# NCTM-MAA Position Statement

## RECOMMENDATIONS for the PREPARATION of high school students for COLLEGE MATHEMATICS COURSES

The following statement, adopted by the Board of Governors of the Mathematical Association of America and the Board of Directors of the National Council of Teachers of Mathematics, is a brief outline of the basic ingredients of adequate preparation for collegiate-level mathematics.* The statement does not break new ground; it reflects standards that have been generally accepted for over a decade. It is intended to support the continuing efforts of conscientious teachers everywhere to provide students with sound and stimulating mathematical training. It is specifically designed to provide a benchmark for our efforts and those of others to assess and react to recent reports of a general decline in the performance of students in mathematics.

A joint committee of the Mathematical Association of America and the National Council of Teachers of Mathematics consulted with secondary school and college teachers in various parts of the country to study recent trends in the preparation of students. The comments from these consultations on which there was strongest consensus are the basis for this statement and its ten recommendations.

The Mathematical Association of America and the National Council of Teachers of Mathematics wish to emphasize that the statement and recommendations, as they refer to secondary school programs, are addressed only to those programs for students planning to go to college and that they are not intended to be more comprehensive. During the past twenty years many important changes have taken place in both the content and teaching of mathematics at the secondary school level. Many excellent new programs have been adopted and taught effectively by teachers in elementary and secondary schools. Nevertheless, any consideration of the relative merits of new versus traditional school curricula has been deliberately avoided. A study of this issue would have exceeded both the charge to the committee and its limited resources. This statement and these recommendations incorporate many of the best features of both of these curricula and are addressed to all mathematics programs regardless of pedagogical heritage.

* *Collegiate mathematics* refers to courses in calculus (or calculus and analytic geometry), probability and statistics, finite mathematics, and higher-level mathematics courses.

### Necessary Course Work

Mathematics is a highly structured subject in which various concepts and techniques are greatly dependent on each other. The concepts of arithmetic and algebra, however, are basic to all of mathematics. Further work in mathematics and in all areas in which mathematics is used as a tool requires correct performance, with understanding, of basic arithmetic operations, the manipulation of algebraic symbols, and an understanding of what the manipulations mean.

Any student who is unable to perform arithmetic calculations and algebraic operations with accuracy and reasonable speed, to understand which operations to use in a given problem, and to determine whether the results have meaning is severely handicapped in the study and applications of mathematics. The prevalence of inexpensive pocket calculators makes the performance of complicated calculations less tedious, but the use of calculators does not lessen the need for students to understand which concepts and operations are needed to solve a problem, to make sensible estimates, and to analyze their results.

For further work in mathematics, and in many other areas from business to psychology, from biology to engineering, the ability to use algebra with skill and understanding is also essential. Having a passing grade in algebra is not enough. Both understanding and competence in the skills of algebra are necessary. Neither conceptual understanding nor technical skill alone will suffice in today's world, let alone in tomorrow's. Algebra is a useful subject which will help to solve problems in the real world. Opportunities to apply algebraic skills should be provided whenever possible, especially to problems that show the utility of mathematics.

Algebra courses in secondary school should include, in addition to the basic topics—

(*a*) polynomial functions;
(*b*) properties of logarithms;
(*c*) exponential and logarithmic functions and equations;
(*d*) arithmetic and geometric sequences and series;
(*e*) the binomial theorem;
(*f*) infinite geometric series;
(*g*) linear and quadratic inequalities.

For most students, adequate coverage of the topics in algebra requires at least two years of study.

Students who will take calculus—and this now includes many students who will take college work in business, premedicine, economics, biology, statistics, engineering, and physical science—may or may not need trigonometry, depending on the type of calculus course appropriate for their particular programs. But they will need a good deal of what is often called

precalculus, including especially a sound understanding of the concept of a function, which is also fundamental for work beyond the most elementary level in probability and computing.

Those students needing trigonometry should study—

(*a*) trigonometric functions and their graphs;

(*b*) degree and radian measure;

(*c*) trigonometric identities and equations;

(*d*) inverse trigonometric functions and their graphs.

For such students, the equivalent of one semester should be devoted to the study of the topics in trigonometry.

All students who go on to take collegiate mathematics will find their college work easier if they have been introduced to some axiomatic system and to deductive reasoning. Traditionally this has been accomplished in a geometry course. Geometry courses in secondary school should include, in addition to basic topics—

(*a*) fundamental properties of geometric figures in three dimensions;

(*b*) applications of formulas for areas and volumes;

(*c*) experience in visualizing three-dimensional figures.

Other courses (the word *course* refers here and elsewhere in this statement to a semester course unless otherwise noted) beyond algebra, trigonometry, and geometry should be available to students who have adequate background and time to take them. A course in coordinate (or analytic) geometry is ideal, since it combines algebra with geometry and provides a useful preparation for calculus. In addition to coordinate geometry, courses in the following topics are valuable: probability, statistics, elementary finite mathematics (or linear algebra), an introduction to computers and computing, and applications of mathematics.

If coordinate geometry is offered, it should include, in addition to the basic topics—

(*a*) conic sections;

(*b*) rational functions and their graphs;

(*c*) polar coordinates;

(*d*) parametric equations and their graphs.

Inductive as well as deductive reasoning, techniques of estimation and approximation, and an awareness of problem-solving techniques, with special emphasis on the transition from the verbal form to the language of mathematics, should be emphasized in all courses.

Calculus, where offered in secondary schools, should be at least a *full year* course and be taken only by those students who are strongly prepared in algebra, geometry, trigonometry, and coordinate geometry.

We recognize that many secondary schools have a curriculum similar to that outlined above. We emphasize again that, in order to be properly prepared for collegiate-level courses in mathematics, students need to develop skills (1) in applying standard techniques and (2) in understanding important concepts.

## Recommendations

The Board of Governors of the Mathematical Association of America and the Board of Directors of the National Council of Teachers of Mathematics make the following recommendations:

1. Proficiency in mathematics cannot be acquired without individual practice. We therefore endorse the common practice of making regular assignments to be completed outside of class. We recommend that parents encourage their children to set aside sufficient time each day to complete these assignments and that parents actively support the request of the teacher that homework be turned in. Students should be encouraged to develop good study habits in mathematics courses at all levels and should develop the ability to read mathematics.

2. Homework and drill are very important pedagogical tools used to help the student gain understanding as well as proficiency in the skills of arithmetic and algebra, but students should not be burdened with excessive or meaningless drill. We therefore recommend that teachers and authors of textbooks step up their search for interesting problems that provide the opportunity to apply these skills. We realize that this is a difficult task, but we believe that providing problems that reinforce manipulative skills should have high priority, especially those that show that mathematics helps solve problems in the real world.

3. We are aware that teachers must struggle to maintain standards of performance in courses at all levels from kindergarten through college and that serious grade inflation has been observed. An apparently growing trend to reward effort or attendance rather than achievement has been making it increasingly difficult for mathematics teachers to maintain standards. We recommend that mathematics departments review evaluation procedures to ensure that grades reflect student achievement. Further, we urge administrators to support teachers in this endeavor.

4. In light of recommendation 3, we also recognize that the advancement of students without appropriate achievement has a detrimental effect on the individual student and on the entire class. We therefore recommend that school districts make special provisions to assist students when deficiencies are *first* noted.

5. We recommend that cumulative evaluations be given throughout each course, as well as at its completion, to *all* students. We believe that the absence of cumulative evaluation promotes short-term learning. We strongly oppose the practice of exempting students from evaluations.
6. We recommend that computers and minicalculators be used in imaginative ways to reinforce learning and to motivate the student as proficiency in mathematics is gained. Calculators should be used to supplement rather than to supplant the study of necessary computational skills.
7. We recommend that colleges and universities administer placement examinations in mathematics prior to final registration to aid students in selecting appropriate college courses.
8. We encourage the continuation or initiation of joint meetings of college and secondary school mathematics instructors and counselors in order to improve communication concerning mathematics prerequisites for careers, the preparation of students for collegiate mathematics courses, joint curriculum coordination, remedial programs in schools and colleges, the exchange of successful instructional strategies, the planning of in-service programs, and other related topics.
9. Schools should frequently review their mathematics curriculum to see that it meets the needs of its students in preparing them for college mathematics. School districts that have not conducted a curriculum analysis recently should do so now, primarily to identify topics in the curriculum that could be either omitted or de-emphasized, if necessary, in order to provide sufficient time for the topics included in this statement. We suggest, for example, that the following could be de-emphasized or omitted from the curriculum:
   (*a*) Logarithmic calculations that can better be handled by calculators or computers
   (*b*) The extensive solving of triangles in trigonometry
   (*c*) Proofs of superfluous or trivial theorems in geometry
10. We recommend that algebraic concepts and skills be incorporated wherever possible into geometry and other courses beyond algebra to help students retain these concepts and skills. ■

---

This position statement was prepared jointly by the National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091, and the Mathematical Association of America, 1225 Connecticut Ave., NW, Washington, DC 20036.