Reinforce division by learning ratios

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At the sixth-grade level, pupils are expected to learn several concepts that are related to ratios in one way or another; for example, *rational number*, *decimal*, *percent*, and *constant*. Learning and relating these concepts is affected by factors which, to many teachers, seem difficult to reconcile. The process of abstracting the concepts, learning symbolic representations of the concepts, and making a shift from concrete or visual to formal or abstract ways of thinking about these concepts are among the factors that must be taken into consideration. With all of this, teachers consistently express concern for how to employ interesting, meaningful ways to teach the concepts without detracting from a need to teach basic skills.

Each of the three activities described below reinforce the basic skills of measuring and computing with decimals by having pupils discover that, in one sense, a ratio is a single number that expresses a relationship between two other numbers. The three activities usually require three to four mathematics periods. In a self-contained classroom they are most successful when the whole class is involved. In other teaching situations, groups of ten to fifteen pupils are best. Pupils should be allowed to share information, and everything should be done to create an atmosphere of cooperative problem-solving.

Activity 1

Begin by asking the students: What are ratios? Encourage the students to express their own ideas of ratios. Then tell the class that during the next few periods they are going to study ratios, and suggest that they try to figure out what the word *ratio* means.

Make available to the class several circular objects, some string that does not stretch, and rulers that have a centimeter scale. Have the students carefully measure the circumferences and diameters of the objects using the string and rulers. The results of each measurement should be recorded in a systematic way. (See table 1.)

Table 1
Measurements of circular objects

Object	*Circumference* (c)	*Diameter* (d)	$\frac{c}{d}$
wheel	20.8	6.5	$\frac{20.8}{6.5} = 3.2$
can			
bottle			
⋮			

When measurements are completed, have the students divide the circumferences by the respective diameters. Then ask the students how the two sets of measurements seem to compare. Let the students discuss the results. Some pupils will know that this number, approximately 3.14, is called pi but the name is not essential in this particular activity.

Activity 2

Make available to the students pieces of ruled paper and rulers with centimeter scales. Have the students draw several squares—the sides of the squares can be any length. Then have the pupils measure the lengths of the sides and of the diagonals of the squares to the nearest tenth of a centimeter and record the information in a table. (See table 2.) When the measurements have been recorded, have the students divide the lengths of the diagonals by the respective lengths of the sides. Again, have the students compare the two sets of measurements. The quotients (ratios) will approximate 1.42. Let the students discuss this result.

Try to have students see that a ratio is a single number that tells how two numbers are related. The number 3.14 tells how the circumference and the diameter of a circle are related. The number 1.42 tells how the diagonal of a square is related to the side of the square. These numbers or ratios are *constant*: the relationship is the same for all circles and for all squares.

Activity 3

Make rulers and paper available to the students. Have each student draw several different rectangles; encourage them to draw rectangles that vary in appearance. Then tell the class that the ancient Greeks were interested in aesthetics—what made things beautiful. Suggest that each student think about rectangles and then draw what he believes would be a beautiful rectangle. This will seem mysterious to the students and the mystery can be played up. Have each student measure the length and width of his "beautiful" rectangle in centimeters and divide the length by the width. Examine the results and select those rectangles for which the ratio of length to width is greater than 1.4 and less than 1.8. Tell the class that these are examples of rectangles that the ancient Greeks thought were beautiful. You might display all of the "beautiful" rectangles along with their respective ratios and let the class discover the critical ratios. Have students check the dimensions of familiar rectangular things like picture frames, stationery, and rooms, and discuss why or why not these things are beautiful by the Greek standards.

It is quite possible that some students will not agree with the Greek idea of what is a beautiful rectangle; a student may prefer long narrow rectangles, for example. (There are also reasons for making rectangles that do not fit the Greek proportions; these reasons may come out in the class discussions.) It will not matter if students vary in their ideas of what is beautiful in a rectangle. The next step of this activity will bring out the mathematical relationships inherent in what is sometimes referred to as the "golden rectangle." Students will see that a rectangle can be beautiful in a mathematical way.

After sufficient discussion, have each student (1) measure the width of his most beautiful rectangle, (2) reproduce the width on each length, and (3) connect the two points so that a square is formed at one end of his rectangle. Tell the students to shade the square as shown in figure 1.

Table 2
Measurements of squares

Length of side (s)	*Length of diagonal (d)*	$\frac{d}{s}$
1	1.4	1.4
2	2.8	1.4
3	.	.
.	.	.
.	.	.
.		

The unshaded part of each figure is another rectangle. Have each student compare the shape of his new rectangle with the shape of his original rectangle. The students

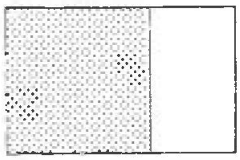
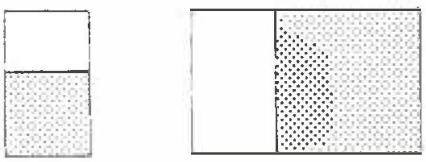


Fig. 1

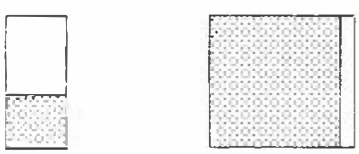
will notice that for rectangles having ratios of length to width between 1.4 and 1.8, the new rectangle has nearly the same form as the original rectangle. (See fig. 2.) For



"Beautiful" rectangles

Fig. 2

other ratios, the new rectangle will look much different than the original rectangle. (See fig. 3.)



"Ugly" rectangles

Fig. 3

Mathematics works in wondrous ways. It just happens that if the ratio of the length to width of a rectangle approximates 1.618, the new rectangle formed by drawing the square will have the same form as the original rectangle.

At this point, you may conclude the activities with a discussion of ratios. I know one teacher who extended the activity by having each student select his favorite width. The student then multiplied this number by 1.618 and used his width and the obtained length (width multiplied by 1.618) to draw a rectangle. He next used his width to draw a square "inside" his rectangle to produce a new rectangle. This process was continued to obtain a drawing like figure 4.

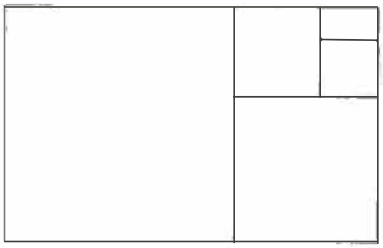


Fig. 4

As you can see, these activities involve the students in much drawing, measuring, and computing while they study ratios. The activities also encourage exploration. Some students may be interested in further research on "golden rectangles." Students may also discover other constant ratios as they find more examples of ratios.

References

National Council of Teachers of Mathematics. *Historical Topics for the Mathematics Classroom:* Thirty-first Yearbook. Washington D.C.: The Council, 1969. ■