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THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

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VOLUME XVII, NUMBER 2, DECEMBER 1977

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From the Editor

Our Apologies for Errors of Commission and Omission

1. Don Hines, treasurer, is the reporter of the NCTM Cincinnati annual meeting notes instead of Bob Holt, president, as given in the September 1977 issue. Both were in attendance and both are capable of fine reporting. Therefore, the error was naturally an easy one to manage.

2. The error of omission was the failure to give *Vector,* the British Columbia Newsletter/Journal, credit for being the source and original publisher of the lesson plans on pages 22 and 23 of the September 1977 issue (Vol. XVII, No. 1).

Looking into the Future

Articles from the 1977 annual conference in Red Deer will be the feature of our February issue. The report on the Math Olympiad will be one of the features in May. Any articles relating to the theme of math contests and/or articles of special interest to senior high level teachers will be featured in this issue.

Ideas for use in your classrooms will be features of every issue. First preference will be given to Alberta teachers sharing with each other simple ideas or more complex ideas that are pertinent to aiding in the enrichment of the curriculum. However, we will use ideas from other sources that may not be as directly related to our course of study but are, nevertheless, equally useful when properly applied to our individual classrooms.

The next NCTM annual meeting is in San Diego, April 12 to 15, 1978. Remember that Bob Holt, our president, has assisted in providing an outstanding program as a member of the program committee.

In this Issue

Lyle Pagnucco, director, is chairman of our "name-of-site" meeting in Calgary in 1979. This is a joint meeting with NCTM that attracts attendance from Western Canada and Northern USA, with international speakers of a quality and number that we are unable to obtain for our regular annual meetings due to difference in size of meetings.

Needed from the Membership

... articles that share your thoughts and ideas with all of us. These may include ideas on "how to teach," "what to teach," "why teach ... ," or on a "pet" subject such as "Pythagoras Theorem - My Approach" or "Pythagoras Theorem - Why Teach It at All."

We need your ideas and "pet" ways of winning math contests. Some of you have success regularly. Some of us never get out of the lower ranks. How about helping us improve? Or, should we place less emphasis on winning contests and more on "basic skills"? Can we prepare for contests and develop basic skills at the same time? Will success in contests correlate with success in life and/or in university life? Many are answering that we can succeed in teaching while succeeding in contests since both challenge our students to acquire the same skills, knowledge, and wisdom. If you agree, you need to share the techniques of your success with those of us still lost in the forest. If you disagree, you need to tell us why, with logical arguments that even a philosophy professor would find worthy of consideration and which we need as much as a "pat-on-the-back."

We need your ideas on "how to teach" one small concept with a game, puzzle, or any other method not routine that works. Commercial ideas are plentiful, but we often find that our own approach is even better. We all know one and one equal two, eleven or ten depending on what restrictions we place on the problem. How do we show Grade I pupils the difference between the sum and the placing of the numerals side-by-side, or how do we explain base two to the junior high student preparing for a career in computer science? How many Grade XII students need more than just the statement one (base 2) + one (base 2) = 10 (base 2) before they can work the problems of this type with understanding? Get the idea? Send in your ideas on any concept now!

You will soon be receiving a new *Monograph*. In 1978 you will receive a monograph on reading in math. (The exact title will be given by Dr. John Percevault, editor, at his convenience or at the time of publication.) During the next six months, your executive will need two things: an editor and a theme for the 1979 monograph. What is the most valuable item or items you can think of for our 1979 theme? Do you have an editor to suggest? We will choose either or both from our own collective ideas unless you help. We prefer *your* theme to one *we* choose since we know yours will be pertinent and we can only say in our limited wisdom that ours is what you want and need. "Don't make us become dictators even as a presidium."

Determinants and the Coordinate Plane

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The purpose of this paper is to demonstrate the use of determinants as a tool for the development of aspects of high school geometry and trigonometry. Although this presentation does not presuppose a knowledge of determinants, prior contact with the subject (e.g. study of the properties, Cramer's rule, determinant equations) will unquestionably enhance the study. The writer experimented with such an introduction to determinants for several years as a ninth and tenth year mathematics teacher, and found this approach valuable from several points of view. Primarily, the mathematics is interesting, highly motivating, and lends itself well to student discovery. Secondly, determinants can be presented profitably to the slow learner as well as to the more capable student, since the expansion of a determinant reinforces the fundamental operations in an almost game atmosphere. This reinforcement may involve directed numbers, monomials and polynomials as elements of a determinant. Moreover, problems dealing with determinant factoring, the solution of linear and quadratic determinant equations, have proven fascinating to students at this level. Thirdly, the geometric and trigonometric applications in this algebraic environment provide the student with a real sense of continuity and unity of subject matter. Finally, determinants are important to the development of branches of higher mathematics. Indeed, if we can lay the foundation of this concept while mastering aspects of high school mathematics, it seems we ought to do so.

In order to position our theory sequentially within the framework of our geometry, we first postulate the area relationship of a rectangle as the product of a pair of adjacent sides, viz., $K_{Rect} = bh$. Since a diagonal of a rectangle divides its area into two right triangles of equal area, it follows that $K_{Rt\Delta} = \frac{1}{2} bh$.

Our first major objective will be to derive the (determinant) area formula for a triangle in the coordinate plane. To begin with, we define second- and third-order determinants as square arrays of elements of the form

$$\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} \quad \text{and} \quad \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

whose values are $x_1y_2 - x_2y_1$ and $x_1y_2z_3 + x_2y_3z_1 + x_3y_1z_2 - x_1y_3z_2 - x_2y_1z_3 - x_3y_2z_1$, respectively.

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the coordinates of the vertices of any triangle ABC in the coordinate plane (see Fig. 1). Through vertices A, B and C, draw lines parallel to the coordinate axes forming rectangle DBEF, so that

$$K_{\Delta ABC} = K_{DBEF} - K_{\Delta I} - K_{\Delta II} - K_{\Delta III}$$

Also,

$$K_{DBEF} = (x_2 - x_1)(y_3 - y_2) = x_2y_3 - x_2y_2 - x_1y_3 + x_1y_2$$

$$K_{\Delta I} = \tfrac{1}{2}(x_2 - x_1)(y_1 - y_2) = \tfrac{1}{2}(x_2y_1 - x_2y_2 - x_1y_1 + x_1y_2)$$

$$K_{\Delta II} = \tfrac{1}{2}(x_2 - x_3)(y_3 - y_2) = \tfrac{1}{2}(x_2y_3 - x_2y_2 - x_3y_3 + x_3y_2)$$

$$K_{\Delta III} = \tfrac{1}{2}(x_3 - x_1)(y_3 - y_1) = \tfrac{1}{2}(x_3y_3 - x_3y_1 - x_1y_3 + x_1y_1)$$

Substituting and collecting terms, we get

$$K_{\Delta ABC} = \tfrac{1}{2}(x_1y_2 + x_2y_3 + x_3y_1 - x_1y_3 - x_2y_1 - x_3y_2)$$

which, by the definition of third-order determinant, may be expressed as

$$K_{\Delta ABC} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \quad [A]$$

This relationship produces a positive or negative area as we take the coordinates in a counterclockwise or clockwise order, respectively. To demonstrate the point, take a conveniently placed unit triangle whose coordinates are (0, 0), (0, 1) and (1, 0) as in Fig. 2. Starting at the origin, and taking the points clockwise

$$K_{\Delta} = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = -\frac{1}{2}$$

If, however, we take the points counterclockwise, then

$$K_{\Delta} = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = +\frac{1}{2}$$

Since in this elementary treatment of area we will be concerned with positive areas only, we will adopt the convention throughout of taking the points in a counterclockwise direction.[1*]

We now direct our attention to the area of the quadrilateral in the coordinate plane. Given four distinct points $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ and $D(x_4, y_4)$, vertices of convex quadrilateral ABCD. If diagonal AC is drawn, two triangles are formed, the sum of whose areas will be the area of the quadrilateral (see Fig. 3), that is

$K_{ABCD} = K_{\Delta ABC} + K_{\Delta ACD}$.

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*Footnote references will be found at the conclusion of the article.

By [A]

$$K_{\Delta ABC} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \quad \text{and} \quad K_{\Delta ACD} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \\ x_4 & y_4 & 1 \end{vmatrix}$$

Expanding these determinants and substituting,

$$K_{ABCD} = \tfrac{1}{2}[(x_1y_2 + x_3y_1 + x_2y_3 - x_3y_2 - x_2y_1 - x_1y_3) + (x_1y_3 + x_4y_1 + x_3y_4 - x_4y_3 - x_3y_1 - x_1y_4)]$$

$$= \tfrac{1}{2}[x_1(y_2 - y_4) + x_2(y_3 - y_1) + x_3(y_4 - y_2) + x_4(y_1 - y_3)]$$

$$= \tfrac{1}{2}[(x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3)]$$

The binomial in the brackets is of the form ad - bc, which by the definition can be written in the determinant form

$$K_{ABCD} = \frac{1}{2}\begin{vmatrix} x_1 - x_3 & y_1 - y_3 \\ x_2 - x_4 & y_2 - y_4 \end{vmatrix}$$

or more compactly

$$K_{ABCD} = \frac{1}{2}\begin{vmatrix} \Delta x_1 & \Delta y_1 \\ \Delta x_2 & \Delta y_2 \end{vmatrix} \quad [B]$$

where the Δ-elements are the differences of the respective x- and y-coordinates of the endpoints of the diagonals of the quadrilateral.

ILLUSTRATIVE EXAMPLE: Show by coordinate methods and determinants the area of a parallelogram is equal to base times height (bh).

Place the parallelogram ABCD in the coordinate plane so that one vertex is at the origin, and one side lies along the x-axis as shown in the figure. The vertices may therefore be expressed as A(0, 0), B(b, 0), C(a + b, h) and D(a, h), where b is a base and h the altitude. (See Fig. 4)

By [B]

$$K_{ABCD} = \frac{1}{2}\begin{vmatrix} (a+b)-0 & h-0 \\ a-b & h-0 \end{vmatrix}$$

$$= \tfrac{1}{2}(bh + ah - ah + bh) = bh$$

Following is a sampling of problems that lend themselves well to solution by this technique:

1) Prove the area of a trapezoid is equal to $\frac{1}{2}h(b_1 + b_2)$.

2) Prove: A line which joins the midpoints of two sides of a triangle cuts off a triangle whose area is one-fourth the area of a given triangle.

3) Prove that a median of a triangle divides it into two triangles of equal area.

4) Prove that a diagonal of a parallelogram divides it into two triangles of equal area.

Since our quadrilateral area formula turns out to be a second-order determinant relationship, we are motivated to re-examine the notion of triangle area. Our purpose is to derive a second-order determinant relationship in places of the third-order form with which we have been involved up to now.

Accordingly, we consider the area of ΔABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ as shown in Fig. 5. Let P be any other point with coordinates (x_p, y_p). We first examine the area of quadrilateral ABPC.

By [B]

$$K_{ABPC} = \frac{1}{2}\begin{vmatrix} x_1 - x_p & y_1 - y_p \\ x_2 - x_3 & y_2 - y_3 \end{vmatrix}$$

Move point P to coincide with point B so that a double point exists at B with coordinates (x_2, y_2). The resultant figure is considered from two points of view: (1) as a triangle, viz., ΔABC, and (2), as a degenerated quadrilateral, viz., quad AB(P)C. Notice that side PC of quad AB(P)C falls on BC, which exists here in the dual capacity as a side of the triangle, and as a diagonal of the quadrilateral. Also, diagonal AP falls on side AB. Since the areas of the two figures, ΔABC and quad AB(P)C, are identical, we may write

$$K_{\Delta ABC} = K_{AB(P)C}$$

Relationship [B] applied to the right-hand member results in a new statement for the area of the triangle, that is

$$K_{\Delta ABC} = \frac{1}{2}\begin{vmatrix} x_1 - x_p & y_1 - y_p \\ x_2 - x_3 & y_2 - y_3 \end{vmatrix}$$

Since the vertex P has been moved to B, it follows that $x_p = x_2$ and $y_p = y_2$ and we may rewrite this last relationship in the form

$$K_{\Delta ABC} = \frac{1}{2}\begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_2 - x_3 & y_2 - y_3 \end{vmatrix}$$

or more compactly

$$K_{\Delta ABC} = \frac{1}{2}\begin{vmatrix} \Delta x_1 & \Delta y_1 \\ \Delta x_2 & \Delta y_2 \end{vmatrix} \quad [C]^2$$

While the right-handed members of [B] and [C] are apparently identical, it must be made clear that in the former, the Δ-elements represent the differences in the x- and y-coordinates of the end-points of the <u>diagonals</u>, whereas in the latter, the Δ-elements refer to the differences in the x- and y-coordinates of the end-points of the <u>sides</u> of the triangle, taken counterclockwise.

ILLUSTRATIVE EXAMPLE: Find the area of the triangle whose vertices are A(1, 0), B(4, 7) and C(6, -2).

$$K_{\Delta ABC} = \frac{1}{2}\begin{vmatrix} 3 & 7 \\ -5 & 2 \end{vmatrix} = \tfrac{1}{2}(6 + 35) = 20\tfrac{1}{2}$$

We now employ the results of the previous section in the development of the linear equation in the determinant form by considering three collinear points as the vertices of a triangle with zero area.

Given two points $A(x_1, y_1)$ and $B(x_2, y_2)$. We wish to consider the equation of the straight line which contains the two points A and B. Take any other point P(x, y) on this line, and consider the figure ABP as a triangle with zero area. It follows from [C] that

$$\begin{vmatrix} \Delta x & \Delta y \\ \Delta x_1 & \Delta y_1 \end{vmatrix} = 0 \quad [D]$$

where $\Delta x = x - x_1$, and $\Delta x_1 = x_1 - x_2$, etc.

ILLUSTRATIVE EXAMPLE: Write the equation of the line which contains the points A(1, - 3) and B(5, 6).

Consider any other point P(x, y) contained in the line AB.

By [D] $\begin{vmatrix} x - 1 & y - (-3) \\ 1 - 5 & -3 - 6 \end{vmatrix} = 0$ Expressed in standard form $9x - 4y - 21 = 0$.

Another application of this concept is the problem of writing the equation of the line through a point $P(x_3, y_3)$ parallel to the line containing the points $A(x_1, y_1)$ and $B(x_2, y_2)$ as shown in fig. 5.

Consider any other point Q(x, y) contained in the required line.

Since

$QP \parallel AB$, then $K_{\Delta ABQ} = K_{\Delta ABP}$.

By relationship [B], $\begin{vmatrix} \Delta x & \Delta y \\ \Delta x_1 & \Delta y_1 \end{vmatrix} = \begin{vmatrix} \Delta x_1 & \Delta y_1 \\ \Delta x_2 & \Delta y_2 \end{vmatrix}$ [E][3]

where $\Delta x = x - x_1$, $\Delta x_1 = x_1 - x_2$, etc.

ILLUSTRATIVE EXAMPLE: Write the equation of the line which passes through the point (3, 2), and is parallel to the line containing the points (-3, 0) and (4, -2).

By [E] $\begin{vmatrix} x - (-3) & y - 0 \\ -3 - 4 & 0 - (-2) \end{vmatrix} = \begin{vmatrix} -3 - 4 & 0 - (-2) \\ 4 - 3 & -2 - 2 \end{vmatrix}$

Expressed in standard form, $2(x + 3) + 7y = 28 - 2$

$$2x + 7y = 20$$

We now direct our attention to the applications of determinants to the subject of trigonometry, more specifically, to the derivation of the trigonometric sum formulas by use of determinants.

Consider the unit circle with the center at the origin, with the angles x, y, and x-y in standard position, and whose rays intersect the circle at A(1, 0), B(cos(x-y), sin(x-y)), C(cos y, sin y) and D(cos x, sin x), as shown in fig. 7. Since angle DOC is equal to x-y, it follows that triangles BOA and DOC are congruent, and thus equal in area. Hence we may write

$$K_{\Delta BOA} = K_{\Delta DOC}$$

By [C]

$$\begin{vmatrix} \cos(x-y) - 0 & \sin(x-y) - 0 \\ 0 - 1 & 0 - 0 \end{vmatrix} = \begin{vmatrix} \cos x - 0 & \sin x - 0 \\ 0 - \cos y & 0 - \sin y \end{vmatrix}$$

so that

$$\begin{vmatrix} \cos(x-y) & \sin(x-y) \\ -1 & 0 \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\cos y & -\sin y \end{vmatrix}$$

which, when expressed in standard form, produces

$$\sin(x-y) = \sin x \cos y - \cos x \sin y.$$

To derive the cosine difference formula we consider triangles DOB and COA in Fig. 7. As in the previous case, these triangles are congruent, and thus equal in area. Hence we may write

$$K_{\Delta DOB} = K_{\Delta COA}$$

By [C]

$$\begin{vmatrix} \cos x - 0 & \sin x - 0 \\ 0 - \cos(x-y) & 0 - \sin(x-y) \end{vmatrix} = \begin{vmatrix} \cos y - 0 & \sin y - 0 \\ 0 - 1 & 0 - 0 \end{vmatrix}$$

so that

$$\begin{vmatrix} \cos x & \sin x \\ -\cos(x-y) & -\sin(x-y) \end{vmatrix} = \begin{vmatrix} \cos y & \sin y \\ -1 & 0 \end{vmatrix}$$

Expressing this in standard form

$$(-\sin(x-y))\cos x + (\cos(x-y))\sin x = \sin y.$$

Since $\sin(x-y) = \sin x \cos y - \cos x \sin y$, we get

$$-(\sin x \cos y - \cos x \sin y)\cos x + (\cos(x-y))\sin x = \sin y$$

and

$$(\cos(x-y))\sin x = \sin y + \sin x \cos x \cos y - \cos^2 x \sin y$$

$$= \sin x \cos x \cos y + \sin y \sin^2 x$$

so that $\cos(x-y) = \cos x \cos y + \sin x \sin y.$

The sum formulas for $\sin(x + y)$ and $\cos(x + y)$ are derived in appropriately analogous ways (see fig. 8).

REFERENCES

1. For a complete discussion of positive and negative areas and volumes see Felix Klein, Elementary Mathematics From an Advanced Standpoint-Geometry, Dover Publications, Inc. 1939, pp. 3-9.

2. This result is a geometric interpretation of the elementary properties of determinants, which allow for the subtraction of rows (or columns) and the reduction of an nth order determinant to an (n - 1)th order determinant, viz.,

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} x_1 - x_2 & y_1 - y_2 & 0 \\ x_2 - x_3 & y_2 - y_3 & 0 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_2 - x_3 & y_2 - y_3 \end{vmatrix}$$

See also <u>Higher Algebra</u>, Hall and Knight, Macmillan and Company, Ltd. 1946, pp. 415-16.

3. If, based on the area equality, we write [E] in an equivalent form

$$\begin{vmatrix} \Delta x & \Delta y \\ \Delta x_1 & \Delta y_1 \end{vmatrix} = \begin{vmatrix} x_3 - x_1 & y_3 - y_1 \\ \Delta x_1 & \Delta y_1 \end{vmatrix}$$

so that

$$\begin{vmatrix} \Delta x & \Delta y \\ \Delta x_1 & \Delta y_1 \end{vmatrix} - \begin{vmatrix} x_3 - x_1 & y_3 - y_1 \\ \Delta x_1 & \Delta y_1 \end{vmatrix} = 0,$$

and if we apply the property of determinants

$$\begin{vmatrix} a & b \\ e & f \end{vmatrix} - \begin{vmatrix} c & d \\ e & f \end{vmatrix} = \begin{vmatrix} a - c & b - d \\ e & f \end{vmatrix}$$

then [E] may be restated in the simpler form

$$\begin{vmatrix} x - x_3 & y - y_3 \\ \Delta x_1 & \Delta y_1 \end{vmatrix} = 0$$

It is suggested the reader do the illustrative problem using this relationship.

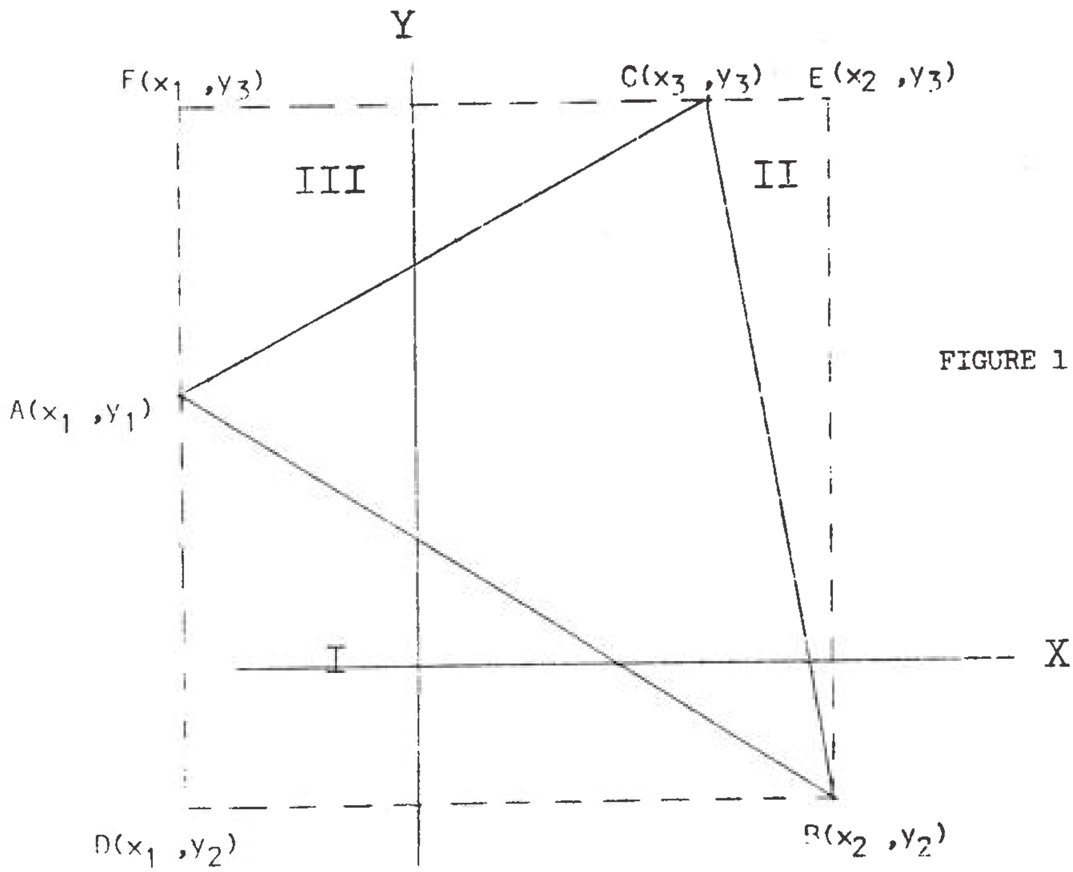


FIGURE 1

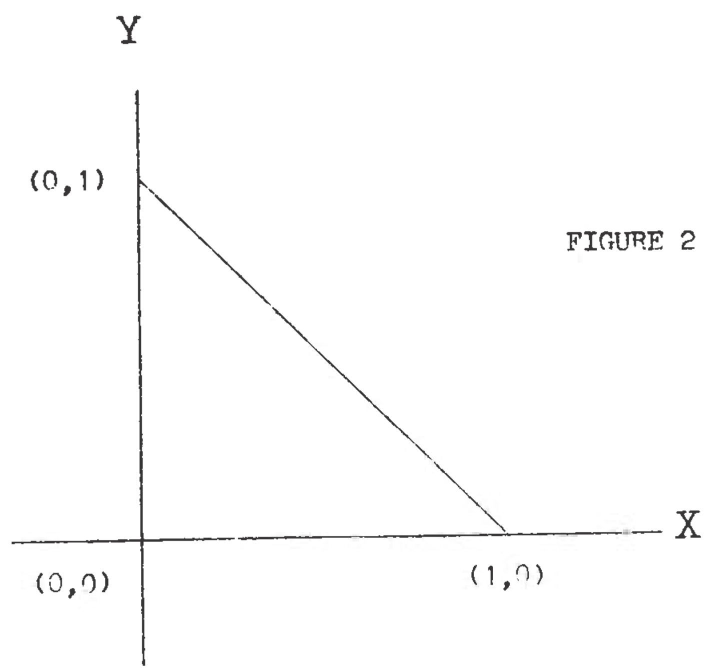


FIGURE 2

FIGURE 3

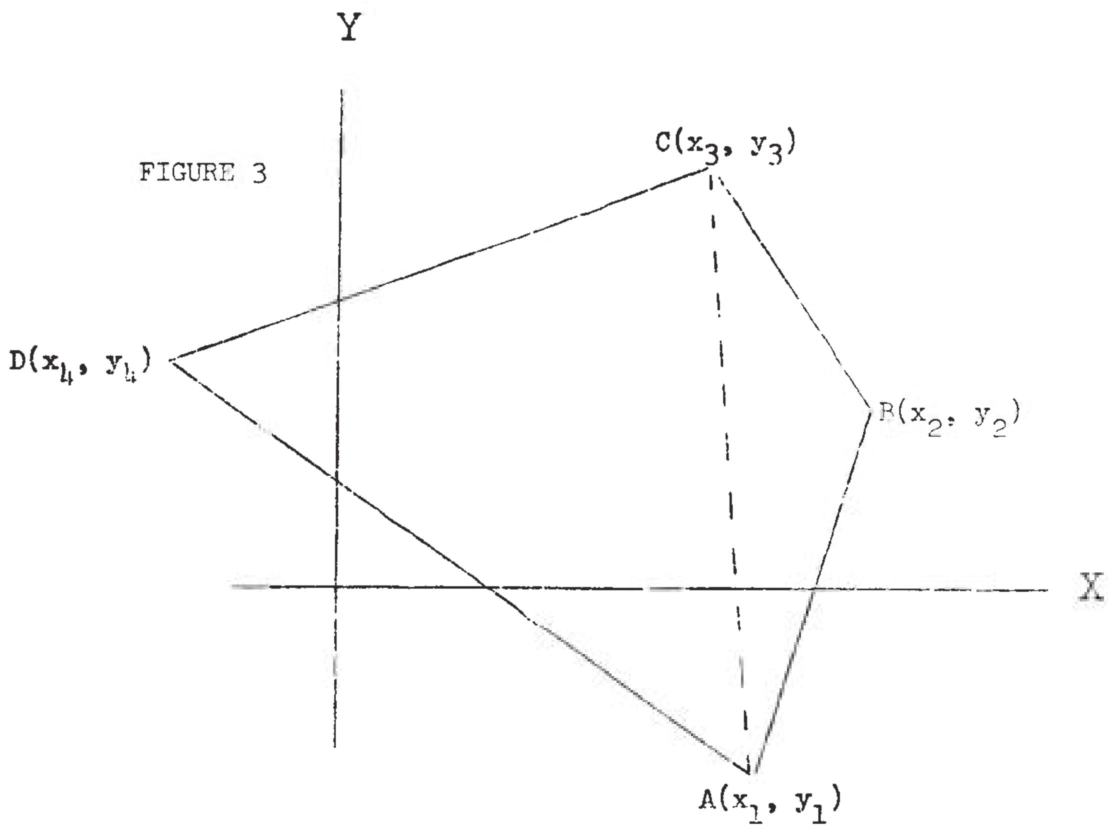
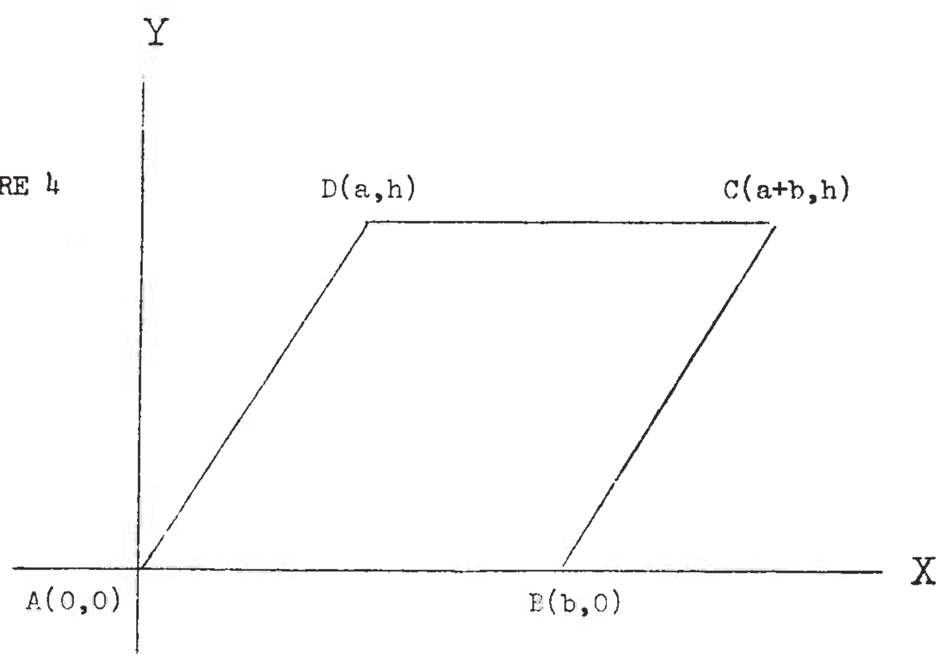


FIGURE 4



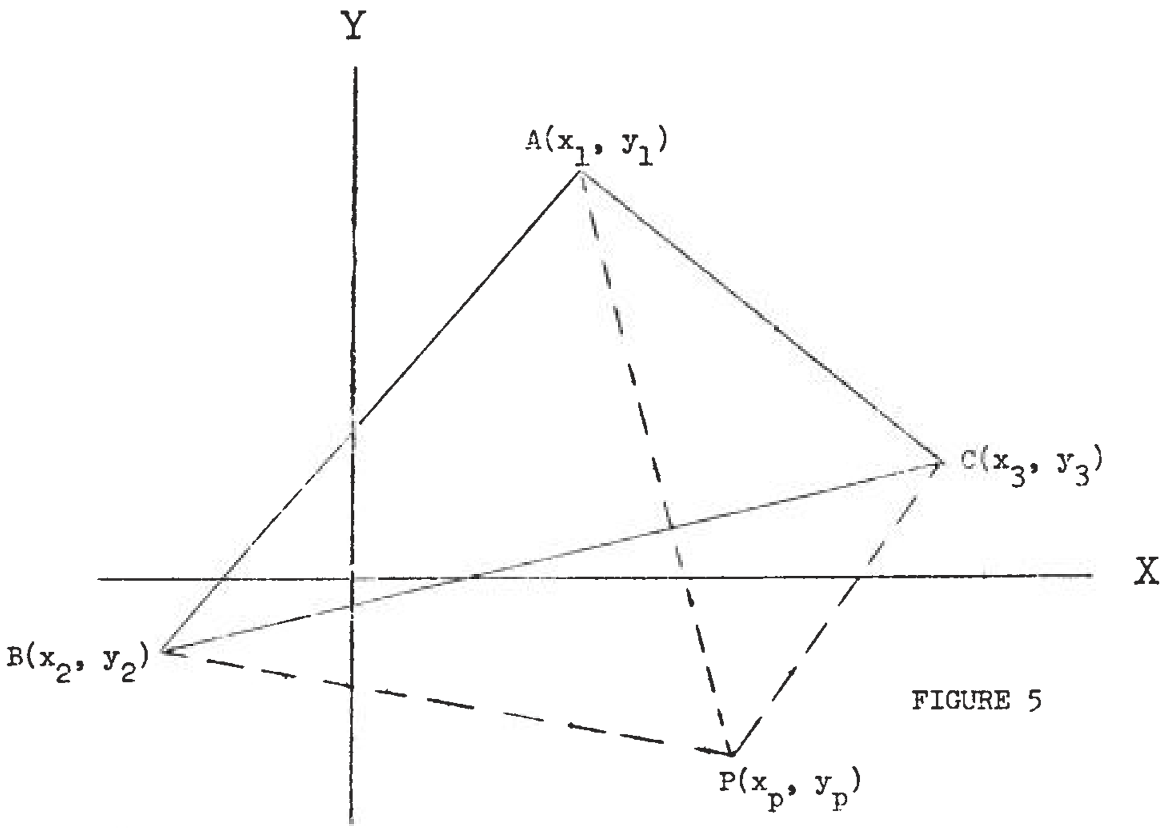


FIGURE 5

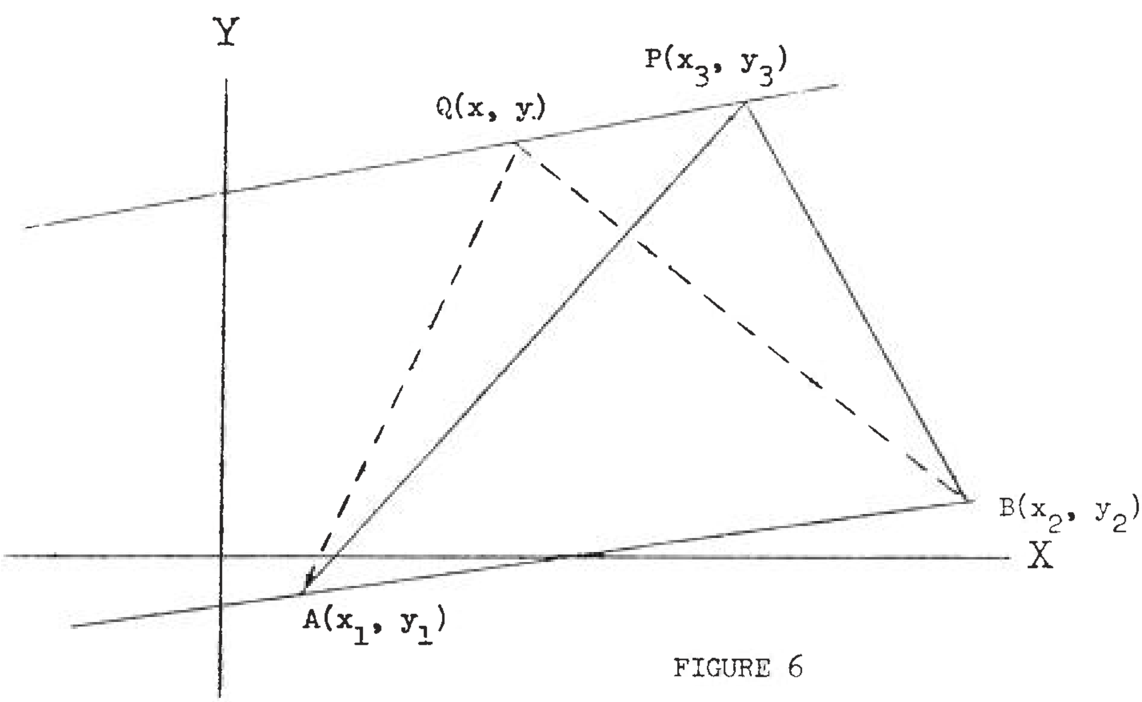
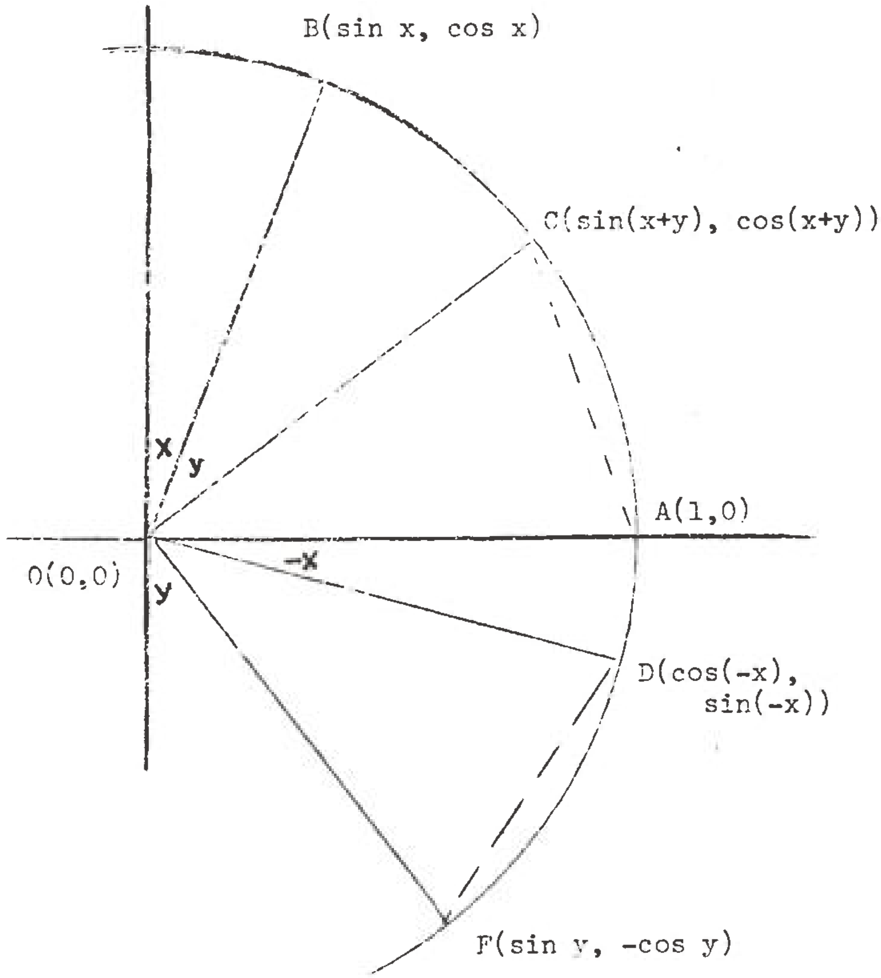


FIGURE 6

FIGURE 8



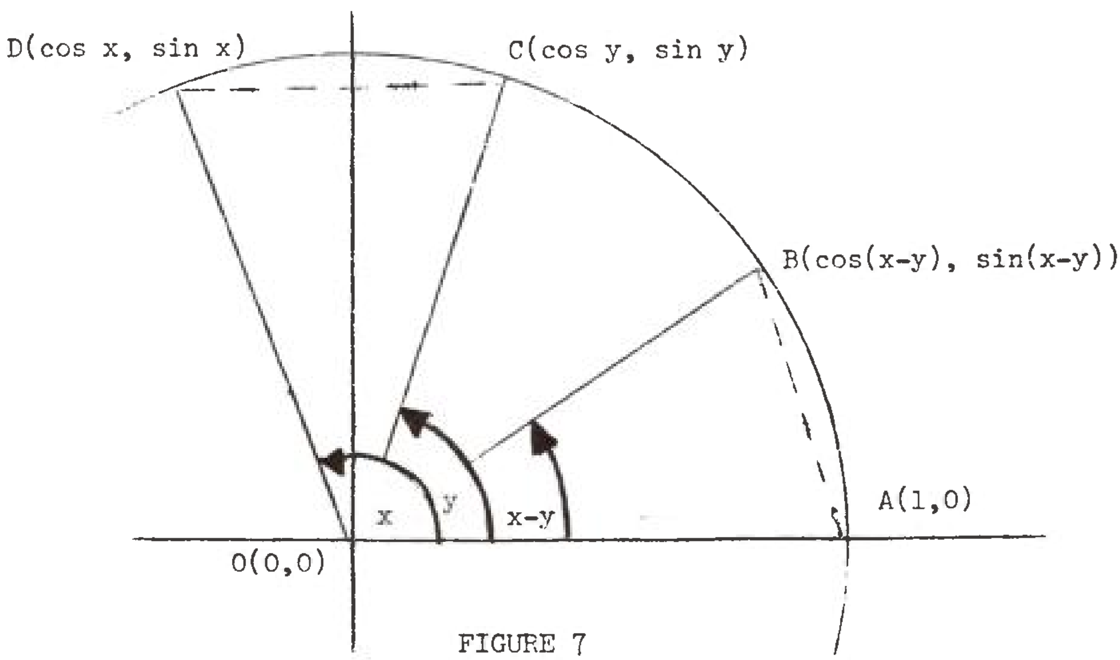


FIGURE 7

Lesson Plans

Reprinted from Vector, *Newsletter/Journal of the British Columbia Association of Mathematics Teachers.*

LESSON PLAN

TANGRAMS

by Dennis Hamaguchi, W.L. Seaton School, Vernon

Fellow Mathgents:

Our superiors have requested a 'mission impossible' to save our country Geometrica from the invaders of Goo Gol Lee. Our people have a hint of the coming danger – a shattered clay tablet was discovered near the capital. Our task is to piece the tablet back to its original shape to avert the danger. This message will not, I repeat, will not, self-destruct in any number of seconds. Good Luck!

Plan:

1. Construct a 15 centimetre square. Label it ABCD.
2. Draw the diagonal $\overline{AC}$.
3. By construction, determine the midpoint of $\overline{AB}$ (call it T), and the midpoint of $\overline{BC}$ (call it U). Join $\overline{TU}$.
4. By construction, determine the midpoint of $\overline{TU}$. Call it V. Join $\overline{DV}$.

   OUR MISSION IS ALMOST COMPLETED.
5. Label the intersection of $\overline{DV}$ and $\overline{AC}$, W.
6. By construction, determine the midpoint of $\overline{AW}$ (call it X). Join $\overline{XT}$.
7. By construction, determine the midpoint of $\overline{CW}$ (call it Y) Join $\overline{VY}$.
8. Cut out your seven pieces. Each shape is called a *tangram*.
9. See how many shapes you can make with all seven pieces. Each time you make a tangram, draw an outline of the shape.

LESSON PLAN

ADDITION SHADE IN

by Grace Dilley

Shade squares in each row so that the column numbers add to the number given at the left.

	128	64	32	16	8	4	2	1
167								
162								
226								
162								
167								

	2048	1024	512	256	128	64	32	16	8	4	2	1
64												
160												
272												
520												
2044												
64												
1612												

THE FIVE SQUARE PUZZLE

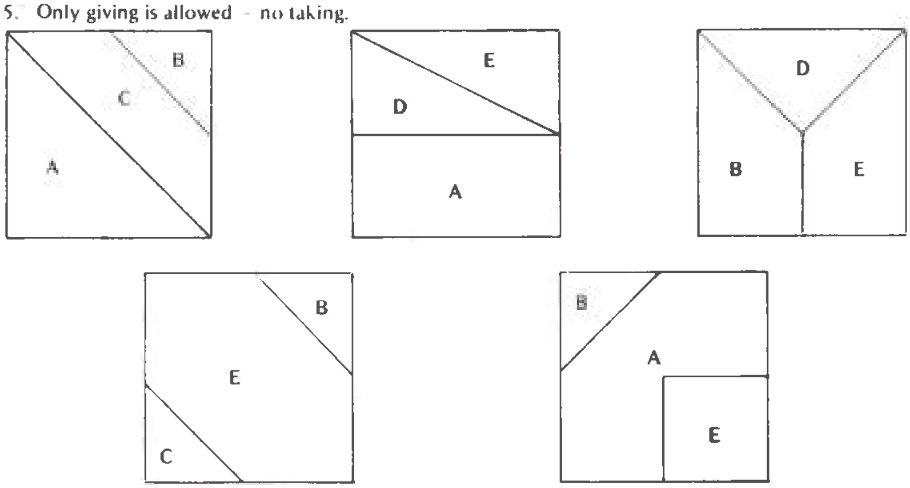
by Grace Dilley

The following diagrams show the pieces needed for one group of five pupils and the way the pieces fit together into five squares. Make the pieces on Bristol board.

Divide the class into groups of five students. At the beginning, the pieces labeled 'A' are given to the student labeled 'A,' the 'B' pieces are given to 'B,' etc., in each group.

RULES

1. Each member must construct one square directly at his work place.
2. No member may talk, signal, or gesture in any way that would provide guidance, direction, or suggestion to any other group member. For example, no member may signal that he wants a piece from another member.
3. Any member may *give* any of his pieces to another person.
4. Each member's pieces must be in front of him at his work place except the one that he is giving to another member.
5. Only giving is allowed – no taking.



LESSON PLAN

FIND A MATCH

by Dennis Hamaguchi

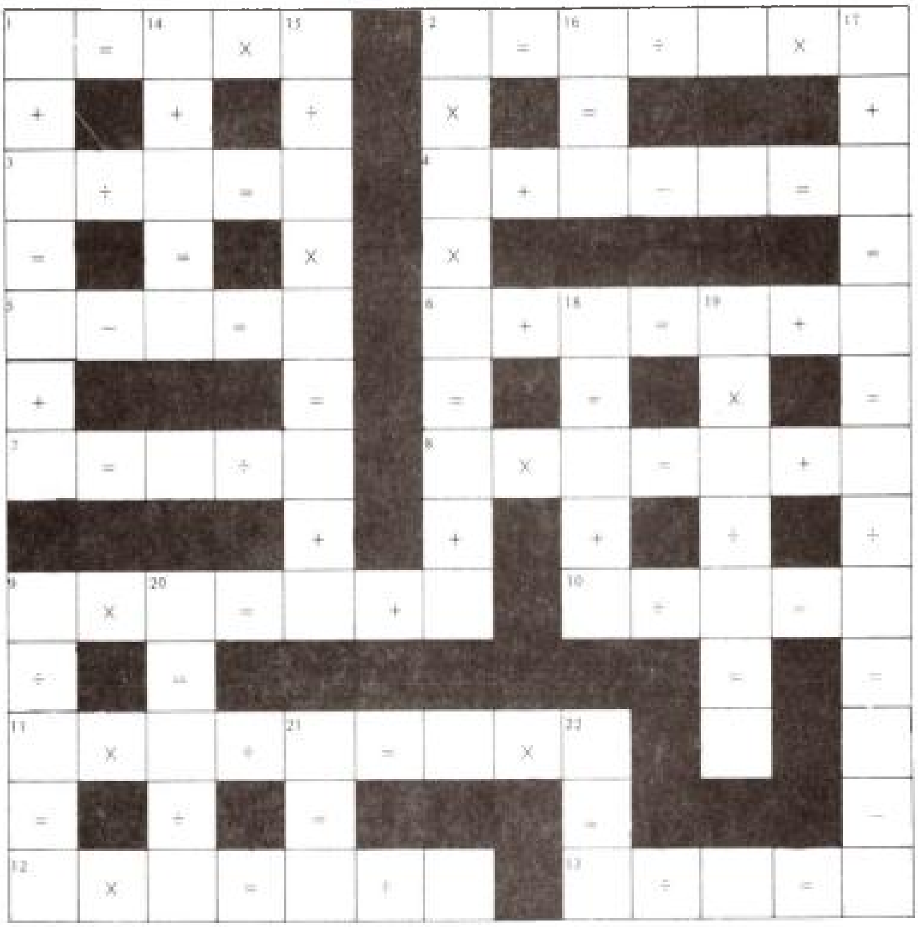
DIRECTIONS: Each of the two blocks below is divided into 18 boxes. Boxes in the top block contain problems, and boxes in the bottom block contain the answers. Work any problem and find your answer in the bottom block. Then write the word from the problem box into the answer box. Keep doing problems, and you will spell out a funny saying.

$\frac{5}{8} - \frac{1}{2}$ THE	$\frac{5}{6} - \frac{1}{3}$ A	$\frac{3}{4} - \frac{1}{12}$ TWICE	$\frac{4}{5} - \frac{2}{3}$ AS	$\frac{13}{10} - \frac{2}{5}$ IS	$\frac{3}{4} - \frac{1}{3}$ WHO
$\frac{7}{8} - \frac{1}{6}$ A	$\frac{3}{2} - \frac{5}{9}$ OCEAN	$\frac{3}{5} - \frac{1}{4}$ BATH	$\frac{17}{12} - \frac{5}{6}$ SAILOR	$\frac{13}{9} - \frac{2}{3}$ A	$\frac{7}{6} - \frac{5}{9}$ TAKING
$\frac{4}{3} - \frac{3}{8}$ KNOWN	$\frac{7}{15} - \frac{1}{6}$ DOUBLE	$\frac{9}{10} - \frac{5}{6}$ CROSSES	$\frac{9}{8} - \frac{7}{12}$ CROSSER	$\frac{7}{9} - \frac{1}{4}$ WITHOUT	$\frac{4}{5} - \frac{1}{6}$ DIRTY

$\frac{17}{24}$	$\frac{7}{12}$	$\frac{5}{12}$	$\frac{1}{15}$	$\frac{1}{8}$	$\frac{17}{18}$
$\frac{2}{3}$	$\frac{19}{36}$	$\frac{11}{18}$	$\frac{1}{2}$	$\frac{7}{20}$	$\frac{9}{10}$
$\frac{23}{24}$	$\frac{2}{15}$	$\frac{7}{9}$	$\frac{19}{30}$	$\frac{3}{10}$	$\frac{13}{24}$

Addition-Subtraction-Multiplication-Division

by Dennis Hamaguchi



The following totals are to be achieved.

ACROSS:
1. Four
2. Two
3. Two
4. Five
5. Five
6. Thirteen
7. One
8. Nine
9. Fifteen
10. One
11. Fifteen
12. Two
13. One

DOWN:
1. Ten
2. Sixteen
9. One
14. Four
15. Ten
16. Nine
17. Seven
18. Five
19. Three
20. Three
21. Two
22. Five

CHRISTMAS

Use several squares to equal 1.

1. Graph the following lines:

   $2x + y = -6$

   $2x - y = 6$

   $y = -2$

   $y = -4$

   Color the area enclosed by these lines RED.

2. Graph $y = -1$; $y = -2$; $x = -1$; $x = 1$

   Color the area enclosed by these lines BROWN.

The graph of a circle $(x - 2)^2 + (y + 3)^2 = 25$
center $(2 - 3)$ radius = 5

3. Graph and color the area inside the following:

   Radius = ½

YELLOW	$(x - 0)^2 + (y - 6)^2 = \frac{1}{4}$
PINK	$(x + 1)^2 + (y - 0)^2 = \frac{1}{4}$
ORANGE	$(x - 1)^2 + (y - 3)^2 = \frac{1}{4}$
LAVENDER	$(x + 2)^2 + (y - 2)^2 = \frac{1}{4}$
	$(x - 3)^2 + (y - 0)^2 = \frac{1}{4}$
BLUE	$(x - 2)^2 + (y - 1)^2 = \frac{1}{4}$
	$(x + 1)^2 + (y - 4)^2 = \frac{1}{4}$

4. Graph $2x + y = 8$; $2x - y = 8$; $y = -1$

   Color the area enclosed by these lines except for the area in 3 above GREEN.

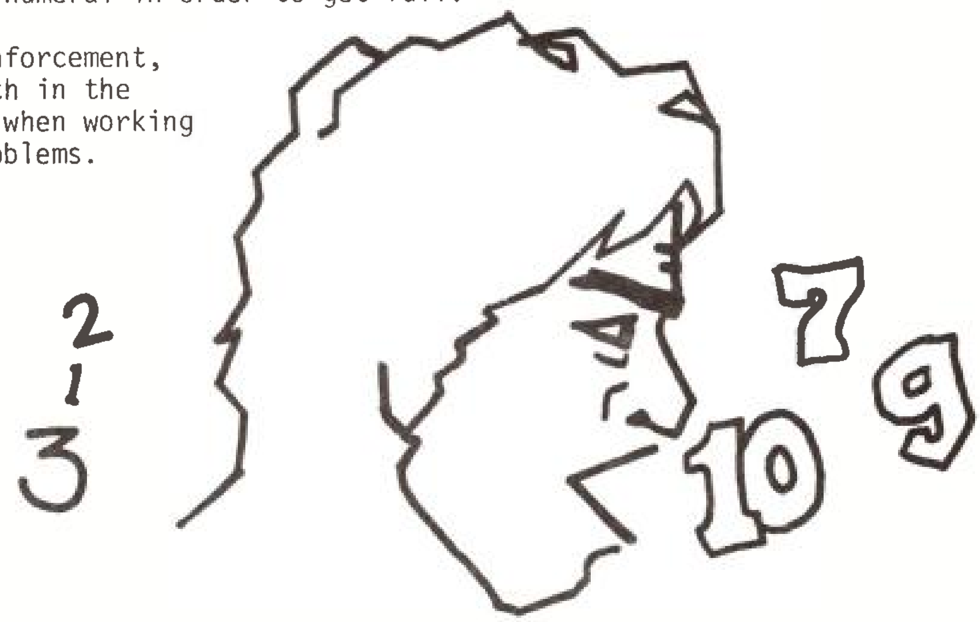
Suggestions and Ideas

Reprinted from The Math Post, *Christmas edition, 1973.*

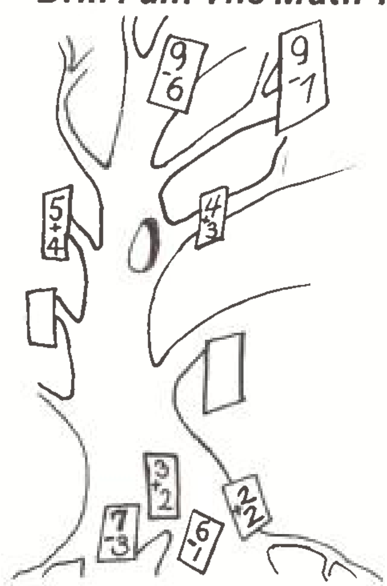
Greater Than or Less Than?

A simple, effective way for children to determine the direction of the greater than/less than symbol. Tell the children a story about a giant with a huge stomach. Mention that he is very special because he eats only numerals. Since his appetite is so great, his mouth always has to be opened toward the biggest numeral in order to get full.

For reinforcement, use teeth in the symbols when working with problems.



Drill Fun: The Math Tree

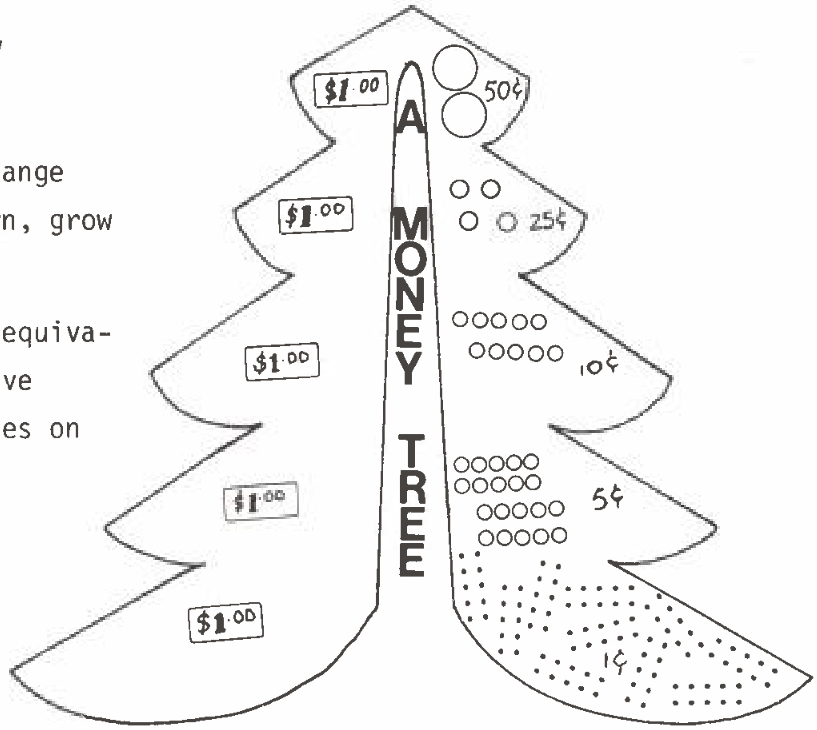


Use flash cards. (Answer may be on the back.) Try to climb the tree. New operations will be placed at the top of the tree. Known ones go at the bottom. As new operations become known, they move to the bottom. Old operations are placed in the tree knot for later review.

Grow a Money Tree!

If learning to make change gets your students down, grow a Money Tree!

Students learn dollar equivalents fast, once they've constructed the branches on this tree.



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