midway Elementary School) and Betty Pullin (Oak Grove Elementary School) in DeKalb County, Georgia, who spent time reviewing this month's IDEAS. A second "thank you" goes to all their students who helped pilot these worksheets. Their help is greatly appreciated.

## I D E A S

Objective: To practice measuring lengths of objects less than 20 cm
Level: 1, 2, 3
Directions for teachers:

1. Give a copy of the worksheet to each student.
2. Tell the children to cut out the paper ruler.
3. Read the directions to the students.
(a) Be sure the students measure the vertical (not slanted) height of each creature.
(b) Be sure the "feet" of each creathre are included in the height.

Going further:

1. Ask which creature is the tallest, shortest, fattest, and so on, or which has the most arms, eyes, feet, and so on.
2. Ask each student to draw a creature and measure its height.
3. Use the ruler to measure objects in the room: for example, eraser, pencil, paper, scissors, crayon.

Measure the height of each creature to the nearest centimetre.


## I D EAS For Teachers

Objective: To develop a frame of reference for centimetre by measuring parts of the body
Level: 2, 3, 4, 5
Directions for teachers:

1. Provide students with measuring tapes marked off in centimetres, or pieces of string and centimetre rulers
2. Give each student a copy of the worksheet.
3. Read the directions with the students and then have the students work in pairs
or small groups to measure each other's body parts.

## Going further:

1. Have the students separate themselves into three groups: (see IDEAS in the Arithmetic Teacher. October 1974)
a. tall rectangles (height > arm span)
b. squares (height $=$ arm span)
c. short rectangles (height < arm span)
2. Find approximate ratios: head to knee, knee to toe, head to toe, smile to toe, and so on.

Measure each body part to the nearest centimetre.


___cm

big toe


## I D E A S ${ }_{\text {For Teachers }}$

Objective: To practice measuring line segments

Level: 4, 5. 6
Directions for teachers:

1. Give each student a copy of the worksheet.
2. Be sure each student has a centimetre ruler.
3. Let the students read the directions and go to work. The students are expected to measure each straight section of the path
to the nearest centimetre and then to add these measurements.

Going further:

1. Ask students to put an "X" on each part of the maze that is 6 cm (or 5 cm or 4 cm) long.
2. Let each student make a maze and have another student find and measure the path.
Answers: first maze, 20 cm ; second maze, 47 cm .

Find each line path from START to FINISH.
Measure the length of each path In centimetres.


## I D E A S ${ }_{\text {For Teachers }}$

Objective: To estimate measurements and to use the relationship between metre and centimetre

Level: 6, 7, 8
Directions for teachers:

1. The students need to have some experience measuring in centimetres and metres before they can be successful with the exercises.
2. Give each student a copy of the worksheet.
3. Students should imagine the actual objects pictured. The best measurement may not be exactly correct, so students should choose the one which is closest.

Ask students to explain how they decided which measurement is best.
4. Ask students to make up exercises like those on the worksheet.
Answers: canoe, 4 m ; whale, 20 m ; penny, 2 cm; glasses, 10 cm ;
$4 \mathrm{~cm}, 8 \mathrm{~m}=800 \mathrm{~cm}, 7 \mathrm{~m}, 100 \mathrm{~cm}$.
Comments: There are several ways to obtain answers to the exercises at the bottom of the page. In the first pair, 4 is less than 40 and cm is a smaller unit than m , so 4 cm is smaller than 40 m . Alternatively change one of the measurements in each pair to the unit of the other measurement; for example, since $7 \mathrm{~m}=700 \mathrm{~cm}$ and $7000 \mathrm{~cm}=$ 70 m , the pair $7000 \mathrm{~cm}, 7 \mathrm{~m}$ is the same as the pair $7000 \mathrm{~cm}, 700 \mathrm{~cm}$ or the pair $70 \mathrm{~m}, 7$ m.
m means metre cm means centimetre 1 m 100 centimetres

CIrcle the best measurement for each object plctured below.


CIrcle the measurement In each palr that represents the shorter length.
$4 \mathrm{~cm}, 40 \mathrm{~m}$
$7000 \mathrm{~cm}, 7 \mathrm{~m}$
$10 \mathrm{~m}, 100 \mathrm{~cm}$

# Call for Research Papers and Proposals 

Reprinted from NCTM BULLETIN FOR LEADERS, April 1977.

Program space is reserved at the 56 th Annual Meeting, scheduled for 12 to 15 April 1978 in San Diego, California, for those interested in research and its implications for curriculum builders and classroom teachers. There will be three types of sessions: reporting, nonreporting, and round table.

Reporting sessions will be short presentations of the findings of recently completed research.

Nonreporting sessions will be of longer duration and will deal with recent developments in the field of research. These should be of interest primarily to those who are actively engaged in research activities and will provide a sounding board for new and interesting ideas.

The round table sessions will be of varying duration and will be an opportunity for a researcher to discuss with other interested participants the findings, results, and implications of recently completed research on a currently important issue.

Those wishing to be considered for a place on the program must submit seven copies of an abstract or proposal - typewritten, double-spaced, and not exceeding 1,000 words. The following should be included in the mailing:

1. The name and professional affiliation of the author. If more than one person is involved, complete information must be given as to the anticipated responsibility of each contributor.
2. The preferred mailing address of the contributor.
3. Title of the paper or presentation.
4. A summary of the paper or outline of the presentation clearly describing the purpose and significance of the research, the conceptual framework, the procedures, the design, the analysis, and the findings and conclusions.
5. A designation as to which of the three types of sessions the proposal is directed.
With each abstract or proposal submitted the sender should enclose two self-addressed postcards; one will be used to acknowledge receipt of the proposal and the other to notify the sender of the decision of the screening committee.

All materials submitted will be reviewed by persons selected by the NCTM Research Advisory Committee. The screening committee consists of these persons and the NCTM San Diego Program Committee member responsible for Research Sessions.

Abstracts of all papers selected for presentation will be available in a document published by the ERIC Center for Science, Mathematics, and Environmental Education.

All materials must be submitted as soon as possible to Jesse A. Rudnick, Mathematics Education, Ritter Hall 334, Temple University, Fhiladelphia, PA 19122. Materials postmarked after 15 May 1977 will not be accepted.

## Math Council Publications Available at Barnett House

Math Council publications that are available at Barnett House are:

1. Source material for classroom activities that are ready for your application.
a. An Active Learning Unit on Real Numbers - $\$ 3.00$
b. Monograph No. 1 - Manipulative Materials for Teaching and Learning Mathematics - \$2.50
c. Monograph No. 3 - Metrication: Activities, Relationships and Humor - $\$ 3.00$
d. Monograph No. 4 - Timeless Activities for Mathematics K - 12 - $\$ 4.00$ All of these except (a) are useful for $K-12$ classes, while (a) is written for secondary classes only. Your editor has used material from each of the above successfully while teaching classes from Grade IV through Grade IX and recommends them highly.
2. Resource publication for assisting the teacher to review his/her technique in teaching.
a. Monograph No. 2 - Mathematics Teaching: The State of the Art - $\$ 2.50$

This publication is a summary of the major topics from the Edmonton MCATA-NCTM "name-of-site" meeting, October 1973. The ideas offered and challenges made are just as refreshing, helpful, and encouraging as they were when presented. They are up-to-date in their useful application.

All of these publications have been sent to members active at the time of publication. They are now available to new members and other teachers for the publication price.

Send your order to: The Alberta Teachers' Association, Barnett House, 11010 - 142 Street, Edmonton T5N 2R1.

# 1977 Alberta High School Prize Exam Resulits 

| Prize | Amt. | Student | School |
| :---: | :---: | :---: | :---: |
| Canadian Math, Congress <br> Scholarship | $\$ 400$ | FENSKE, Keith W, | Harry Ainlay High School <br> Edmonton, Alberta |
| Nickel Foundation <br> Scholarship | $\$ 400$ | BERTRAND, Daniel | Camille Lerouge Collegiate <br> Red Deer, Alberta |
| Third Highest | $\$ 150$ | PEZZANI, Glenn | Harry Aislay High School <br> Edmonton, Alberta |
| Fourth Highest | $\$ 150$ | WONG, David | Lethbridge Collegiate Inst. <br> Lethbridge, Alberta |

SPECIAL PROVINCIAL PRIZES
Highest Grade 12 student (below first 4):
\$ 75 TOTMAN, Ian W. Fort Saskatchewan Sr. High Fort Saskatchewan

Highest Grade 10/ll student (below first 4):
\$ 75 McINTOSH, Lawrence P. Bishop Carroll High School Calgary, Alberta

DISTRICT PRIZES

| District No. Amt. | Name | School |
| :---: | :--- | :--- |
| 1 | $\$ 50$ | MORRILL, Cameron | | Edwin Parr Composite |
| :--- |
| 2 |

PART I ANSWER SHEET

To be filled in by the Candidate.

PRINT:
Last Name First Name Initial

Candidate's Address Town/City

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Name of School
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Grade
ANSNERS:


To be completed by the Department of Mathenatics, University of Alberta:

| Points | Points Correct | Number Wrong |
| :--- | :--- | :--- |
| $1-20$ | $5 \times=$ | $1 \times=$ |
| Totals | $\mathrm{C}=$ |  |
| SCORE $=\mathrm{C}-\mathrm{W}=$ |  |  |

Do all problems. Each problem is worth five points. TIME: 60 Minutes

1. If $a>b>0$, then
(A) $\frac{a+1}{a}>\frac{b+1}{b}$
(B) $\frac{a+1}{a} \geq \frac{b+1}{b}$
(C) $\frac{a+1}{a} \leq \frac{b+1}{b}$
(D) $\frac{a+1}{a}<\frac{b+1}{b}$
(E) none of the preceding are true.
2. Let $A B$ be a diameter of a circle of radius $l$ and let $C$ be a point on the circumference such that $\overline{\mathrm{AC}}=\overline{\mathrm{BC}}$. Then the length $\overline{\mathrm{AC}}$ is equal to
(A) 2
(B) $1 / 2$
(C) $\sqrt{2}$
(D) $1 / \sqrt{2}$
(E) none of these.
3. Out of 100 people, 60 report that they receive the daily news by watching television, whereas 70 read the newspaper. Of those that read the newspaper, $70 \%$ also watch television. The number not receiving any news by television or newspaper is
(A) 15
(B) 19
(C) 23
(D) 27
(E) none of these.
4. $\left(64^{.9}\right)(32)^{-.08}$ equals
(A) 64
(B) 32
(C) $\quad 24$
(D) 8
(E) none of these.
5. Let $f(x)$ be a non-constant polynomial with real coefficients. If $f(x)=f(x-1)$ for all $x$ then $f(x)$
(A) has exactly one root
(B) cannot exist
(C) has exactly two roots
(D) has either no roots or an
infinite number of roots
(E) satisfies none of the preceding.
6. If $k$ is a real number such that $0<k<1$, then the roots of the quadratic equation $k x^{2}-3 x+k=0$ satisfy
(A) both are positive
(B) both are negative
(C) both are zero
(D) one is positive and one is negative
(E) none of the preceding
7. Let $\&$ be a line in the real plane passing through the points $(1,1)$ and $(3,5)$. Then $\ell$ passes through the point $(2, y)$ where
(A) $y=4$
(B) $\mathrm{y}=2$
(C) $y=3$
(D) $\mathrm{y}=5$
(E) $y$ is none of the preceding.
8. $\triangle A B C$ is an equilateral triangle with sides of length 1 , and $D E \| C B$. If the area of $\triangle A D E$ is equal to the area of the trapezoid DEBC , then the length $\overline{\mathrm{DE}}$ equals

(A) $1 / 2$
(B) $1 / 3$
(C) $1 / \sqrt{2}$
(D) $\frac{\sqrt{2}-1}{\sqrt{2}}$
(E) $\frac{\sqrt{3}-1}{\sqrt{3}}$
9. The inequality $(x+1)(x-1) \geq x^{2}$ is valid
(A) for all real $x$
(B) fur no real $x$
(C) for all $x: 1$
(D) for all $\mathrm{x}<0$
(E) for none of the preceding.
10. Suppose a bowl contains 3 red balls and 3 yellow balls. The probability that two balls drawn out at random without replacement will both be red is
(A) $1 / 2$
(B) $1 / 4$
(C) $1 / 3$
(D) $1 / 6$
(E) none of these.
11. $\frac{1}{1+\frac{1}{1+\frac{1}{1+1}}}$ equals
(A) 1
(B) $1 / 2$
(C) $2 / 3$
(D) $3 / 5$
(E) $5 / 8$
12. $\frac{x y-x^{2}}{x y-y^{2}}-\frac{x y}{x^{2}-y^{2}}$ can be simplified to
(A) $\frac{x^{3}}{y^{3}-y x^{2}}$
(B) $\frac{x^{2}}{y^{2}-x^{2}}$
(C) $\frac{x^{2}+y^{2}}{x^{2}-y^{2}}$
(D) $\frac{x^{4}+x y^{3}}{\left(x^{2}-y^{2}\right)\left(x y-y^{2}\right)}$
(E) none of these.
13. If the radius of a sphere is increased $100 \%$, the volume is increased by
(A) $100 \%$
(B) $200 \%$
(C) $300 \%$
(D) $400 \%$
(E) none of these.
14. $x^{4}+16$ equals
(A) $\left(x^{2}+4\right)\left(x^{2}+4\right)$
(B) $\left(x^{2}+4\right)\left(x^{2}-4\right)$
(C) $\left(x^{2}-4 x+4\right)\left(x^{2}+4 x+4\right)$
(D) $\left(x^{2}-2 x \sqrt{2}+4\right)\left(x^{2}+2 x \sqrt{2}+4\right)$
(E) nome of these.
15. The price of a book has been reduced by $20 \%$. To restore it to its former value, the last price must be increased by
(A) $25 \%$
(B) $10 \%$
(C) $15 \%$
(D) $20 \%$
(E) none of these.
16. $O A B C$ is a rectangle inscribed in a quadrant of a circle of radius 10 . If $\overline{O A}=5$, then $\overline{A C}$ equals
(A) $5 \sqrt{2}$
(B) $\sqrt{75}$
(C) 8
(D) 12
(E) none of these.

17. The lengths of the medians of a right triangle which are drawn from the vertices of the acute angles are $\sqrt{73}$ and $2 \sqrt{13}$. The length of the third median is
(A) $\sqrt{73+52}$
(B) $\sqrt{73}+2 \sqrt{13}$
(C) 5
(D) 10
(E) none of these.
18. A car travels 240 miles from one town to another at an average speed of 30 miles per hour. On the return trip the average speed is 60 miles per hour. The average speed for the round trip is
(A) 35 mph
(B) 40 mph
(C) 45 mph
(D) 50 mph
(E) 55 mph
19. $\log _{3} 6+\log _{3}(3 / 2)$ equals
(A) $5 / 2$
(B) 3
(C) 2
(D) 1
(E) 0
20. The slope of the line passing through the points $(3,4)$ and $(1,9)$ is
(A) $-5 / 2$
(B) $5 / 2$
(C) 5
(D) -2
(E) 6
21. Prove that $3^{n}+1$ is not divisible by 8 for any positive integer $n$.
22. Let $k$ be any nonzero real number and let $a, b$, and $c$ be the roots of the cubic equation $4 x^{3}-32 x^{2}-k^{2} x+8 k^{2}=0$. Show that the quadratic equation $x^{2}+2 k x-a b c=0$ cannot have real roots.
23. Sketch accurately the set of all points which satisfy $[|x|]+|y|=2$. Note that for a real number $r$, $[r]$ is defined to be the largest integer $n$ such that $n \leq r$.
24. For any three consecutive natural numbers, prove that the cube of the largest cannot be the sum of the cubes of the other two.
25. Prove that $\sin \left(\frac{\pi}{24}\right)=(\sqrt{2+\sqrt{2}}-\sqrt{3} \sqrt{2-\sqrt{2}}) / 4$.
26. Prove that a triangle with sides of lengths 3,4 and 5 respectively, is a right triangle.
27. If $a, b$, and $c$ denote the lengths of the sides of a triangle, prove that $\sqrt{a(b+c-a)}, \sqrt{b(c+a-b)}$ and $\sqrt{c(a+b-c)}$ are also lengths of the sides of a triangle.
28. Show that the maximum value of $x+y+z$ subject to $x \geq 0, y \geq 0$, $z \geq 0, x+y \leq 2$ and $3 x-y+z \leq 1$ is 5 .
29. Suppose four lines $\ell, \ell_{1}, \ell_{2}$, and $\ell_{3}$ meet in a point.

Suppose $\ell_{1}, \ell_{2}$, and $\ell_{3}$ lie in the same plane, and $\ell$ makes equal angles with $\ell_{1}, \ell_{2}$ and $\ell_{3}$. Prove that $\ell$ is perpendicular to the plane containing $\ell_{1}, \ell_{2}$ and $\ell_{3}$ (see diagram).

10. If the real numbers $a, b, c, x, y, z$ satisfy $a z-2 b y+c x=0$ and $a c-b^{2}>0$, then prove that $x z-y^{2} \leq 0$.

## SOLUTIONS

1971 Alberta High School
PRIZE EXAMINATION IN MATHEMATICS

PART I - KEY


| D | A | E | D | A | E | C | B | C | A |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |

PART II - ANSWERS

1. Clearly $3^{n}+1$ is not divisible by 8 for $n=1,2$. Let $m$ be the smallest positive integer for which $3^{m}+1$ is divisible by 8 . Thus there is an integer $k$ such that $3^{m}+1=8 k$. Adding 8 to both sides gives $3^{m}+9=8 k+8$, or $9\left(3^{m-2}+1\right)=8(k+1)$. This shows that $3^{m-2}+1$ is also divisible by 8 , so that either $m$ was not the smallest such integer or $m$ is 1 or 2 . This contradiction proves the original assertion.
2. The cubic $4 x^{3}-32 x^{2}-k^{2}+8 k^{2}=0$ factors into $\left(4 x^{2}-k^{2}\right)(x-8)=0$. The roots of this equation are $x=8, x=\frac{k}{2}$ and $x=\frac{-k}{2}$. The product of the roots is $a b c=-2 k^{2}$. This means that the quadratic equation become $x^{2}+2 k x+2 k^{2}=0$. The roots of this equation are $-k \pm k \sqrt{-1}$ which are not real for any nonzero real value of $k$.

3. Let $n$ be a natural number such that $n^{3}+(n+1)^{3}=(n+2)^{3}$. Thus $n^{3}+n^{3}+3 n^{2}+3 n+1=n^{3}+6 n^{2}+12 n+8$ and $n$ satisfies $n^{3}-3 n^{2}-9 n-7=0$. This shows that $\frac{7}{n}=n^{2}-3 n-9$ is an integer so that $n$, being a natural number must be 1 or 7 . However $1^{3}+2^{3}=9 \neq 3^{3}=27$ and $7^{3}+8^{3}=855 \neq 9^{3}=729$, proving the assertion.
4. $\quad \sin \left(\frac{\pi}{24}\right)=\sin \left(\frac{\pi}{6}-\frac{\pi}{8}\right)=\sin \left(\frac{\pi}{6}\right) \cos \left(\frac{\pi}{8}\right)-\cos \left(\frac{\pi}{6}\right) \sin \left(\frac{\pi}{8}\right)$
and so

$$
\sin \left(\frac{\pi}{24}\right)=\frac{1}{2} \cos \left(\frac{\pi}{8}\right)-\frac{\sqrt{3}}{2} \sin \left(\frac{\pi}{8}\right) .
$$

But

$$
\cos ^{2}\left(\frac{\pi}{8}\right)=\cos ^{2}\left(\frac{1}{2} \frac{\pi}{4}\right)=\frac{1+\cos \left(\frac{\pi}{4}\right)}{2}=\frac{1+\frac{\sqrt{2}}{2}}{2}=\frac{2+\sqrt{2}}{4}
$$

and so

$$
\sin ^{2}\left(\frac{\pi}{8}\right)=1-\cos ^{2}\left(\frac{\pi}{8}\right)=1-\frac{2+\sqrt{2}}{4}=\frac{2-\sqrt{2}}{4} .
$$

Since $\cos \left(\frac{\pi}{8}\right)$ and $\sin \left(\frac{\pi}{8}\right)$ are positive,

$$
\cos \left(\frac{\pi}{8}\right)=\frac{\sqrt{2+\sqrt{2}}}{2} \text { and } \sin \left(\frac{\pi}{8}\right)=\frac{\sqrt{2-\sqrt{2}}}{2}
$$

Therefore,

$$
\sin \left(\frac{\pi}{24}\right)=\frac{1}{2} \frac{\sqrt{2+\sqrt{2}}}{2}-\frac{\sqrt{3}}{2} \frac{\sqrt{2-\sqrt{2}}}{2}=\frac{\sqrt{2+\sqrt{2}}-\sqrt{3} \sqrt{2-\sqrt{2}}}{4}
$$

6. Let $\triangle A B C$ have sides $A B, B C$, and $C A$ of lengths 3,4 , and 5 respectively. By the law of cosines

$$
\begin{aligned}
\cos (\langle\mathrm{ABC}) & \left.\left.=\frac{1}{2 \overline{\mathrm{AB}} \overline{\mathrm{BC}}} \right\rvert\,(\overline{\mathrm{AC}})^{2}-(\overline{\mathrm{BC}})^{2}-(\overline{\mathrm{AB}})^{2}\right) \\
& =\frac{1}{2 \cdot 3 \cdot 4}\left(5^{2}-4^{2}-3^{2}\right)=0
\end{aligned}
$$



Since $0<\left\langle A B C<180^{\circ}\right.$, $A B C$ must be $90^{\circ}$.
7. Three positive real numbers $x, y$ and $z$ are lengths of the sides of a triangle if and only if $x+y>z, x+z>y$ and $y+z>x$. Suppose that $x^{2}, y^{2}$ and $z^{2}$ are lengths of the sides of a triangle.

$$
(x+y)^{2}=x^{2}+y^{2}+x y>x^{2}+y^{2}>z^{2} .
$$

Since $x, y$ and $z$ are positive, $x+y>z$. Similarly $x+z>z$
and $y+z>x$. The conclusion is that if $x, y$, and $z$ are positive, and if $x^{2}, y^{2}$, and $z^{2}$ are the lengths of the sides of a triangle, then $x, y$ and $z$ are the length of the sides of $a$ triangle. The problem will be complete if we now show that $a(b+c-a)=(\sqrt{a(b+c-a)})^{2}, \quad b(c+a-b)=(\sqrt{b(c+a-b)})^{2}$, and $c(a+b-c)=(\sqrt{c(a+b-c)})^{2}$ are lengths of the sides of a triangle whenever $a, b$ and $c$ are. However $a>0 \quad b+c>a$ implies $a(b+c-a)>0$. Similarly $b(c+a-b)>0$ and $c(a+b-c)>0$. Finally $a(b+c-a)+b(c+a-b)-c(a+b-c)=c^{2}-(a-b)^{2}$. Since $c>a-b$ then $c^{2}-(a-b)^{2}>0$, and
$a(b+c-a)+b(c+a-b)>c(a+b-c)$. Similarly
$a(b+c-a)+c(a+b-c)>b(c+a-b)$ and
$b(c+a-b)+c(a+b-c)>a(b+c-a)$, proving the problem.
8. Suppose $x, y$ and $z$ satisfy the constraints $x \geq 0, y \geq 0$, $z \geq 0, x+y \leq 2$ and $3 x-y+z \leq 1$. Adding the last two inequalities gives $4 x+z \leq 3$ so that $z \leq 3-4 x$. Since $x \geq 0$, this implies that $z \leq 3$. Since $x+y \leq 2, x+y+z \leq 5$. However the values $x=0, y=2$, and $z=3$ satisfy the given conditions and sum to 5 . Therefore the maximum value of $x+y+z$ subject to the given conditions is 5 .
9. Let $\pi_{1}$ be the plane containing $\ell_{1}, \ell_{2}$ and $\ell_{3}$, and let 0 be the point of intersection of $\ell_{1}, \ell_{2}, \ell_{3}$ and $\ell$. Let $\ell_{4}$ be a line in $\pi_{1}$ bisecting the angle between $\ell_{1}$ and $\ell_{2}$, and let $\pi_{2}$ be the plane through $\ell_{4}$ perpendicular to $\pi_{1}$. We claim that $\Omega$ lies on $\pi_{1}$. Let $P \neq 0$ be a point on $\ell$.

Drop perpendiculars from $P$ to $\ell_{1}$ and $\ell_{2}$ respectively, and suppose these perpendiculars meet $\ell_{1}$ and $\ell_{2}$ at $Q$ and $R$ respectively. $\pi_{2}$ must intersect one of the segments $P R$ and $P Q$ so suppose $\pi_{2}$ intersects $P R$ at the point $X$


Since $\angle P O Q=\angle P O R, \quad \angle O Q P=\angle O R P=90^{\circ}$ and $\overline{O P}=\overline{O P}, \triangle O P Q$ is congruent to $\triangle O P R$. Thus $\overline{O Q}=\overline{O R}$, and $\overline{P Q}=\overline{P R}$. Let $R Q$ intersect $\ell_{4}$ at $Y$. Since $\triangle O Y Q$ is congruent to $\triangle O R Y$, $\overline{Y Q}=\overline{Y R}$. Since $\left\{X Y Q=\left\langle X Y R=90^{\circ}, \quad \overline{X Y}=\overline{X Y} \quad\right.\right.$ and $\overline{Y Q}=\overline{Y R}$, $\triangle X Y R$ is congruent to $\triangle X Y Q$ so $\overline{X R}=\overline{X Q}$. However $\overline{P R}=\overline{P X}+\overline{X R}=\overline{Q P}$, so $\overline{Q P}=\overline{P X}+\overline{Z Q}$. This can only occur if $X=P$, proving that $\ell$ is on $\pi_{2}$. If $\pi_{3}$ is the plane meeting the bisector of the angle between the lines $\ell_{2}$ and $\ell_{3}$, and perpendicular to $\pi_{1}$, then a similar argument shows that $\ell$ is on $\pi_{3}$. Since two distinct planes meet in a line, $\ell$ is thus the intersection of $\pi_{2}$ and $\pi_{3}$ which is perpendicular to $\pi_{1}$.
10. Given: $a z-2 b y+c x=0$

$$
\begin{equation*}
a c>b^{2} \tag{1}
\end{equation*}
$$

Suppose $\quad x z>y^{2}$
From (2) and (3) acxz $>b^{2} y^{2}$
From (1) $(a z+c x)^{2}=4 b^{2} y^{2}$ which by (4) $(a z+c x)^{2}<4 a c x z$. Thus $(a z+c x)^{2}-4 a c x z=(a z-c x)^{2}<0$. This is impossible since $a z-c x$ is a real number. Thus (3) does not hold and $x z-y^{2} \leq 0$.

## Mathematics Council Constitution

FEES
Membership fees may be established by resolution at the annual general meeting of this Council.
v FINANCES
The Executive Committee shall have power to collect fees and to make expenditures. A financial statement shall be submitted to the annual general meeting.
NAME
The name of this organization shall be Mathematics Council of The Alberta Teachers' Association (MCATA).

## OBJECT

The object of this organization shall be to promote and advance the teaching of mathematics throughout the Province, especially in the elementary and secondary schools.

MEMBERSHIP
(a) Any member of The Alberta Teachers' Association or non-member covered by the Teachers' Retirement Fund.
(b) Any certificated teacher in private schools.
(c) Any member of a university in Alberta or Department of Education.

OFFICERS
The officers of this Council shall consist of a president, a vice-president, a past president, a secretary and a treasurer to be elected for a term of one year, by distributed ballot, and a member appointed by the Provincial Executive Council of The Alberta Teachers' Association.

The term of office for elected officers is July 1 to June 30.
II EXECUTIVE COMMITTEE

The Executive Committee shall consist of the officers and the following members to be appointed by the officers:
(a) six directors for a one-year term appointed in such a way as to ensure that at least three members of the executive committee are representatives of each of elementary and secondary teachers, and that at least one member is a representative of each of junior high and senior high teachers,
(b) one representative from the Department of Education for a two-year term,
(c) two representatives from universities in Alberta, one from a Faculty of Education and one from a Department of Mathematics, provided that two different universities are represented, each for a two-year term, and
(d) one newsletter editor who also serves as publicity chairman, for a one-year term.

One member of the Executive Committee shall be designated as representative to the National Council of Teachers of Mathematics.

All appointments terminate as of June 30.
VIII COMMITTEES
The Executive may appoint from time to time such committees as are necessary to carry on work of the Council.

IX LIAISON
Any communication regarding policy which this Council wishes to make with any organization, government department or other agency, within or without the Province, shall be conducted through the Provincial Executive Council of The Alberta Teachers' Association or other regular channels of the Association.

X REPORTS
This Council shall submit annually a written report of its activities and an audited financial statement to The Alberta Teachers' Association by October 1 of each year. The activities reported shall be for the preceding year.

XI AMENDMENTS
After three months' notice of motion to amend the constitution has been given to each member, this constitution may be amended by a two-thirds majority vote of the members present at any annual general meeting of this Council, subject to ratification by the Provincial Executive Council of The Alberta Teachers' Association.

## XII GENERAL MEETINGS

An annual general meeting shall be held each year, and conducted by t'ne officers of the previous year. At least thirty days' notice shall be given for all general meetings.

XIII DISSOLUTION
In the event of dissolution of the Mathematics Council, all funds will be turned over to The Alberta Teachers' Association.

Ratified by Table Officers of Provincial
Executive Council on December 9, 1976.

## Mathematics Council Executive 1976-77

|  | Res / Bus |  | Res/Bus |
| :---: | :---: | :---: | :---: |
| President |  | Director |  |
| Or. K. Allen Neufeld | 436-0571 | Ms. Pat Beeler. | 938-4901 |
| Dept. of Elem. Education | 432-4188 | John Ware Jr. High School | 253-5320 |
| Edmonton, Alberta |  | 10020 - 19 Street S.W. |  |
| T6G 2G5 |  | Calgary, Alberta |  |
|  |  | T2V 1R2 |  |
| Vice President |  |  |  |
| Mr. Robert Holt | 462-1847 | Director |  |
| Dept. of Elementary | 429-5621 | Mr. Francis Somerville | 249-8056 |
| Education |  | Faculty of Education | 284-5465 |
| University of Alberta |  | University of Calgary |  |
| Edmonton, Alberta |  | Calgary T2N 1N4 |  |
| T6G 2G5 |  |  |  |
|  |  | Director |  |
| Past President |  | Ms. Audrey Brattberg | 424-8769 |
| Or. W. George Cathcart | 435-1949 | Stratford Jr High School | 484-3381 |
| Dept. of Elem. Education | 432-4153 | 8715-153 Street |  |
| University of Alberta |  | Edmonton, Alberta |  |
| Edmonton, Alberta |  | T5R 1P1 |  |
| T6G 2E1 |  |  |  |
|  |  | ATA Representative |  |
| Secretary |  | Mr. D.J. Corse | 434-ก487 |
| Dr. Arthur Jorgensen | 723-5370 | 11010-142 Street | 453-2411 |
| Box 2069 | 723-4471 | Edmonton, Alberta |  |
| Edson, Alberta |  | T5N 2R1 |  |
| TOE OPO |  |  |  |
|  |  | Representative |  |
| Treasurer |  | Mr. Arthur Peddicord | 435-7075 |
| Mr. Donald H. Hinde | 782-6849 | Department of Education | 427-2952 |
| Box 741 | 782-3812 | 3rd. F1., Edward's Prof. Bldg. |  |
| Lacombe, Alberta |  | 10053-111 Street |  |
| TOC 1S0 |  | F.dmonton, Alberta T5K 2 H 8 |  |
| Director |  | Representative |  |
| Mr. Earle Warnica | 328-8005 | Mr. John Percevault | 328-1259 |
| Lethbridge School District | 327-4521 | Faculty of Education | 329-2257 |
| \#51. 433-15 Street S. |  | University of Lethbridge |  |
| Lethbridge, Alberta |  | Lethbridge, Alberta |  |
| T1J 225 |  | TlK 3M4 |  |
| Director |  | Representative |  |
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## Mathematics Council, ATA $17^{\text {th }}$ Annual Conference

## October 14 \& 15 - Red Deer Lodge

Feature Speakers \& Sessions-
ROBERT WIRTZ, Lecturer and writer from Carmel, California, will present the Friday evening session and a session for elementary teachers on Saturday.

DR. ERIC MACPHERSON, Dean of the Faculty of Education, University of Manitoba, will present the luncheon address and a session for secondary teachers.

Idea Exchanges - nine mini-sessions at all levels.
Pre-Registration - (Prjor to September 30)
Detach and mail to: D. Hinde, Box 741 Lacombe, Alberta, TOC 1S0

MATHEMATICS COUNCIL, ATA—— $17^{\text {th }}$ ANNUAL CONFERENCE

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CONFERENCE REGISTRATION
(including luncheon):

| BEFORE | Sept. 30 | - | AFTER |
| :---: | :---: | :---: | :---: |
| MCATA Member | \$16.00 |  | \$18.00 |
| Non-Member (incl. | \$23.50 |  | \$25.50 | membership)

Student Members $\$ 7.00 \ldots \$ 9.00$
Non-Members (incl. \$11.00 $\quad \$ 13.00$ membership)
Additional luncheon \$ 7.00 each $\qquad$ tickets
MCATA MEMBERSHIP
Regular \$7.50
Student \$4.00
New $\qquad$ , Renewal $\qquad$
NCTM MEMBERSHIP (Optional)
With Arithmetic Teacher \$13.00
With Mathematics Teacher \$13.00
With Both Journals \$20.00
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