Rhombi on the Hundred Square: Sums, Patterns, and Proof

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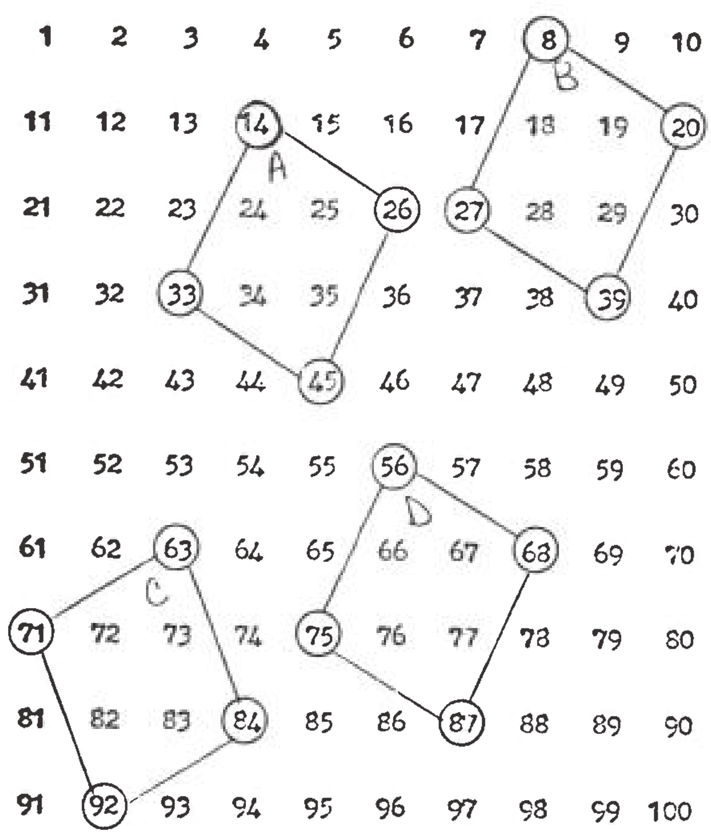
The Hundred Square is a rich source of number patterns. Seeking these patterns gives the student experience in formalizing and verifying conjectures. A valuable by-product is the maintenance of computational skills.

Activity I: Consider rhombi drawn on the Hundred Square as shown in Figure I.

Figure I



1. Add the four numbers which represent the vertices of each rhombus; call the sum V.

2. Add the four interior numbers of each rhombus; call this sum I.

The results of the above calculations appear in Table I.

Table I

Rhombus	V	I
A	118	118
B	94	94
C	310	310
D	286	286

Observe that for each rhombus, the sums V and I are the same; the ratio of these corresponding sums is 1/1. To see if this pattern is true for all rhombi of this size, check other rhombi positioned differently on the Hundred Square.

Activity II: Consider the rhombi drawn on Figures II and III. Note that these rhombi have three circled numbers per side.

Figure II

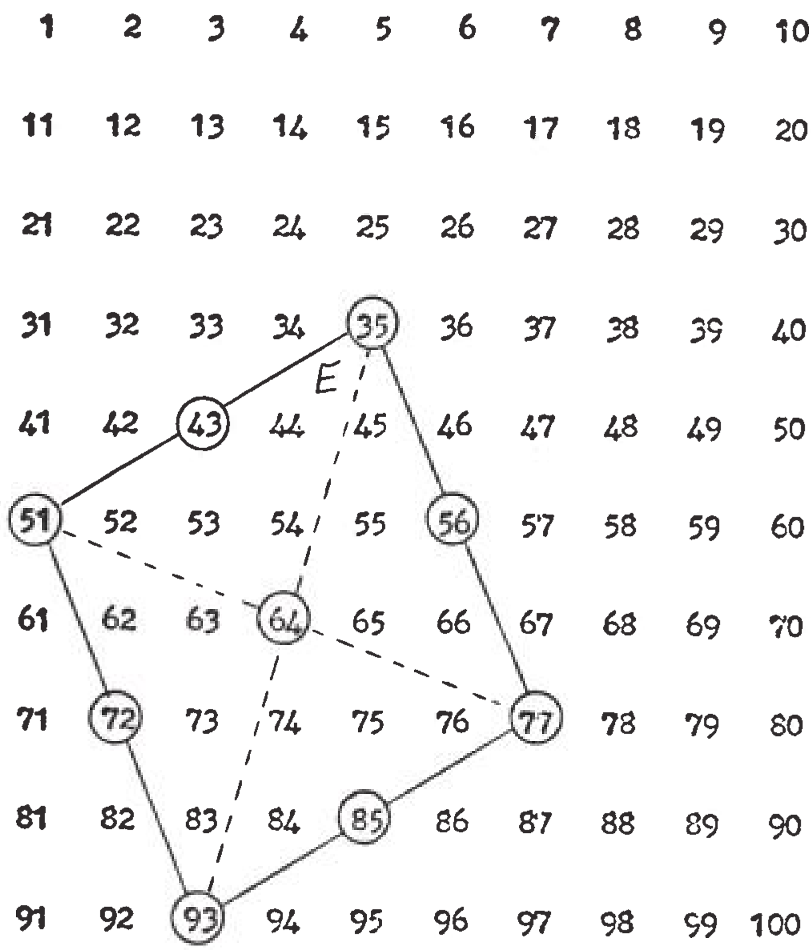
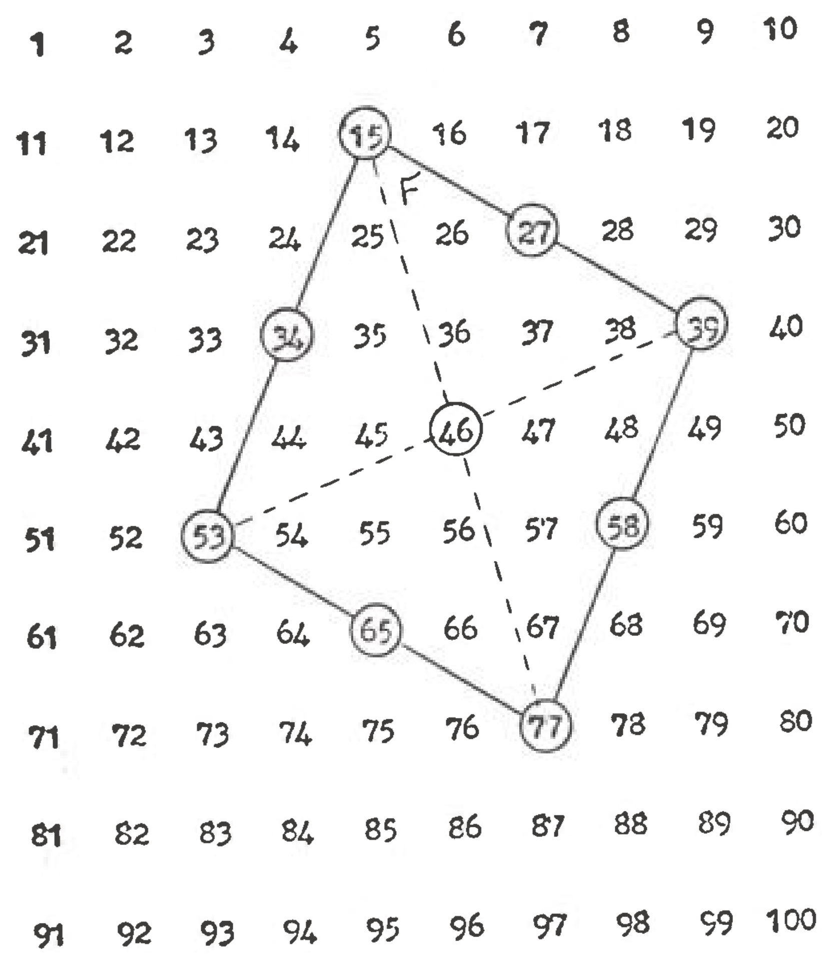

1 2 3 4 5 6 7 8 9 10
11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30
31 32 33 34 35 36 37 38 39 40
E
41 42 43 44 45 46 47 48 49 50
51 52 53 54 55 56 57 58 59 60
61 62 63 64 65 66 67 68 69 70
71 72 73 74 75 76 77 78 79 80
81 82 83 84 85 86 87 88 89 90
91 92 93 94 95 96 97 98 99 100

Figure III



1. Add the four numbers which represent the vertices of each rhombus; call this sum V.

2. Add the eight numbers which lie on the perimeter; call this sum P.

3. Add the seventeen interior numbers; call this sum I.

4. Let C be the "center number" of each rhombus.

5. Find I - C.

The results of these calculations appear in Table II.

Table II

Rhombus	V	P	I	C	I - C
E	256	512	1088	64	1024
F	184	368	782	46	736

Observations:

1. $V = 4C$ or $\frac{V}{C} = \frac{4}{1}$; the sum of the vertices is four times the center number.

2. $\frac{P}{I} = \frac{8}{17}$. Both 512/1088 and 368/782 can be renamed 8/17. Note that there are eight numbers on the perimeter of the rhombus and 17 numbers in the interior of the rhombus.

3. $\frac{I - C}{C} = \frac{16}{1}$. Both 1024/64 and 736/46 can be renamed 16/1. Note that there are 16 numbers in the interior of the rhombus excluding the center number.

Be sure to check these patterns for other rhombi of this size positioned elsewhere on the Hundred Square.

We shall verify pattern 3; that is, $\frac{I - C}{C} = 16$.

Figure IV: A General Hundred Square

1	11	. . .	b			
11						
.						
.						
10a+1			10a+b			

10a+b	10a+(b+1)	10a+(b+2)	10a+(b+3)	10a+(b+4)	10a+(b+5)	10a+(b+6)
10(a+1)+b	10(a+1)+(b+1)	10(a+1)+(b+2)	10(a+1)+(b+3)	10(a+1)+(b+4)	10(a+1)+(b+5)	10(a+1)+(b+6)
10(a+2)+b	10(a+2)+(b+1)	10(a+2)+(b+2)	10(a+2)+(b+3)	10(a+2)+(b+4)	10(a+2)+(b+5)	10(a+2)+(b+6)
10(a+3)+b	10(a+3)+(b+1)	10(a+3)+(b+2)	10(a+3)+(b+3)	10(a+3)+(b+4)	10(a+3)+(b+5)	10(a+3)+(b+6)
10(a+4)+b	10(a+4)+(b+1)	10(a+4)+(b+2)	10(a+4)+(b+3)	10(a+4)+(b+4)	10(a+4)+(b+5)	10(a+4)+(b+6)
10(a+5)+b	10(a+5)+(b+1)	10(a+5)+(b+2)	10(a+5)+(b+3)	10(a+5)+(b+4)	10(a+5)+(b+5)	10(a+5)+(b+6)
10(a+6)+b	10(a+6)+(b+1)	10(a+6)+(b+2)	10(a+6)+(b+3)	10(a+6)+(b+4)	10(a+6)+(b+5)	10(a+6)+(b+6)

The sum I - C is depicted as follows:

$$\begin{array}{l}
10(a + 1) + (b + 2)\\
10(a + 1) + (b + 3)\\
10(a + 2) + (b + 2)\\
10(a + 2) + (b + 3)\\
10(a + 2) + (b + 4)\\
10(a + 2) + (b + 5)\\
10(a + 3) + (b + 1)\\
10(a + 3) + (b + 2)\\
10(a + 3) + (b + 4)\\
10(a + 3) + (b + 5)\\
10(a + 4) + (b + 1)\\
10(a + 4) + (b + 2)\\
10(a + 4) + (b + 3)\\
10(a + 4) + (b + 4)\\
10(a + 5) + (b + 3)\\
10(a + 5) + (b + 4)\\
\hline
160a + 480 + (16b + 48)
\end{array}$$

$$\begin{aligned}
\frac{I - C}{C} &= \frac{160a + 480 + 16b + 48}{10(a + 3) + (b + 3)}\\
&= \frac{16(10a + b + 33)}{10a + b + 33}\\
&= 16
\end{aligned}$$

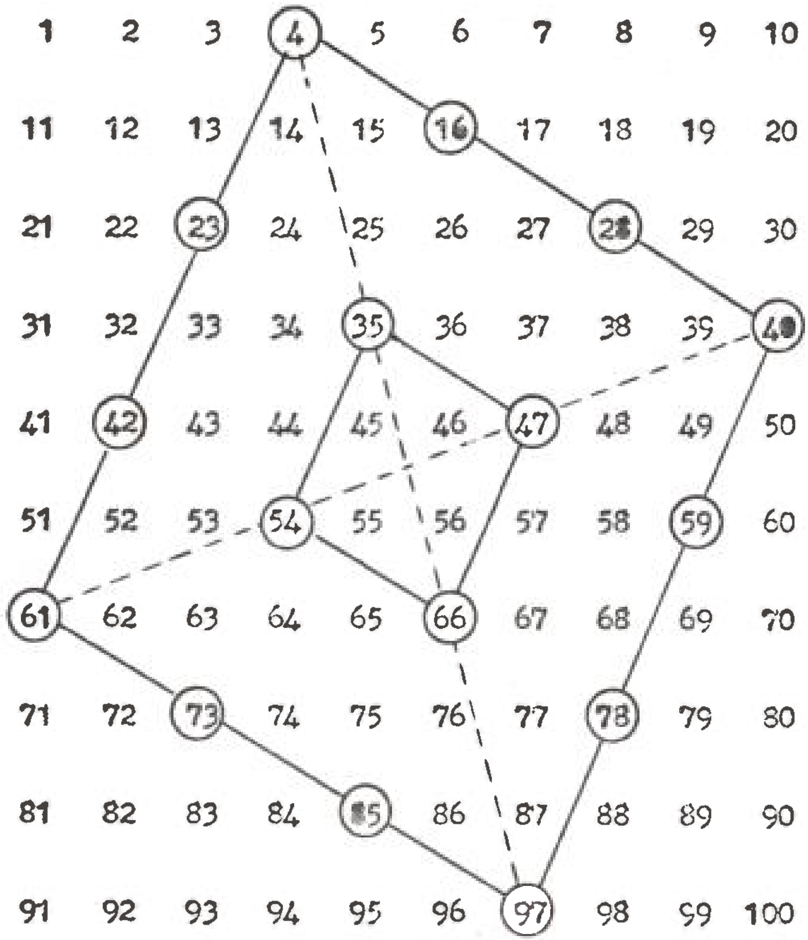
Questions for the reader and his/her students:

1. Do the patterns found in Activities I and II hold on the Addition Table, Multiplication Table, and the Subtraction Table?

2. Can you prove the other patterns?

3. Can you find similar patterns on the following nested rhombus configuration? There are four circled numbers per side. There is no center number; there is a center rhombus.

Figure V


1 2 3 4 5 6 7 8 9 10
11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30
31 32 33 34 35 36 37 38 39 40
41 42 43 44 45 46 47 48 49 50
51 52 53 54 55 56 57 58 59 60
61 62 63 64 65 66 67 68 69 70
71 72 73 74 75 76 77 78 79 80
81 82 83 84 85 86 87 88 89 90
91 92 93 94 95 96 97 98 99 100