Mathematics Talks for High Schools

Because of the interest which was shown in this program last year, members of the Mathematics Department at the University of Alberta will be available again this year to give talks to high school students and/or teachers. Anyone interested in scheduling any of these talks is asked to contact Ivan Baggs, Department of Mathematics, University of Alberta, Edmonton, or phone 432-3385. A list of speakers, titles and summaries is given below.

1. *Set Theory - Where did it come from? Why was it so important? How important is it now?* - by Professor R.D. Bercov

   Summary: George Cantor created set theory in the years 1874 to 1895 to give mathematical precision to the concept of infinity. Hilbert, Russell, and others attempted to use the theory as the basis for an axiomatic treatment of all of mathematics. That this approach could not be successful has been made clear by the work of, among others, Gödel in the 1940s and Cohen in the 1960s. The desirability of formulating so much of mathematics in the language of sets must surely now be reconsidered.

2. *Galois - A hundred years ahead of his time* - by Professor R.D. Bercov

   Summary: At the beginning of the 19th century, formulae had been known for about 300 years giving the roots of equations of degree at most 4 in terms of the coefficients (for example, the quadratic formula $x = \frac{-b \pm\sqrt{b^2 - 4ac}}{2a}$). The search for the formula for the fifth degree equation came to an end with the brilliant work of Abel who showed that there is no such formula, and Galois who changed the nature of algebra by his work on the theory of equations. Unfortunately for both Galois and mathematics, this work was only understood and appreciated after his death.

3. *Canonical Calculation* - by Professor H.I. Freedman

   Summary: History of calculation and calculating instruments from ancient through medieval up to the present time. Topics include finger reckoning, ancient Roman hand calculators, bank accounts in the middle ages, the first mechanical adding machine. Slides and models utilized.

4. *Transcendental Π-ditation* - by Professor H.I. Freedman

Summary: A chronology of π from ancient times to the present. Topics include methods of calculating π, π in the Bible, irrationality of π, squaring the circle, anecdotes on π. Charts and slides utilized.

5. *Mathematical Biology* - by Professor H.I. Freedman

Summary: How mathematics can be used as an aid in explaining biological phenomena. Why is a louse not as big as a house? How do genes distribute from one generation to the next? This talk is intended for an audience of "better" students.

6. *Large Numbers* - by Professor H.I. Freedman

Summary: How does one write a large number? Can you ever own a complete set of bingo cards? What is a googol? Charts utilized.

7. *How Much Uncertainty?* - by Professor S.G. Ghurye

8. *Linear Difference Equations* - by Professor S.G. Ghurye

9. $f(u + v) = f(u)f(v)$ - by Professor S.G. Ghurye

10. *The Art of Problem-Solving* - by Professor M.S. Klamkin

Summary: The art of problem-solving is related to that of problem proposing. For by considering other problems related to the one to be solved, we are often led to a key idea which unlocks its solution. This is illustrated by an elementary geometric minimization problem which is rather difficult to solve. To obtain its solution, we will consider the general patterns of "level lines," "relaxation," "homotheticity," and "symmetry." Each pattern in turn will be illustrated by several other problems.

11. *Problem-solving via Transforms* - by Professor M.S. Klamkin

Summary: Here we take a broad viewpoint in solving problems via transformations. We illustrate (with slides) the philosophy and applications of transform theory by a series of problems starting off with some simple ones in arithmetic and geometry. We then consider some other problems in algebra, probability, number theory, combinatorics and physics. The choice of problems illustrated will depend on the background of the audience attending.

12. *Mathematical Creativity* - by Professor M.S. Klamkin

Summary: Although the psychological aspects of creativity in mathematics are important, we shall dwell mainly on the mathematical aspects. We will show how one can start with some rather elementary mathematical results and often end up with some rather sophisticated results.

13. *On the Teaching of Mathematics so as to be Useful* - by Professor M.S. Klamkin

14. *Vector Proofs in Solid Geometry* - by Professor M.S. Klamkin

Summary: In solving problems, one usually has the choice of using analytical geometric, synthetic geometric, or vectorial methods. We discuss, with many illustrations, the advantages and disadvantages of these three general approaches. For many problems, especially higher dimensional ones, it seems that the vectorial approach is a good compromise insofar as the case of setting up the problem is concerned as well as its subsequent solution.

15. *Optimization by Means of Level Lines and Inequalities* - by Professor M.S. Klamkin

Summary: We show, with many illustrations, how one can solve maximum and minimum problems with dispatch using level lines and/or inequalities. Also one can obtain the condition for the maximum or minimum of a rational function of one variable, in a given interval, without calculus.

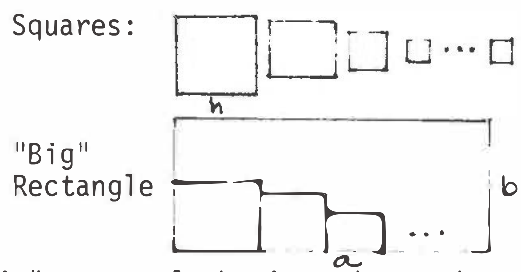
16. *Mathematical Competitions* - by Professor M.S. Klamkin

17. *Probability, Gambling, and If You Can Keep Your Shirt When All About You ...* - by Professor D.L. McLeish

Summary: What are the chances that all the monkeys in the world will eventually type *Macbeth*? What happens when the Law of Averages is repealed? Which bets pay the most in roulette? Various fads, fallacies and statistical fibs circulated by everyone from the most humble gambler to the parapsychologist to the most auspicious of our governmental agencies will be discussed.

18. *Packing Squares Into A Rectangle* - by Professor A. Meir

Summary: Suppose a set of squares is given with sides $x_1, x_2, \ldots, x_n$. Say the largest among the numbers $x_1, x_2, \ldots, x_n$ is equal to h and the total area of all squares is A. We wish to "pack" these squares into a "big" rectangle of size $a \times b$, without overlap, sides parallel to the sides of the "big" rectangle.



How big must the "big" rectangle be in order to be able to pack the squares into it? Answer: If $(a-h)(b-h) \geq A - h^2$, then we can always do it.

19. *Mathematics and Statistics Programmes Offered at the University of Alberta* - by Professor J.S. Muldowney and Dr. Ivan Baggs

<ins>Summary:</ins> This talk describes the various programmes leading to the B.Sc. and B.A. degrees in Mathematics and Statistics. The career opportunities for graduates of these programmes are also discussed. We will also describe some of the mathematics courses required and some of the mathematics options available to students wishing to major in a discipline other than mathematics.

20. *Why do Fisheries Need to be Controlled?* - by Professor R.D. Routledge

<ins>Summary:</ins> It is now obvious that many of our fish stocks have been fished too heavily. Simple mathematics will be used to provide an explanation for this phenomenon. The graphs of two functions will be used to find the level of fishing that produces the greatest profit, and then to demonstrate the tendency to exceed this level in the absence of strict controls. Specific reference will be made to the Nova Scotia lobster fishery. It will be shown that more sophisticated mathematics is needed to understand the behavior of this fishery in more detail.

21. *Probability in Gambling, Politics, and the Scheduling of Buses* - by Professor R.D. Routledge

<ins>Summary:</ins> When the eighteenth century gambler Le Chevalier de Méré found that his faulty intuition was costing him his fortune, he wrote to the mathematician Pascal for advice. This prompted Pascal and Fermat to begin the modern study of probability. This theory has been applied to such diverse problems as the determination of fair bets, the interpretation of opinion surveys, and the scheduling of buses.

22. *Testing Your Intuition* - by Professor J.G. Timourian

<ins>Summary:</ins> A collection of problems is stated for which the obvious "solutions" are wrong. The moral is that it is a good idea to guess at the solution to a problem, as long as you then work to demonstrate that your solution is correct.

23. *Calculus* - by Professor J.G. Timourian

Summary: A quick, one-class-period introduction to calculus and the problems it was invented to solve.

24. *Are Imaginary Numbers Really Imaginary?* - by Professor J.G. Timourian

Summary: How to break away from the confines of the line. Addition and multiplication of points in the plane.

25. *Multiplying Things Which Are Not Numbers* - by Professor J.G. Timourian

Summary: A discussion of matrix-vector and matrix-matrix multiplication, with some applications in geometry and/or business.

26. *Counting* - by Professor J.G. Timourian

Summary: The integers, the even integers, the points on a line 1 cm long, and the points on a line a metre long are all examples of infinite sets. Are all these infinities the same? Is there any sense to saying there are "more" integers than even integers? Or that there are "more" points on a line than there are integers? These topics are discussed in a historical context.

27. *Opinion Polls - Are They Reliable?* - by Dr. I. Baggs

Summary: Are the results of a "man-in-the-street" poll or a "phone-in" poll reliable indicators of public opinion? Why? Is there a more reliable method of conducting a poll? Is it possible to measure the accuracy of an opinion poll? How large a sample of the population should a poll include? Time permitting, different methods of conducting polls will be discussed and examples will be given of their uses.

28. *Is Mathematics Useful?* - by Dr. I. Baggs

Summary: This question will be considered by looking at some "real-world" problems and games. Examples which may be discussed include a simple model for the analysis and prediction of petroleum supplies; crop yield as a function of a given fertilizer; lotteries; bingo games; poker games; instant insanity and so forth. It will be indicated how methods used to analyze these problems apply to more general settings.

29. *The Different Levels of Infinity* - by Professor H.H. Brungs

Summary: A simple device, called equivalence of sets, can be used to distinguish different levels of infinity. In this context one can show that there are as many rational numbers as there are integers, and that there are as many points in the whole plane as there are in the interval [0,1]. But there are more points on the line than there are rational numbers, and the set of subsets of any set contains more elements than the set itself.

30. *Symmetries* - by Professor H.H. Brungs

Summary: Symmetries are abundant in art and nature - Eskimo prints, sunflowers, or snowflakes are examples. We discuss symmetries as mappings, which leads to groups of symmetries. A particularly simple example is the dihedral group D_n as the group of symmetries of the regular n-gon. Considering groups of symmetries in the 3-space leads to the five regular solids.

31. *Why The Cube Can Not Be Doubled* - by Professor H.H. Brungs

Summary: Given any length a, can we construct a length b just using straight edge and compass such that $b^3 = 2a^3$? Translation of this problem into algebra will lead to the consideration of fields, and then the solution of the above problem is surprisingly simple. ■