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THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

VOLUME XVII, NUMBER 3, FEBRUARY 1978

MCATA Annual Conference, October 1977

This issue has some noted changes. These come from suggestions made at the Annual Conference. Further short notes of ideas, games, and other information are needed from members to continue this new format.

delta-k

THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

VOLUME XVII, NUMBER 3 FEBRUARY 1978

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Excerpts from a Speech Given by Dr. Eric MacPherson at the MCATA Annual Conference

Dr. MacPherson is Dean of the Faculty of Education at the University of Manitoba.

October 14, 1977:

- Societal agencies of medicine and education are growing more rapidly than the GNP. This necessitates a slowdown of program expansion and restrictions that require adjustment to selective priorities, because of lack of funding for expansion.
- Program reductions and/or personnel restrictions are going to occur, and public criticism is a smokescreen to reduce the pain of restrictive growth.
- Change of curriculum has been in a state of flux so long that younger teachers are unaware of the fact that curriculum change has always been normal.
- Desire for simplicity calls for a return to the basics, as change has created a less simple way of life.
- The end of the Baby Boom resulted in a decline in lower levels for achievement tests. A return to a higher level is claimed to be a result of the demand for a return to the basics.
- Curriculum change is no longer important, as people say "forget it" and return to the basics.
- The need for skills in complex arithmetic problems is changing as the calculator replaces the need for human calculation. There is now a need for a greater emphasis on understanding the methods of problem-solving.
- Let parents furnish calculators rather than having the school provide them. Teachers need to learn how to use them and where to use them. Using class

sets of identical calculators, start experimenting slowly, because response to the use of calculators at the elementary level is now chilly.

- Look at the basics of new math and use practically.

- The sociology of the secondary classroom has changed drastically.

- A false concept is that an easy type of secondary program has failed to meet the needs for university.

- The true ratio of top students is as high as ever. Lower standards of excellence are created for those below top level.

- The danger now is a lack of challenge for top achievement because of mass education.

- How are we going to adapt to mass education of today to justify the future of secondary education?

  We must adjust our secondary program to provide for the needs of academics and non-academics, with more than one stream of secondary mathematics. We mustn't segregate excessively as is done in Europe.

- Curriculum decisions are going to be made in the classroom, as research can never solve, but can only assist in solving, the problem.

<u>October 15, 1977:</u>

Early 1800s: mass education

1890-1910: There was a progression from Dickens-type schools to mass education, from children being treated as adults to children being treated as persons of their own age level.

1920s: There were first attempts made at evaluation during this period, but these were not sufficiently carried through. Therefore, some changes in the results of the measurements of today would be less criticized had the results of the '20s been carried through properly.

1950-1970: A fundamental change in curriculum content was necessary as there was a demand for change to meet a changing philosophy of life during this period.

It is time to stabilize the curriculum with changes quietly made and less dramatically innovative. The "back to basics" cries mean: do the job and keep quiet about changes you have to make, because the public is now suspicious rather than praising.

Recognize the problem and work together or "repetition," "regurgitation," "remediation" will return to haunt us.

Pay more attention to problem-solving and less to extensive standardized skills. Teaching methods that help in problem-solving in any discipline help in problem-solving in life. ■

Your Chance to Contribute to the Arithmetic Teacher

The Board of Education for the City of Hamilton
100 Main Street West
Hamilton, Ont.
L8N 3L1

1977 11 23

Mr. Ed Carriger
R.R. #1, Site 2, Box 3
Bluffton, Alberta

Dear Mr. Carriger:

As you may be aware, the Arithmetic Teacher publishes a page entitled "What's Going On ..." each month.

As one of only two Canadian reporters, I would like to hear about any activities and projects which may be of interest to the many readers of the Arithmetic Teacher.

Thank you for your kind consideration in this matter, and please send the information to:

Bob Robinson
"What's Going On" Reporter
Mathematics Department
Hamilton Board of Education
100 Main Street West
Hamilton, Ontario
L8N 3L1

Sincerely,

R. Robinson
NCTM Co-ordinator for OAME

Please forward to your editor or Bob Robinson your ideas. Your editor has the forms. (See following page for sample form.)

Sample Reporting Form for "What's Going On ..."

Describe any innovative projects or activities in local school systems, statewide projects, projects sponsored by area colleges and universities and interesting developments in your state or local Councils of Teachers of Mathematics.

TITLE OF PROJECT

PURPOSE

DESCRIPTION

Name, address and phone number of person from whom additional information can be obtained:

Has the person listed been informed that information about the project is being submitted to "What's Going On ..."? Yes ______ No ______

Are there materials available for distribution? Yes ______ No ______

Mail to: Bob Robinson, Mathematics Department, Hamilton Board of Education, 100 Main Street West, Hamilton, Ontario L8N 3L1. ■

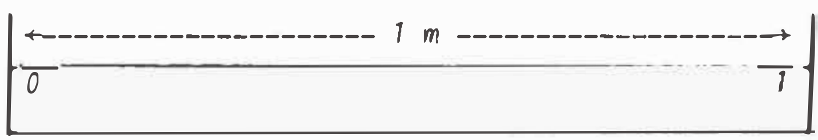
IDEAS for Junior High

Tom Kieren

Folding Fractions

Material needed: Calculator tape, colored pencils or pens.

1. Measure a strip of calculator tape exactly one metre long. Cut it so the ends are perpendicular to its length.



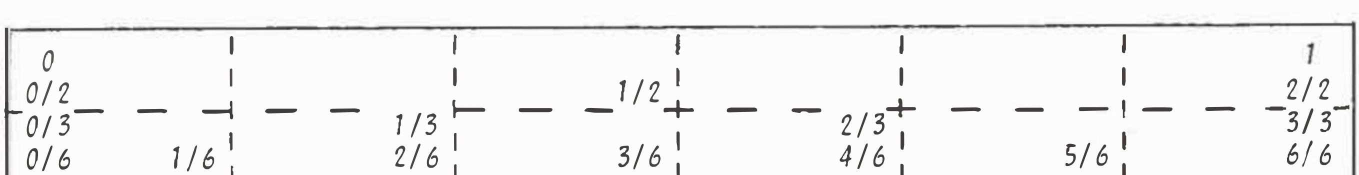
Label the ends 0 and 1.

2. Fold the strip exactly in two and label the appropriate points 0/2, 1/2, 2/2 with a colored pen.

3. Now fold your strip exactly in three parts. Label the appropriate points 0/3, 1/3, 2/3, 3/3 using a different colored pen.

4. Use your "third" folds to fold the strip exactly in six parts of the same size. Are any folds the same as previous folds? Why?

   Label the folds 0/6, 1/6, 2/6, 3/6, 4/6, 5/6, 6/6. Use a different colored pen. Your strip should look like:



5. Do the following folding and labeling.

   a) Fold your strip into 4 parts of the same size. Label 0/4, 1/4, 2/4, 3/4, 4/4 using a new color.

   b) Fold your strip into 8 parts and label in the same way as above using a new color.

   c) Fold your strip into 16 parts and label using a new color.

   d) Fold your strip into 12 parts and label using a new color.

6. Use your strip to answer the following.

   a) Which is larger: 2/3 or 5/8? How can you tell?

   b) Arrange in order from smallest to largest:

      2/3, 5/16, 5/6, 7/8, 1/2, 3/8, 1/12, 1/6

      How do you make decisions about these?

   *c) Where would 7/32 fit in the list? How could you tell?

   d) List fractions which are the same as or equivalent to:

      1/2 :

      How did you know?

   e) Make lists of equivalent fractions.

      3/8, _____, _____

      0, 0/2, _____, _____, _____, _____, _____

      3/4, _____, _____, _____, _____

      2/6, _____, _____, _____, _____

      4/12, _____, _____, _____, _____

      *6/24, _____, _____, _____, _____

      *2/5, _____, _____, _____, _____

f) How would you find more fractions to put on the 1/3 fold?

Would 18/27 be on the 1/3 fold? Why?

Would 10/40 be on the 1/3 fold? Why?

*Complete this general label for the 1/3 fold.

$$\frac{\quad}{3 \times k}$$

7. a) Invent a way of using your strip to add fractions.

Can you get results greater than 1?

b) Invent a way of using your strip to subtract fractions. ■

Mathematics Talks for High Schools

Because of the interest which was shown in this program last year, members of the Mathematics Department at the University of Alberta will be available again this year to give talks to high school students and/or teachers. Anyone interested in scheduling any of these talks is asked to contact Ivan Baggs, Department of Mathematics, University of Alberta, Edmonton, or phone 432-3385. A list of speakers, titles and summaries is given below.

1. *Set Theory - Where did it come from? Why was it so important? How important is it now?* - by Professor R.D. Bercov

   Summary: George Cantor created set theory in the years 1874 to 1895 to give mathematical precision to the concept of infinity. Hilbert, Russell, and others attempted to use the theory as the basis for an axiomatic treatment of all of mathematics. That this approach could not be successful has been made clear by the work of, among others, Gödel in the 1940s and Cohen in the 1960s. The desirability of formulating so much of mathematics in the language of sets must surely now be reconsidered.

2. *Galois - A hundred years ahead of his time* - by Professor R.D. Bercov

   Summary: At the beginning of the 19th century, formulae had been known for about 300 years giving the roots of equations of degree at most 4 in terms of the coefficients (for example, the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$). The search for the formula for the fifth degree equation came to an end with the brilliant work of Abel who showed that there is no such formula, and Galois who changed the nature of algebra by his work on the theory of equations. Unfortunately for both Galois and mathematics, this work was only understood and appreciated after his death.

3. *Canonical Calculation* - by Professor H.I. Freedman

   Summary: History of calculation and calculating instruments from ancient through medieval up to the present time. Topics include finger reckoning, ancient Roman hand calculators, bank accounts in the middle ages, the first mechanical adding machine. Slides and models utilized.

4. *Transcendental* Π*-ditation* - by Professor H.I. Freedman

Summary: A chronology of π from ancient times to the present. Topics include methods of calculating π, π in the Bible, irrationality of π, squaring the circle, anecdotes on π. Charts and slides utilized.

5. *Mathematical Biology* - by Professor H.I. Freedman

Summary: How mathematics can be used as an aid in explaining biological phenomena. Why is a louse not as big as a house? How do genes distribute from one generation to the next? This talk is intended for an audience of "better" students.

6. *Large Numbers* - by Professor H.I. Freedman

Summary: How does one write a large number? Can you ever own a complete set of bingo cards? What is a googol? Charts utilized.

7. *How Much Uncertainty?* - by Professor S.G. Ghurye

8. *Linear Difference Equations* - by Professor S.G. Ghurye

9. $f(u + v) = f(u)f(v)$ - by Professor S.G. Ghurye

10. *The Art of Problem-Solving* - by Professor M.S. Klamkin

Summary: The art of problem-solving is related to that of problem proposing. For by considering other problems related to the one to be solved, we are often led to a key idea which unlocks its solution. This is illustrated by an elementary geometric minimization problem which is rather difficult to solve. To obtain its solution, we will consider the general patterns of "level lines," "relaxation," "homotheticity," and "symmetry." Each pattern in turn will be illustrated by several other problems.

11. *Problem-solving via Transforms* - by Professor M.S. Klamkin

Summary: Here we take a broad viewpoint in solving problems via transformations. We illustrate (with slides) the philosophy and applications of transform theory by a series of problems starting off with some simple ones in arithmetic and geometry. We then consider some other problems in algebra, probability, number theory, combinatorics and physics. The choice of problems illustrated will depend on the background of the audience attending.

12. *Mathematical Creativity* - by Professor M.S. Klamkin

Summary: Although the psychological aspects of creativity in mathematics are important, we shall dwell mainly on the mathematical aspects. We will show how one can start with some rather elementary mathematical results and often end up with some rather sophisticated results.

13. *On the Teaching of Mathematics so as to be Useful* - by Professor M.S. Klamkin

14. *Vector Proofs in Solid Geometry* - by Professor M.S. Klamkin

Summary: In solving problems, one usually has the choice of using analytical geometric, synthetic geometric, or vectorial methods. We discuss, with many illustrations, the advantages and disadvantages of these three general approaches. For many problems, especially higher dimensional ones, it seems that the vectorial approach is a good compromise insofar as the case of setting up the problem is concerned as well as its subsequent solution.

15. *Optimization by Means of Level Lines and Inequalities* - by Professor M.S. Klamkin

Summary: We show, with many illustrations, how one can solve maximum and minimum problems with dispatch using level lines and/or inequalities. Also one can obtain the condition for the maximum or minimum of a rational function of one variable, in a given interval, without calculus.

16. *Mathematical Competitions* - by Professor M.S. Klamkin

17. *Probability, Gambling, and If You Can Keep Your Shirt When All About You ...* - by Professor D.L. McLeish

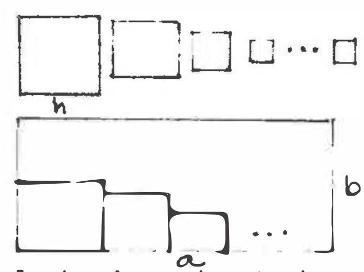
Summary: What are the chances that all the monkeys in the world will eventually type *Macbeth*? What happens when the Law of Averages is repealed? Which bets pay the most in roulette? Various fads, fallacies and statistical fibs circulated by everyone from the most humble gambler to the parapsychologist to the most auspicious of our governmental agencies will be discussed.

18. *Packing Squares Into A Rectangle* - by Professor A. Meir

Summary: Suppose a set of squares is given with sides $x_1, x_2, \ldots, x_n$. Say the largest among the numbers $x_1, x_2, \ldots, x_n$ is equal to h and the total area of all squares is A. We wish to "pack" these squares into a "big" rectangle of size $a \times b$, without overlap, sides parallel to the sides of the "big" rectangle.

Squares:

"Big"
Rectangle



How big must the "big" rectangle be in order to be able to pack the squares into it? Answer: If $(a-h)(b-h) \geq A - h^2$, then we can always do it.

19. *Mathematics and Statistics Programmes Offered at the University of Alberta* - by Professor J.S. Muldowney and Dr. Ivan Baggs

<u>Summary:</u> This talk describes the various programmes leading to the B.Sc. and B.A. degrees in Mathematics and Statistics. The career opportunities for graduates of these programmes are also discussed. We will also describe some of the mathematics courses required and some of the mathematics options available to students wishing to major in a discipline other than mathematics.

20. *Why do Fisheries Need to be Controlled?* - by Professor R.D. Routledge

<u>Summary:</u> It is now obvious that many of our fish stocks have been fished too heavily. Simple mathematics will be used to provide an explanation for this phenomenon. The graphs of two functions will be used to find the level of fishing that produces the greatest profit, and then to demonstrate the tendency to exceed this level in the absence of strict controls. Specific reference will be made to the Nova Scotia lobster fishery. It will be shown that more sophisticated mathematics is needed to understand the behavior of this fishery in more detail.

21. *Probability in Gambling, Politics, and the Scheduling of Buses* - by Professor R.D. Routledge

<u>Summary:</u> When the eighteenth century gambler Le Chevalier de Méré found that his faulty intuition was costing him his fortune, he wrote to the mathematician Pascal for advice. This prompted Pascal and Fermat to begin the modern study of probability. This theory has been applied to such diverse problems as the determination of fair bets, the interpretation of opinion surveys, and the scheduling of buses.

22. *Testing Your Intuition* - by Professor J.G. Timourian

<u>Summary:</u> A collection of problems is stated for which the obvious "solutions" are wrong. The moral is that it is a good idea to guess at the solution to a problem, as long as you then work to demonstrate that your solution is correct.

23. *Calculus* - by Professor J.G. Timourian

<u>Summary:</u> A quick, one-class-period introduction to calculus and the problems it was invented to solve.

24. *Are Imaginary Numbers Really Imaginary?* - by Professor J.G. Timourian

<u>Summary:</u> How to break away from the confines of the line. Addition and multiplication of points in the plane.

25. *Multiplying Things Which Are Not Numbers* - by Professor J.G. Timourian

<u>Summary:</u> A discussion of matrix-vector and matrix-matrix multiplication, with some applications in geometry and/or business.

26. *Counting* - by Professor J.G. Timourian

<u>Summary:</u> The integers, the even integers, the points on a line 1 cm long, and the points on a line a metre long are all examples of infinite sets. Are all these infinities the same? Is there any sense to saying there are "more" integers than even integers? Or that there are "more" points on a line than there are integers? These topics are discussed in a historical context.

27. *Opinion Polls - Are They Reliable?* - by Dr. I. Baggs

<u>Summary:</u> Are the results of a "man-in-the-street" poll or a "phone-in" poll reliable indicators of public opinion? Why? Is there a more reliable method of conducting a poll? Is it possible to measure the accuracy of an opinion poll? How large a sample of the population should a poll include? Time permitting, different methods of conducting polls will be discussed and examples will be given of their uses.

28. *Is Mathematics Useful?* - by Dr. I. Baggs

<u>Summary:</u> This question will be considered by looking at some "real-world" problems and games. Examples which may be discussed include a simple model for the analysis and prediction of petroleum supplies; crop yield as a function of a given fertilizer; lotteries; bingo games; poker games; instant insanity and so forth. It will be indicated how methods used to analyze these problems apply to more general settings.

29. *The Different Levels of Infinity* - by Professor H.H. Brungs

<u>Summary:</u> A simple device, called equivalence of sets, can be used to distinguish different levels of infinity. In this context one can show

that there are as many rational numbers as there are integers, and that there are as many points in the whole plane as there are in the interval [0,1]. But there are more points on the line than there are rational numbers, and the set of subsets of any set contains more elements than the set itself.

30. *Symmetries* - by Professor H.H. Brungs

Summary: Symmetries are abundant in art and nature - Eskimo prints, sunflowers, or snowflakes are examples. We discuss symmetries as mappings, which leads to groups of symmetries. A particularly simple example is the dihedral group D_n as the group of symmetries of the regular n-gon. Considering groups of symmetries in the 3-space leads to the five regular solids.

31. *Why The Cube Can Not Be Doubled* - by Professor H.H. Brungs

Summary: Given any length a, can we construct a length b just using straight edge and compass such that $b^3 = 2a^3$? Translation of this problem into algebra will lead to the consideration of fields, and then the solution of the above problem is surprisingly simple. ■

NCTM-MAA Position Statement

RECOMMENDATIONS for the PREPARATION of high school students for COLLEGE MATHEMATICS COURSES

The following statement, adopted by the Board of Governors of the Mathematical Association of America and the Board of Directors of the National Council of Teachers of Mathematics, is a brief outline of the basic ingredients of adequate preparation for collegiate-level mathematics.* The statement does not break new ground; it reflects standards that have been generally accepted for over a decade. It is intended to support the continuing efforts of conscientious teachers everywhere to provide students with sound and stimulating mathematical training. It is specifically designed to provide a benchmark for our efforts and those of others to assess and react to recent reports of a general decline in the performance of students in mathematics.

A joint committee of the Mathematical Association of America and the National Council of Teachers of Mathematics consulted with secondary school and college teachers in various parts of the country to study recent trends in the preparation of students. The comments from these consultations on which there was strongest consensus are the basis for this statement and its ten recommendations.

The Mathematical Association of America and the National Council of Teachers of Mathematics wish to emphasize that the statement and recommendations, as they refer to secondary school programs, are addressed only to those programs for students planning to go to college and that they are not intended to be more comprehensive. During the past twenty years many important changes have taken place in both the content and teaching of mathematics at the secondary school level. Many excellent new programs have been adopted and taught effectively by teachers in elementary and secondary schools. Nevertheless, any consideration of the relative merits of new versus traditional school curricula has been deliberately avoided. A study of this issue would have exceeded both the charge to the committee and its limited resources. This statement and these recommendations incorporate many of the best features of both of these curricula and are addressed to all mathematics programs regardless of pedagogical heritage.

* *Collegiate mathematics* refers to courses in calculus (or calculus and analytic geometry), probability and statistics, finite mathematics, and higher-level mathematics courses.

Necessary Course Work

Mathematics is a highly structured subject in which various concepts and techniques are greatly dependent on each other. The concepts of arithmetic and algebra, however, are basic to all of mathematics. Further work in mathematics and in all areas in which mathematics is used as a tool requires correct performance, with understanding, of basic arithmetic operations, the manipulation of algebraic symbols, and an understanding of what the manipulations mean.

Any student who is unable to perform arithmetic calculations and algebraic operations with accuracy and reasonable speed, to understand which operations to use in a given problem, and to determine whether the results have meaning is severely handicapped in the study and applications of mathematics. The prevalence of inexpensive pocket calculators makes the performance of complicated calculations less tedious, but the use of calculators does not lessen the need for students to understand which concepts and operations are needed to solve a problem, to make sensible estimates, and to analyze their results.

For further work in mathematics, and in many other areas from business to psychology, from biology to engineering, the ability to use algebra with skill and understanding is also essential. Having a passing grade in algebra is not enough. Both understanding and competence in the skills of algebra are necessary. Neither conceptual understanding nor technical skill alone will suffice in today's world, let alone in tomorrow's. Algebra is a useful subject which will help to solve problems in the real world. Opportunities to apply algebraic skills should be provided whenever possible, especially to problems that show the utility of mathematics.

Algebra courses in secondary school should include, in addition to the basic topics—

- (*a*) polynomial functions;
- (*b*) properties of logarithms;
- (*c*) exponential and logarithmic functions and equations;
- (*d*) arithmetic and geometric sequences and series;
- (*e*) the binomial theorem;
- (*f*) infinite geometric series;
- (*g*) linear and quadratic inequalities.

For most students, adequate coverage of the topics in algebra requires at least two years of study.

Students who will take calculus—and this now includes many students who will take college work in business, premedicine, economics, biology, statistics, engineering, and physical science—may or may not need trigonometry, depending on the type of calculus course appropriate for their particular programs. But they will need a good deal of what is often called precalculus, including especially a sound understanding of the concept of a function, which is also fundamental for work beyond the most elementary level in probability and computing.

Those students needing trigonometry should study—

(*a*) trigonometric functions and their graphs;
(*b*) degree and radian measure;
(*c*) trigonometric identities and equations;
(*d*) inverse trigonometric functions and their graphs.

For such students, the equivalent of one semester should be devoted to the study of the topics in trigonometry.

All students who go on to take collegiate mathematics will find their college work easier if they have been introduced to some axiomatic system and to deductive reasoning. Traditionally this has been accomplished in a geometry course. Geometry courses in secondary school should include, in addition to basic topics—

(*a*) fundamental properties of geometric figures in three dimensions;
(*b*) applications of formulas for areas and volumes;
(*c*) experience in visualizing three-dimensional figures.

Other courses (the word *course* refers here and elsewhere in this statement to a semester course unless otherwise noted) beyond algebra, trigonometry, and geometry should be available to students who have adequate background and time to take them. A course in coordinate (or analytic) geometry is ideal, since it combines algebra with geometry and provides a useful preparation for calculus. In addition to coordinate geometry, courses in the following topics are valuable: probability, statistics, elementary finite mathematics (or linear algebra), an introduction to computers and computing, and applications of mathematics.

If coordinate geometry is offered, it should include, in addition to the basic topics—

(*a*) conic sections;
(*b*) rational functions and their graphs;
(*c*) polar coordinates;
(*d*) parametric equations and their graphs.

Inductive as well as deductive reasoning, techniques of estimation and approximation, and an awareness of problem-solving techniques, with special emphasis on the transition from the verbal form to the language of mathematics, should be emphasized in all courses.

Calculus, where offered in secondary schools, should be at least a *full year* course and be taken only by those students who are strongly prepared in algebra, geometry, trigonometry, and coordinate geometry.

We recognize that many secondary schools have a curriculum similar to that outlined above. We emphasize again that, in order to be properly prepared for collegiate-level courses in mathematics, students need to develop skills (1) in applying standard techniques and (2) in understanding important concepts.

Recommendations

The Board of Governors of the Mathematical Association of America and the Board of Directors of the National Council of Teachers of Mathematics make the following recommendations:

1. Proficiency in mathematics cannot be acquired without individual practice. We therefore endorse the common practice of making regular assignments to be completed outside of class. We recommend that parents encourage their children to set aside sufficient time each day to complete these assignments and that parents actively support the request of the teacher that homework be turned in. Students should be encouraged to develop good study habits in mathematics courses at all levels and should develop the ability to read mathematics.

2. Homework and drill are very important pedagogical tools used to help the student gain understanding as well as proficiency in the skills of arithmetic and algebra, but students should not be burdened with excessive or meaningless drill. We therefore recommend that teachers and authors of textbooks step up their search for interesting problems that provide the opportunity to apply these skills. We realize that this is a difficult task, but we believe that providing problems that reinforce manipulative skills should have high priority, especially those that show that mathematics helps solve problems in the real world.

3. We are aware that teachers must struggle to maintain standards of performance in courses at all levels from kindergarten through college and that serious grade inflation has been observed. An apparently growing trend to reward effort or attendance rather than achievement has been making it increasingly difficult for mathematics teachers to maintain standards. We recommend that mathematics departments review evaluation procedures to ensure that grades reflect student achievement. Further, we urge administrators to support teachers in this endeavor.

4. In light of recommendation 3, we also recognize that the advancement of students without appropriate achievement has a detrimental effect on the individual student and on the entire class. We therefore recommend that school districts make special provisions to assist students when deficiencies are *first* noted.

5. We recommend that cumulative evaluations be given throughout each course, as well as at its completion, to *all* students. We believe that the absence of cumulative evaluation promotes short-term learning. We strongly oppose the practice of exempting students from evaluations.
6. We recommend that computers and minicalculators be used in imaginative ways to reinforce learning and to motivate the student as proficiency in mathematics is gained. Calculators should be used to supplement rather than to supplant the study of necessary computational skills.
7. We recommend that colleges and universities administer placement examinations in mathematics prior to final registration to aid students in selecting appropriate college courses.
8. We encourage the continuation or initiation of joint meetings of college and secondary school mathematics instructors and counselors in order to improve communication concerning mathematics prerequisites for careers, the preparation of students for collegiate mathematics courses, joint curriculum coordination, remedial programs in schools and colleges, the exchange of successful instructional strategies, the planning of in-service programs, and other related topics.
9. Schools should frequently review their mathematics curriculum to see that it meets the needs of its students in preparing them for college mathematics. School districts that have not conducted a curriculum analysis recently should do so now, primarily to identify topics in the curriculum that could be either omitted or de-emphasized, if necessary, in order to provide sufficient time for the topics included in this statement. We suggest, for example, that the following could be de-emphasized or omitted from the curriculum:
   (*a*) Logarithmic calculations that can better be handled by calculators or computers
   (*b*) The extensive solving of triangles in trigonometry
   (*c*) Proofs of superfluous or trivial theorems in geometry
10. We recommend that algebraic concepts and skills be incorporated wherever possible into geometry and other courses beyond algebra to help students retain these concepts and skills. ■

This position statement was prepared jointly by the National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091, and the Mathematical Association of America, 1225 Connecticut Ave., NW, Washington, DC 20036.

MISSOULA MEETING

16 - 18 MARCH 1978 • MISSOULA, MONTANA

Host: Montana Council of Teachers of Mathematics

Missoula began as a western settlement called Hellgate, named for the winds that blow in mid-winter (not early spring). The site of the opening session for this March meeting is the Village Motor Inn at the mouth of Hellgate Canyon and on the bank of the beautiful Clark Fork River.

The conference will be a roundup of nationally known mathematics educators who will provide a high-caliber conference for the Northwest. A sampling of the principal speakers, some of whom are greenhorns and tenderfeet, include Lola (Calamity June) May, John (the Duke) Egsgard, (Dandy) Don Kamp, and (Buffalo) Bob Wirtz. Other outlaws on the program are Boyd (Spud) Henry and Iris (the Washington Belle Starr) Dayoub.

An added feature of the roundup will be an administrators' conference on Friday. There will also be many interdisciplinary sessions for elementary and junior high school teachers. Western hospitality and free green beer (courtesy of MCTM on Thursday night) await you, as do ski areas and good sessions. So grab your thirty-eight-litre hats and head for the West for the mathematics rendezvous of the year!

Hotel rooms have been reserved for convention registrants at the Village Motor Inn and the Red Lion Motor Inn. Please make your reservations early; there is a state basketball tournament in Missoula the weekend of our meeting.

All convention activities will be held at Sentinel High School (S) and the Village Motor Inn (V). The program will begin on Thursday, 16 March, at 4:00 p.m. and end on Saturday, 18 March, at 2:00 p.m.

The *complete program booklet* was mailed in December 1977 to NCTM members in Montana, Idaho, Washington, Oregon, Wyoming, British Columbia, Alberta, and Saskatchewan. Others may request copies from the NCTM Headquarters Office, 1906 Association Dr., Reston, VA 22091.

Halve Your Meter

Charles B. Rhinehart
Music Director
Campus School
State University College
Oswego, New York 13126

Reprinted with permission from School Science and Mathematics, *the Official Journal of the School Science and Mathematics Association Inc., Volume LXXIV, Number 5, Whole No. 655, May-June 1974.*

What can a music teacher add when his elementary school decides to "go metric" for a month?

The *modus operandi* for the interdisciplinary approach in our school was for each teacher to design activities, from the perspective of his bailiwick, which lead to the mastery of certain competencies decided upon by the teachers as a group. One of these competencies involved the students learning some of the language of the metric system. My response to this was, "Why not use the device of rhyme and tune to learn the new metric vocabulary?"

"Halve Your Meter" is the result of such an attempt. To our surprise it has been an instant hit in music classes in Grades I through IV. It is our hope that familiarity with the metric language will provide a first step in understanding the metric system.

If students want to make their *own* songs, a math or science teacher could have his students write down their words, or better yet, have them record their own songs on tape. From such a tape many music teachers are able, and willing, to notate the students' work.

From the Standpoint of a music teacher looking at the metric system - how much more pleasant to have a spoonful of sugar to make the medicine go down.

Halve Your Meter

■

Teaching Ideas

Reprinted from The Math Post, *Volume 9, Issue 1*

Show Down

Objectives: Given cards with the answers to the basic facts of a given operation, the student will match one answer at a time to the problem by covering the correct square.

Variation: Play the game with partners. Players take turns turning over a card and covering square.

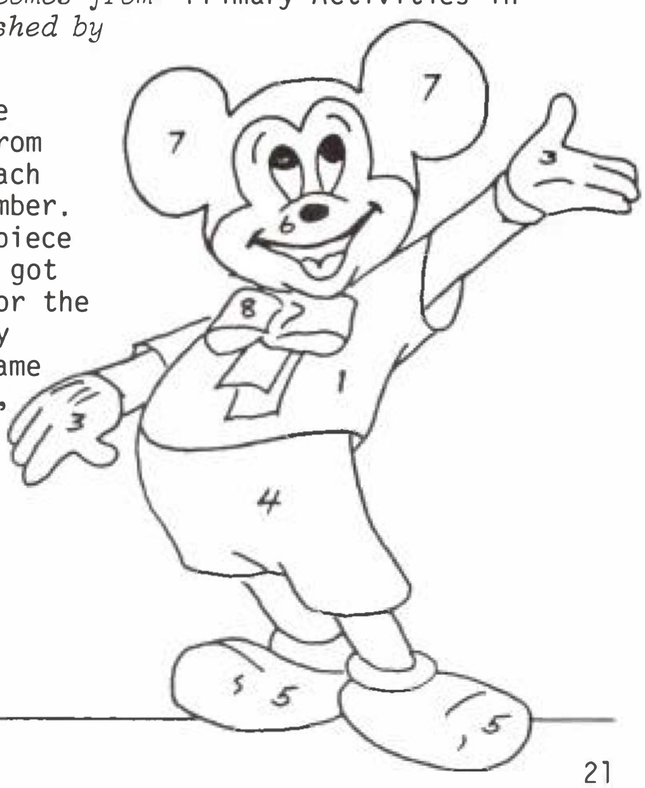
Materials: A playing board with 56 squares (8 rows of 7 squares); each square contains a basic fact of a given operation. Two sets of 56 cards - on each card is written an answer to match each fact on the board. (The operation of basic facts and the size of the game board can be varied.)

Procedure: Two players face each other on opposite sides of the game board. Each takes a set of cards and turns them face down. At an agreed time both begin turning over cards and covering the square to match the answer. (Only one card can be turned at a time.) If a player cannot find a place for a card, he puts it in a discard pile, then turns up another card. Play continues until the board is covered or both players have no cards left. The player with the greatest number of squares covered is the winner.

Mickey Mouse

This primary number recognition activity comes from Primary Activities in Mathematics *by Dr. Donald Buckeye; published by Midwest Publications.*

The game is played with two or more people. Cut out a body of Mickey Mouse from cardboard and cut it into pieces. Have each piece of the body labelled a different number. The student rolls the dice and takes the piece of body which has the same number that he got on the dice. The object of the game is for the student to put together the body of Mickey Mouse. The first to do this wins. The game can also be played by subtracting, adding, multiplying and dividing the numbers on the dice.



Divide Mickey into several pieces: ears, head, bowtie, hands, body, pants, shoes, and so on. Give each piece a different number.

Beetles

This first-year math game is sent to us by Wendy Both of Canyon Meadows Elementary School.

Use a die among 4 players. Take it in turns to throw the die. You must throw a 6 to begin (draw the body). To this you can add 2s (for legs) and 1 for the tail. You need a 5 for the head before you can add 3s (feelers) or 4s (eyes).

When a player has completed (drawn) a beetle, he calls "Beetle!" Everyone stops, counts the number of parts of the beetle drawn, and writes his total in the "score" column.

Start game #2. When nine games have been completed, add the scores to determine the Grand Total.

6 - body	3 - feeler
5 - head	2 - leg
4 - eye	1 - tail

1.	2.	3.	SCORE 1. ___ 2. ___ 3. ___
4.	5	6.	4. ___ 5 ___ 6. ___
7	8.	9.	7. ___ 8 ___ 9 ___ Total ___

A local bookstore is selling copies of Deal Me In *by Margie Golick. This book describes card games that help develop arithmetic skills for children five years to adult. The following games are found in the book.*

Tough Beans

A full deck of 52 cards is used. Each card is counted for its face value: Jacks = 11; Queens = 12; Kings = 13. The object of the game is to be the first to get rid of your cards.

Eight cards are dealt to each player. Then the pack is placed in the middle as a stock and the top card is turned face-up.

The player to the dealer's left plays face-up any card overlapping the turned-up card so that the denominations of both are visible. Next player must put down any number of cards whose total denominational value is equal to the sum or the difference of the two face-up cards. For example, if the face-up cards are King and 9, the next player must play cards totalling 22 (sum) or 4 (difference). After showing players his cards, he stacks them so that only one card is visible, and lays them on the face-up cards, again overlapping so there are again two cards (the last one played by the previous player, and the top card played by the player who has just completed his play) whose sum or difference must be arrived at by the next player.

When a player cannot arrive at the necessary total from the cards in his hand, he must draw from the stock until he is able to do so.

Up and Down

Numerical sequence regardless of suit and numerical sequence within suits are the important considerations in this game.

The deck is dealt out between the two players. Cards are left in a face-down pile before each player. The object of the game is to be the first to get rid of all your cards.

Players take turns facing up their cards - one at each turn - onto a pile in front of their face-down stacks. If a card is an Ace, it is put in the center of the table. Cards faced up may be played to the center (building upwards in suit sequence on the Aces) or on an opponent's face-up pile in sequence up or down regardless of suit. Thus, if one player has a Queen on the top of his face-up pile, the other player may place any Jack or King; if he has a 5 on top, any 4 or 6 may be played on it.

If a face-up card can be played, either to the center or on opponent's pile, player can have another turn and continue to face up cards until he can make no more plays. The top card of the face-up pile is always available for play.

When the face-down stack is exhausted, the face-up pile is turned over and play continues.

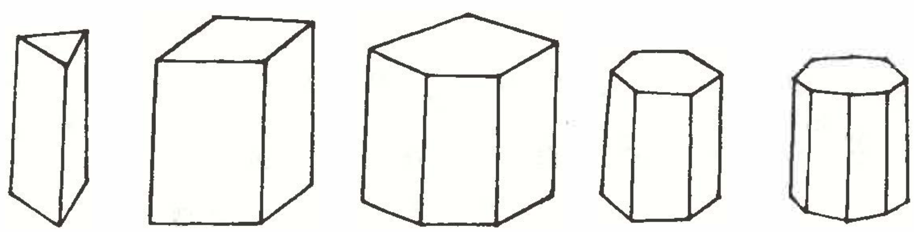
Change Anyone?

Everyone you ask says that he doesn't have the necessary coins for making change for a dollar bill. A few people, of course, just do not have enough coins. You look at the handful of coins held out by a friend and see that although they total more than $1.00, no combination of those coins will equal one dollar exactly. What is the largest amount of money you can have in coins, and still not be able to change $1.00?

Answer: four 1¢; four 10¢; three 25¢

Prisms

Use these models of prisms to help complete a chart like the one below. A prism is named by the shape of its base.

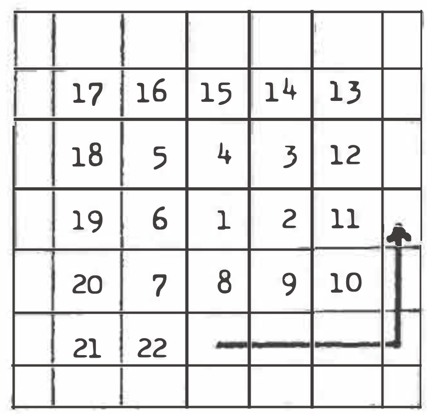


Prisms	Number of Faces	Number of Vertices	Number of Edges
Triangular			
Square-based			
Pentagonal			
Hexagonal			
Heptagonal			
Octagonal			
Decagonal			

1. Can you see a pattern? If so, what is it?
2. Without looking at models, complete the chart for: heptagonal, decagonal prisms.

Patterns

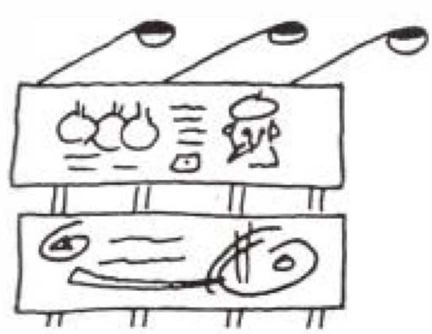
1. Have students use 0.5 cm graph paper and continue numbering around the spiral.
2. Ask students to discover patterns.
   a. Is there a ray which contains only the squares?
   b. Is there a ray which contains only the squares of odd numbers?
   c. Does one ray contain only primes?
   d. Can the corner numbers be found without counting each square?
   e. 16 + 4 = 5 + 15
   Does this pattern always hold? Ever hold?



16	15
5	4

Daffynition

A double-decker billboard?



Answer: Cosine

Reprinted from the Mathematics Student, *Vol. 22, Oct., 1974*

Mathematics Contest

A student mathematics contest held annually at Villanova University had two problems on the 1975 paper which were missed by most participants. Try them and Good Luck!

1. A motorist mounted snow tires on his car that were one inch more in diameter than the regular tires. The diameter of the regular tires was 20 inches. How many fewer revolutions per mile does each wheel make?

2. The sides of a trapezoid have measures of 20, 21, 21, 50 going in one direction. The measures of the parallel sides are 21 and 50. Find the area of the trapezoid to the nearest tenth.

And They Aren't Easy!

The problems below were part of the application form for the Student Science Training Program held at the University of Chicago in the summer of 1975. And they aren't easy! Try them.

1. The integer 25 is a "perfect square" since it is a square of another integer, namely the integer 5. The least positive remainder of 25 upon division by 8 is 1. Find, if possible, an integer which is a perfect square and which has the least positive remainder 3 upon division by 8.

2. Let A,B,C,D and E be five towns such that the straight line distance between A and B is 30 miles, between B and C is 80 miles, between C and D is 236 miles, between D and E is 86 miles and between E and A is 40 miles. Can you find the distance between C and E without a map?

3. In a very hotly fought battle, at least 70 percent of the combatants lost an eye; at least 75 percent an ear; at least 80 percent an arm; at least 85 percent a leg. How many lost all four members?

4. Nine students use a fraternity dining room which has four tables with three settings on each table. How should one vary the seating arrangement so that in four days each student should share a table at dinner with each other student?

Four Rows, Same Totals

Fill in the squares following these directions:

1. Use any numbers you want, so long as each figure is less than 15.
2. A number can be used just one time, and cannot duplicate a number already shown here.
3. When finished, the two long horizontal rows and the two long vertical rows will add up to identical totals, creating a Magic Cross.

		2	
5			
			12
	15		

Answer:

	3	*2*	
5	*10*	*11*	*8*
9	*6*	*7*	*12*
	15	*14*	

A set of four booklets by Swartz and Gardner and published by Houghton Mifflin entitled Measurement Concepts and Applications *offers a variety of interesting metric activities. The following activities come from Book 3 in this series.*

Rates and Shapes

An object gives off heat through its surface. So maybe the surface area of the object will affect its rate of heat loss. You can check up on this. Get two light plastic containers. One container should have a large diameter. The other should have a much smaller diameter. Pour equal volumes of hot water into the containers.

Read the temperature in each container every two minutes for ten minutes and make a record of the results.

Measure the surface area of the water that is in contact with the air.

Narrow container: diameter ________ cm
surface area of water ________ cm^3

Wide container: diameter ________ cm
surface area of water ________ cm^3

Does the surface area of the water affect the rate of heat loss? How?

More Bounce to the Gram

Does the height that a ball bounces depend on how far it falls? Does it depend on the kind of ball that's bouncing?

Drop a ball from a height of one metre. Have a friend watch the ball and place a clothespin on the metre stick at the height to which the ball bounced. Repeat the experiment until the clothespin is at the right place. Record the bounce height.

Repeat the experiment using drop heights of 50 cm, 150 cm and 200 cm.

Make a graph of the results.

Who is Ahead?

Last year another company in town offered you an interesting new job. However, it required taking a 10 percent decrease in salary. To make the switch didn't make sense, so you declined the job.

Your friend George was restless to try new areas, so he took the job. He got a 10 percent lower salary, and you received a 10 percent salary increase for loyalty.

This year, George did so well he received a 10 percent increase. Your company was hit by hard times, so you had to take a 10 percent reduction in salary.

Financially, who is ahead? You both started into last year with $10,000 salary apiece.

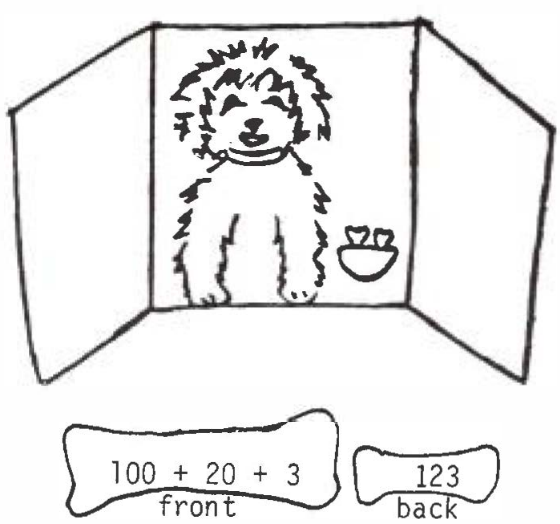
Answer: Both make $9,900

More Teaching Ideas

Reprinted from The Math Post, *Volume 7, Issue 2*

Mrs. Delores Schreiner, Assistant Principal of Marlborough Elementary School, is a veritable store of ideas. The following pages illustrate some of these. Other ideas will appear in subsequent issues of The Math Post.

Numeration



Feed the Dog a Bone

(two players)

Tape three pieces of pulpboard together and make a dog's (dinosaur, elephant and so on) picture on the middle sheet. Cut out the space for a mouth. Make cards shaped like a bone and place in the dog's dish.

First child sits behind the dog's picture and checks the answers. Second child feeds the bone saying the re-naming for 100 + 20 + 3 as 123. (First child checks the back; if not correct, he tosses the bone back to the second child.)

This can be played with bones which have

a. operations
b. fractions
c. geometric shapes
d. definitions (1 mile = ___ft.)

They can keep track of how many bones they *can* feed the dog to find a winner.

Game is over when the bones are all fed.

Greater - Less - Equal

(two players)

Develop a *set* of cards to 100 (about 50 cards).

Make some of the numbers the same,

for example $\boxed{42}$ $\boxed{49}$ $\boxed{94}$ ■ $\boxed{42}$ and so on $\boxed{\begin{smallmatrix}>\\<\\=\end{smallmatrix}}$ front of card

back of card

Deal the whole deck face down to each person. First player predicts his numeral will be >, < or =. Both players then lay down their top card. If the player's prediction is accurate, he gathers up both cards and puts them at the bottom of his deck. Each player takes turns predicting. The player with the most cards wins.

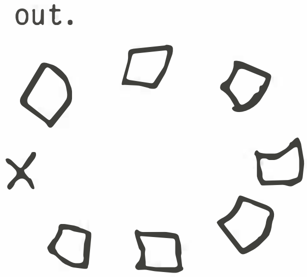
Skunked

(6 - 8 players + teacher)

Drill game.

Make a deck of cards with the numbers 25, 50, 75 and so on. Draw a picture of a skunk on one card and mix into the deck.

Have the children sit around in a circle with the teacher. Deal all the cards out.



The child with the first numeral in the counting by 25s (10s, 5s) sequence lays down his card, calling out 25; the second follows with 50, and so forth.

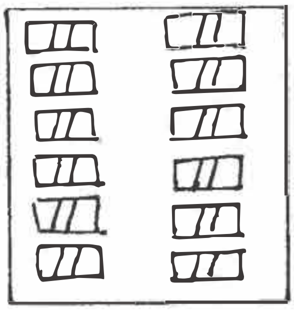
If a child plays the wrong card *or* if a child halts the play by not playing out the next numeral, the teacher calls "skunked," and the person holding the skunk card passes it to the child in error. (The teacher must of course determine *who* has the card which is needed to continue the play).

Each time the play is halted, the skunk card is passed to the child in error.

Whoever has the skunk card at the end of the game is the loser.

Throw-a-Hundred (Thousand - whatever!)

Make a stencil for place value to 100.



Materials - one die
- one pencil

1st player throws the die and enters the numeral in the *ones* place.
2nd player throws the die and enters the numeral in the ones place.
(See who is winning!)

The players go on to fill in the tens, then the hundreds. The person with the largest number wins and puts a check ✓ by the number on his sheet. The game is over when the paper is full. The person with the most ✓'s wins.

Think

(two to four players)

Make 2 decks of cards (15 each).

Front of decks:

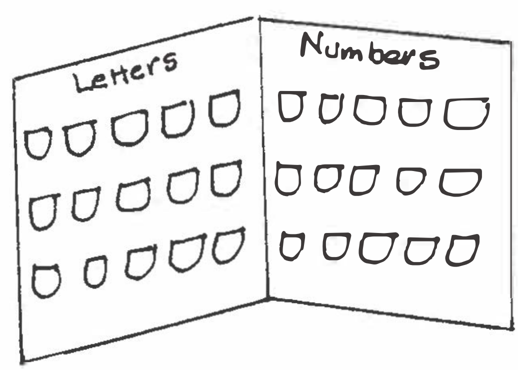
1st deck (15 cards)

A B C D E and so on to O

2nd deck (15 cards)

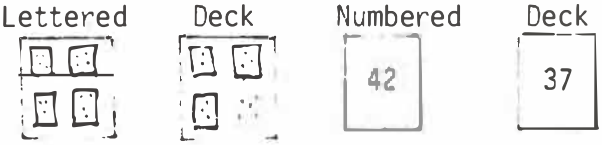
1 2 3 4 5 and so on to 15

Make a chart with 15 pockets on each side (use library card pockets).



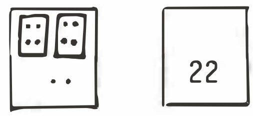
Back side of decks:

Lettered Deck Numbered Deck



Mix the cards up in each deck so that
A will *not* match 1
B will *not* match 2 and so on.

With the cards in the pockets on each half of the chart, the children take turns trying to turn over a letter card and a number card which are pairs, for example:



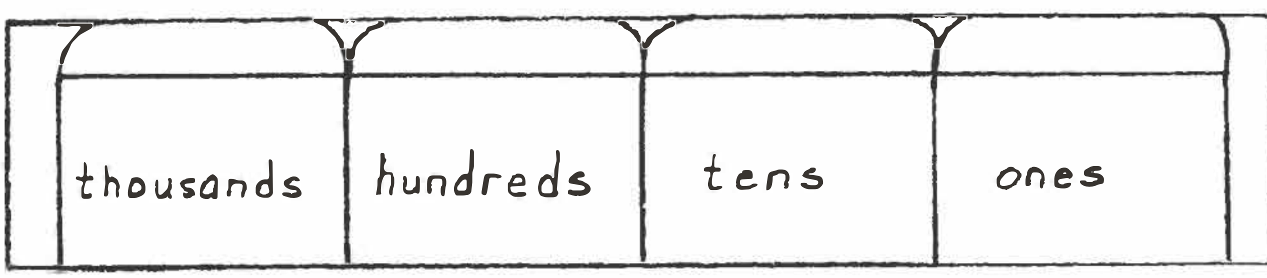
= a pair

If the two cards turned up do not match, the cards are put back into the pockets, and the next child has a turn. The children try and remember where the cards are placed in the pockets.

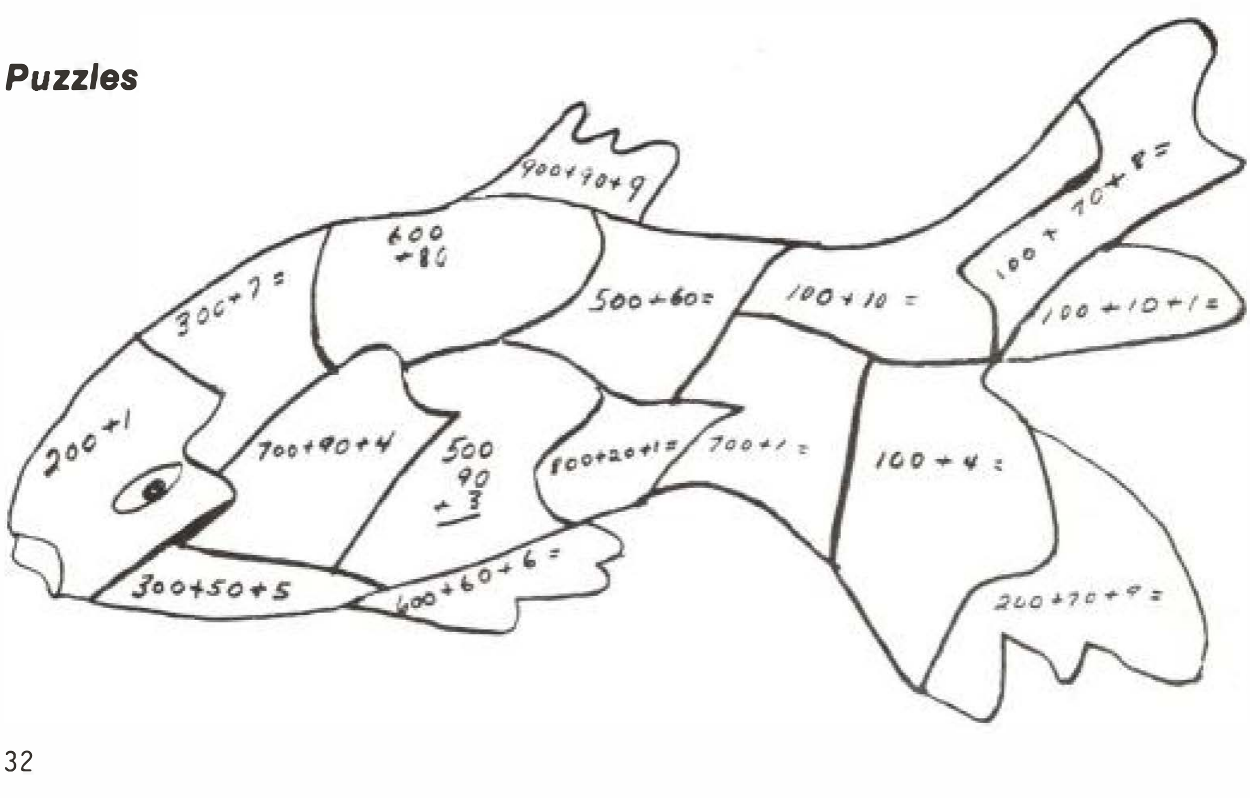
This game tests: a. knowledge of numbers
b. memory

The child with the most cards wins.

A handy, easily-made aid for teaching regrouping can be made from a piece of cardboard and a few library card pockets.

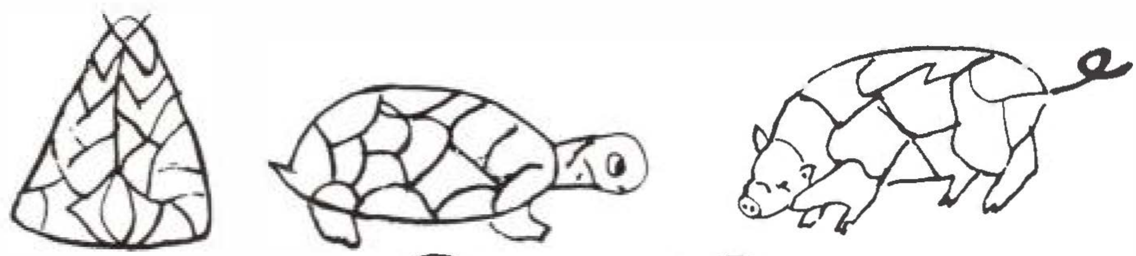


Puzzles



Cut up a second fish with answers 355, 821, and so on, and use this as a puzzle.

other ideas

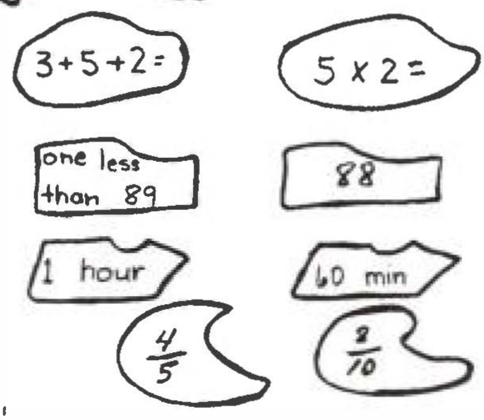


Use with operations:

Use with equalities:

Use with definitions:

Use with fractions:



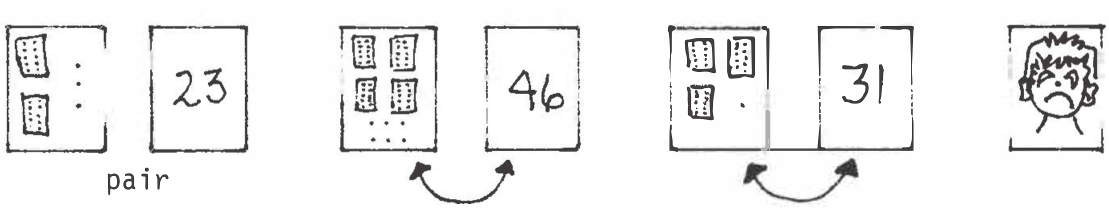
Use with *your* imagination!

A cook wanted to measure four millilitres of oil out of a container but he had only a five-millilitre and a three-millilitre bottle. How did he manage it?

Sad Face (Like Old Maid)

(four players)

Make a deck of cards (28 cards) plus one sad face for this numeration game.



Pass out 7 cards to each - one person will have 8.

Have the children match the pair in their hands and discard. The remainder are kept in the hand.

Player *one* picks one card from the child to his right. If the card he picks matches one in his hand, he discards them, turns, and player *two* picks a card from his hand. This play continues until one card is left - the sad face. The person with sad face is the loser.

You can use the game for

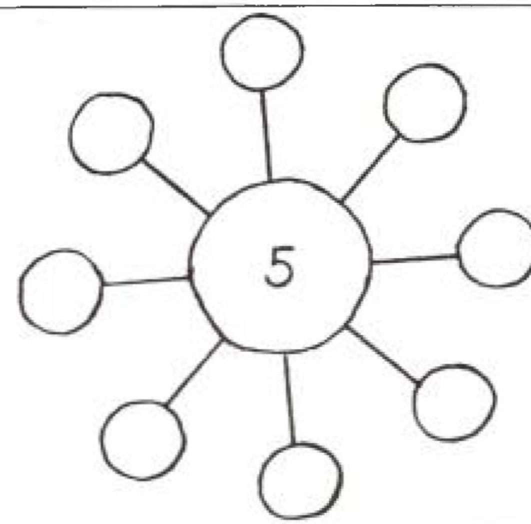
a.	operations		a.	homonyms
b.	time		b.	synonyms
c.	money		c.	contractions
d.	measurement	and	d.	antonyms
e.	geometry		e.	compound words
f.	fractions		f.	?????????

Another brain-teaser from "Math à la Mode"

In each circle place one of the following numbers:

1,2,3,4,5,6,7,8,9

so that the sum of the three numbers in each line is 15.



Active Involvement in Arithmetic Learning

At the annual MCATA conference in Jasper, Robert Eicholtz, full-time writer of mathematics curriculum materials, gave an address on active involvement in arithmetic learning. He cited examples of activities which should be part of daily lessons. Involvement should be obtained in free and open-ended ways.

As a basis for the activities, Mr. Eicholtz uses Piaget's learning theories which are interpreted in the following ways:

Three steps in the formation of a concept are:

1. preliminary stage
   - this is the active involvement stage
   - play, explore, investigate
   - involvement is manipulative, investigative and open-ended

2. structural stage
   - the coming together of ideas

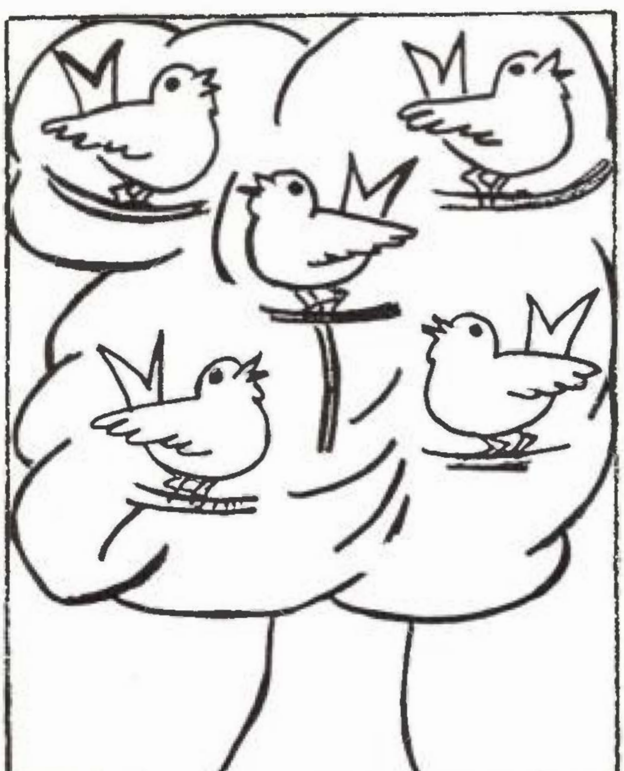
3. practice stage
   - practice what has been learned

A five-point teaching strategy based on these stages involves:

1. preparation
2. investigation — active involvement
3. discussion
4. utilization
5. extension

Following are some of the examples presented at the conference:

First Year - Mastery of Basic Facts



Each student chooses four circle counters and four triangle counters.

The teacher asks:

How many birds are in the tree?

Hide the birds with your counters.

How many circles did you use?

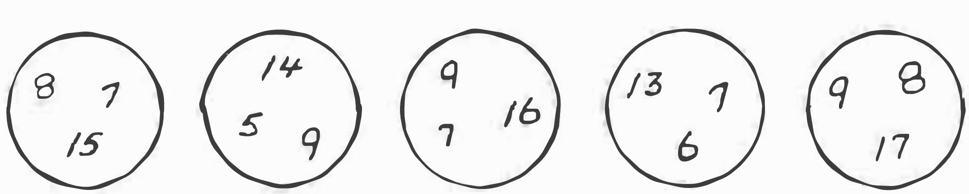
How many triangles did you use?

Find a different way to hide the birds with your counters.

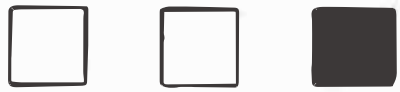
How many different ways can you hide the birds with your counters? (open-ended)

Note: In his address, Mr. Eicholtz used mice instead of birds.

Second Year - Number Combinations



Memorize one of these sets of numbers.
Put your numbers on paper this size.

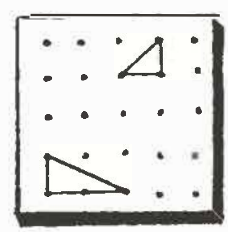


Can you place your papers on the spaces below to make equations? Record the equations you find.

$$\square + \square = \square$$

Fifth Year - Areas of Triangles

Two right triangles of different shapes are shown on the geoboard.



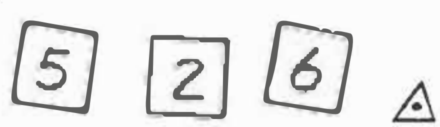
How many triangles of the same size and shape can you find on a geoboard?

How many triangles of different sizes or shapes can you find on a geoboard?

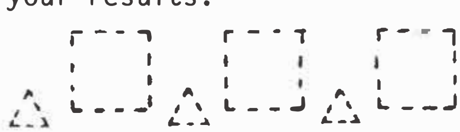
Show your triangles on paper.

Sixth Year - Comparison of Decimals

Make 4 slips of paper like these.



How many ways can you place all your slips on the spaces so that the inequality is true? Record your results.

$6.52 >$ 

Daffynitions

PRISM - a place for bad mathematicians
MATRICES - what mathematicians sleep on

Calculator Calisthenics

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Learn the Facts! Multiplication, That Is!

This game uses the hand-held calculator to help the elementary pupils learn the multiplication facts. It is proving more effective than flash cards and motivates much more interest than ordinary drill procedures. It is very simple and requires minimal involvement or supervision by the teacher. The game can be used for an entire class where there is one hand-held calculator available for each pair of students. However, it is equally effective when used at a learning center where only two or three calculators are available.

Have two students assigned to each calculator (Student X and Student Y). Student X is on Team 1 and Student Y is on Team 2. This way half of the class constitutes Team 1 and the other half Team 2.

Each pair has a score sheet marked "Team 1" and "Team 2."

Place the calculator on the desk so both players can see the keyboard and readout.

Each Student X enters any multiplication problem he chooses into the calculator, say 6 x 8.

Student Y observes the problem entered and states the answer "48," and then Student Y presses the "=" key. The answer given agrees with the calculator readout "48." Team 2 gets one point.

Student X enters another problem, say 7 x 9. Student Y answers "73" and presses the "=" key. The answer in the readout "63" does not agree with the answer given. Team 1 gets one point.

Student X enters three more problems (five in all). Then Student Y enters five problems for Student X to answer. They continue in this manner until the teacher states that time is up.

The teacher now adds all Team 1 scores and all Team 2 scores to determine which team is ahead.

The teacher can set the guidelines at the beginning of the game to suit her pupils' needs. For example, she can limit the problems given to those using the digits 0, 1, 2, 3, 4, 5, or she can limit it to one-digit factors. One teacher told the class to use two-digit factors and said the student giving the answer could use paper and pencil before answering. The game can be used also for the addition facts in the primary grades. The general format of the game is adaptable to many teacher options.

Once the game is begun, the students do it all themselves. They choose the problems. They determine which team gets the point. It is interesting that they tend to use the very facts where drill is most needed. Since the object of the game is to give the opponent a difficult problem which he might miss, the students tend to drill on the very facts they consider the most difficult. They can ask a problem even if they are not certain of the answer since the calculator will supply the correct answer.

This game is the outgrowth of the comments of a distraught fourth-grade teacher. When I visited her classroom a few weeks ago, she told me that she would not use calculators in her mathematics lessons until her students learned their multiplication facts. She told me if I came up with a way to teach the facts, she would use the calculators. This challenge led to this simple little game. It has been used now by several teachers in the fourth, fifth, and sixth grades in a large metropolitan school system and all report that the pupils really enjoy the game and become involved. We have reason to believe that a few minutes of playing this game each day will noticeably improve retention of the muliplication facts.

So teachers, let those little electronic wonders work to your advantage and your students' pleasure. Happy Calculator Calisthenics!! ■

Reinforce division by learning ratios

WILLIAM R. ARNOLD

Currently an associate professor of elementary mathematics education at the University of Northern Colorado in Greeley, William Arnold has had experience teaching on both the elementary and junior high levels. He has also served as a consultant in several school districts.

At the sixth-grade level, pupils are expected to learn several concepts that are related to ratios in one way or another; for example, *rational number*, *decimal*, *percent*, and *constant*. Learning and relating these concepts is affected by factors which, to many teachers, seem difficult to reconcile. The process of abstracting the concepts, learning symbolic representations of the concepts, and making a shift from concrete or visual to formal or abstract ways of thinking about these concepts are among the factors that must be taken into consideration. With all of this, teachers consistently express concern for how to employ interesting, meaningful ways to teach the concepts without detracting from a need to teach basic skills.

Each of the three activities described below reinforce the basic skills of measuring and computing with decimals by having pupils discover that, in one sense, a ratio is a single number that expresses a relationship between two other numbers. The three activities usually require three to four mathematics periods. In a self-contained classroom they are most successful when the whole class is involved. In other teaching situations, groups of ten to fifteen pupils are best. Pupils should be allowed to share information, and everything should be done to create an atmosphere of cooperative problem-solving.

Activity 1

Begin by asking the students: What are ratios? Encourage the students to express their own ideas of ratios. Then tell the class that during the next few periods they are going to study ratios, and suggest that they try to figure out what the word *ratio* means.

Make available to the class several circular objects, some string that does not stretch, and rulers that have a centimeter scale. Have the students carefully measure the circumferences and diameters of the objects using the string and rulers. The results of each measurement should be recorded in a systematic way. (See table 1.)

Table 1
Measurements of circular objects

Object	*Circumference* (c)	*Diameter* (d)	$\frac{c}{d}$
wheel	20.8	6.5	$\frac{20.8}{6.5} = 3.2$
can			
bottle			
⋮			

When measurements are completed, have the students divide the circumferences by the respective diameters. Then ask the students how the two sets of measurements seem to compare. Let the students discuss the results. Some pupils will know that this

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number, approximately 3.14, is called pi but the name is not essential in this particular activity.

Activity 2

Make available to the students pieces of ruled paper and rulers with centimeter scales. Have the students draw several squares—the sides of the squares can be any length. Then have the pupils measure the lengths of the sides and of the diagonals of the squares to the nearest tenth of a centimeter and record the information in a table. (See table 2.) When the measurements have been recorded, have the students divide the lengths of the diagonals by the respective lengths of the sides. Again, have the students compare the two sets of measurements. The quotients (ratios) will approximate 1.42. Let the students discuss this result.

Try to have students see that a ratio is a single number that tells how two numbers are related. The number 3.14 tells how the circumference and the diameter of a circle are related. The number 1.42 tells how the diagonal of a square is related to the side of the square. These numbers or ratios are *constant*: the relationship is the same for all circles and for all squares.

Activity 3

Make rulers and paper available to the students. Have each student draw several different rectangles; encourage them to draw rectangles that vary in appearance. Then tell the class that the ancient Greeks were interested in aesthetics—what made things beautiful. Suggest that each student think about rectangles and then draw what he believes would be a beautiful rectangle. This will seem mysterious to the students and the mystery can be played up. Have each student measure the length and width of his "beautiful" rectangle in centimeters and divide the length by the width. Examine the results and select those rectangles for which the ratio of length to width is greater than 1.4 and less than 1.8. Tell the class that these are examples of rectangles that the ancient Greeks thought were beautiful. You might display all of the "beautiful" rectangles along with their respective ratios and let the class discover the critical ratios. Have students check the dimensions of familiar rectangular things like picture frames, stationery, and rooms, and discuss why or why not these things are beautiful by the Greek standards.

It is quite possible that some students will not agree with the Greek idea of what is a beautiful rectangle; a student may prefer long narrow rectangles, for example. (There are also reasons for making rectangles that do not fit the Greek proportions; these reasons may come out in the class discussions.) It will not matter if students vary in their ideas of what is beautiful in a rectangle. The next step of this activity will bring out the mathematical relationships inherent in what is sometimes referred to as the "golden rectangle." Students will see that a rectangle can be beautiful in a mathematical way.

After sufficient discussion, have each student (1) measure the width of his most beautiful rectangle, (2) reproduce the width on each length, and (3) connect the two points so that a square is formed at one end of his rectangle. Tell the students to shade the square as shown in figure 1.

Table 2
Measurements of squares

Length of side (s)	*Length of diagonal (d)*	$\frac{d}{s}$
1	1.4	1.4
2	2.8	1.4
3	.	.
.	.	.
.	.	.
.		

The unshaded part of each figure is another rectangle. Have each student compare the shape of his new rectangle with the shape of his original rectangle. The students

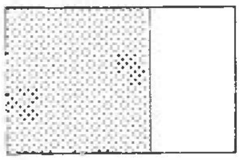
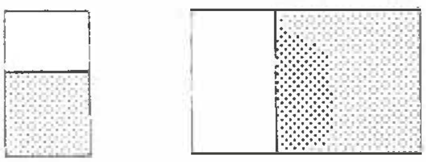


Fig. 1

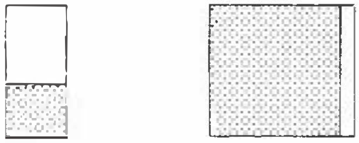
will notice that for rectangles having ratios of length to width between 1.4 and 1.8, the new rectangle has nearly the same form as the original rectangle. (See fig. 2.) For



"Beautiful" rectangles

Fig. 2

other ratios, the new rectangle will look much different than the original rectangle. (See fig. 3.)



"Ugly" rectangles

Fig. 3

Mathematics works in wondrous ways. It just happens that if the ratio of the length to width of a rectangle approximates 1.618, the new rectangle formed by drawing the square will have the same form as the original rectangle.

At this point, you may conclude the activities with a discussion of ratios. I know one teacher who extended the activity by having each student select his favorite width. The student then multiplied this number by 1.618 and used his width and the obtained length (width multiplied by 1.618) to draw a rectangle. He next used his width to draw a square "inside" his rectangle to produce a new rectangle. This process was continued to obtain a drawing like figure 4.

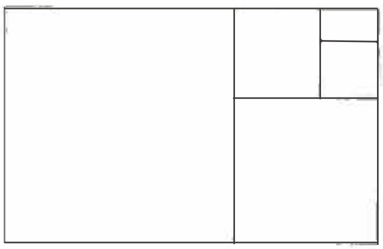


Fig. 4

As you can see, these activities involve the students in much drawing, measuring, and computing while they study ratios. The activities also encourage exploration. Some students may be interested in further research on "golden rectangles." Students may also discover other constant ratios as they find more examples of ratios.

References

National Council of Teachers of Mathematics. *Historical Topics for the Mathematics Classroom: Thirty-first Yearbook.* Washington D.C.: The Council, 1969. ■

Procedures for designing your own metric games for pupil involvement

CECIL R. TRUEBLOOD and
MICHAEL SZABO

Currently an associate professor in mathematics education at Pennsylvania State University, Cecil Trueblood is particularly interested in the teaching of mathematics in the elementary school. Michael Szabo, also at Pennsylvania State, is an associate professor of science education. His educational interests center on instructional development, individualized instruction, and complex problem-solving.

Although much has been written on the values of mathematical games in the elementary grades and many game books have been published, little has been written that would help classroom teachers design, produce, and evaluate games for use in their classroom. The focus of this article is to present a set of seven criteria that were developed in a summer workshop for inservice elementary teachers who decided that they wanted to be able to produce metric games and related activities that would fit into their "metrication" program.

The teachers in the workshop began by asking a practical question: Why should I be interested in producing my own metric games? They concluded that the game format provided them with specific activities for pupils who did not respond to the more typical patterns of instruction. They felt that in the game format they could provide activities of a higher cognitive level for pupils who had difficulty responding to material requiring advanced reading skills.

The teachers then asked a second question: Does the literature on the use of mathematics games contain any evidence that would encourage busy classroom teachers to use planning time to develop their own games? The available professional opinion supported the following conclusions:

1. Games can be used with modest success with verbally unskilled and emotionally disturbed students, and students for whom English is a second language.
2. Games have helped some teachers deal with students who present discipline problems because they are bored with the regular classroom routine.
3. Games seem to fit well into classrooms where the laboratory or learning-center approach is used. This seems related to the feature that games can be operated independent of direct teacher control thus freeing the teacher to observe and provide individual pupils with assistance on the same or related content.

Plan for development

If for any of the reasons just cited you are interested in designing and evaluating several of your own metric games, how should you begin? Simply use the following checklist as a step-by-step guide to help you generate the materials needed to create your game. Use the exemplar that follows the checklist as a source for more detailed

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suggestions. Each item in the checklist has been keyed to the exemplar to facilitate cross referencing.

CHECKLIST GUIDE

- ___ Write down what you want your students to learn from playing your game. (Establish specific outcomes)
- ___ Develop the materials required to play the game. (Make simple materials)
- ___ Develop the rules and procedures needed to tell each player how to participate in the game. (Write simple rules and procedures)
- ___ Decide how you want students to obtain knowledge of results. (Provide immediate feedback)
- ___ Create some way for chance to enter into the playing of the game. (Build in some suspense)
- ___ Pick out the features that can be easily changed to vary the focus or rules of the game. (Create the materials to allow variation)
- ___ Find out what the students think of the game and decide whether they learned what you intended them to learn. (Evaluate the game)

The exemplar

Establish specific outcomes

By carefully choosing objectives that involve both mathematics and science processes—such as observing, measuring, and classifying—the teachers created a game that involves players in the integrated activities. This approach reinforces the philosophy that science and mathematics can be taught together when the activities are mutually beneficial. That is, in many instances integrated activities can be used to conserve instructional time and to promote the transfer of process skills from one subject area to the other. The exemplar's objectives are labeled to show their relationship to science and mathematical processes.

1. Given a set of common objects, the students estimate the objects' weight correct to the nearest kilogram. (Observation and estimation)

2. Given an equal-arm balance, the students weigh and record the weights of common objects correct to the nearest centigram. (Measurement)

3. Given an object's estimated and observed weight correct to the nearest centigram, the student computes the amount over or under his estimate. (Computation and number relationships)

Make simple materials

The following materials were constructed or assembled to help students attain the objectives previously stated in an interesting and challenging manner.

1. Sets of 3-by-5 cards with tasks given on the front and correct answers and points to be scored on the back. (See fig. 1.)

2. A cardboard track (see fig. 2) made from oak tag. Shuffle the *E*'s (estimate cards), *O*'s (observed cards), and the *D*'s (difference cards) and place them on the gameboard in the places indicated.

3. An equal-arm balance that can weigh objects up to 7 kilograms.

4. A pair of dice and one different colored button per player.

5. A set of common objects that weigh less than 7 kilograms and more than 1 kilogram.

6. Student record card. (See fig. 3.)

Write simple rules and procedures

The rules and procedures are crucial to making a game self-instructional. In the following set of directions notice how a student leader and an answer card deck serve to ease the answer processing needed to keep the game moving smoothly from one player to another. It is essential to keep the rules simple and straightforward so that play moves quickly from one student to the other.

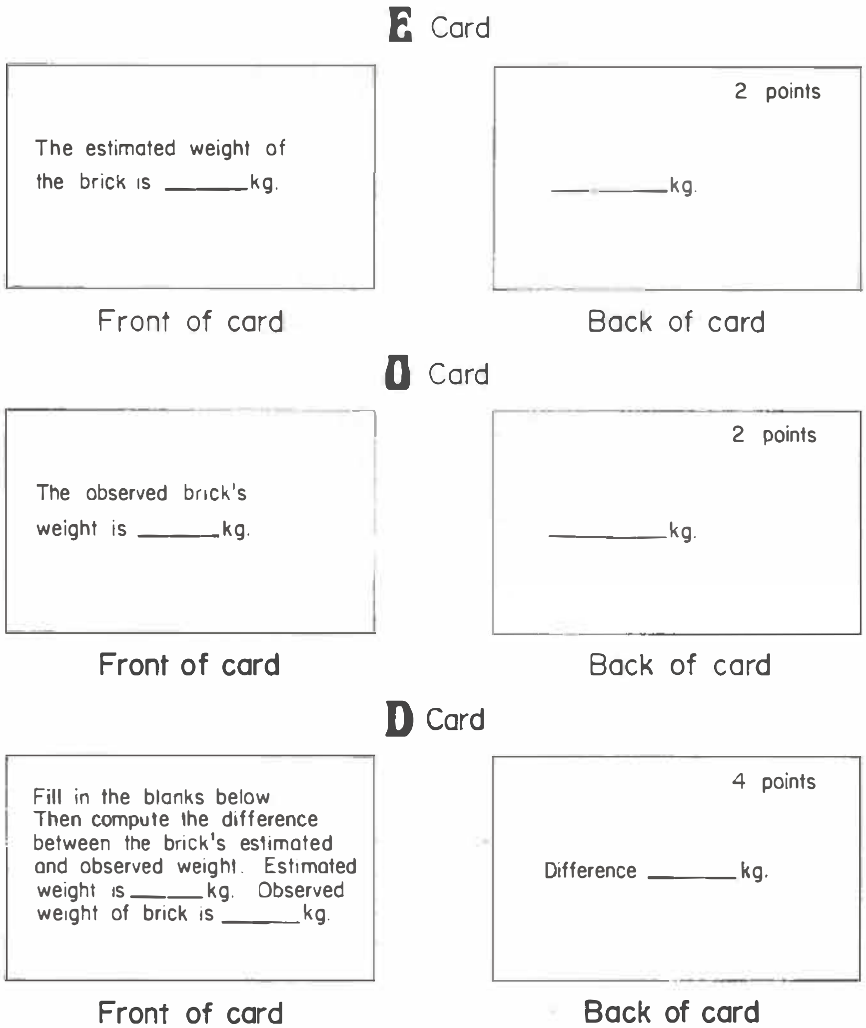

Fig. 1

1. Number of players, two to six.

2. The student leader or teacher aide begins by rolling the dice.

The highest roll goes first. All players start with their buttons in the "Start Here" block. The first player rolls one die and moves his button the number of spaces indicated on the die. If he lands on a space containing an *E*, *O*, or *D* he must choose the top card in the appropriate deck located in the center of the playing board or track and perform the task indicated. (In the example shown in figure 1 this would be Card E_3.) The player then records the card number, his answer, and the points awarded by the student leader on his record card. The student leader checks each player's answer and awards the appropriate number of points by reading the back side on the task card. He then places that card on the bottom of the appropriate deck and play moves to the right of the first player. The player who reaches "Home" square with the highest number of points is the winner. At the end of the play each player turns in his score card to the student leader who gives them to the teacher.

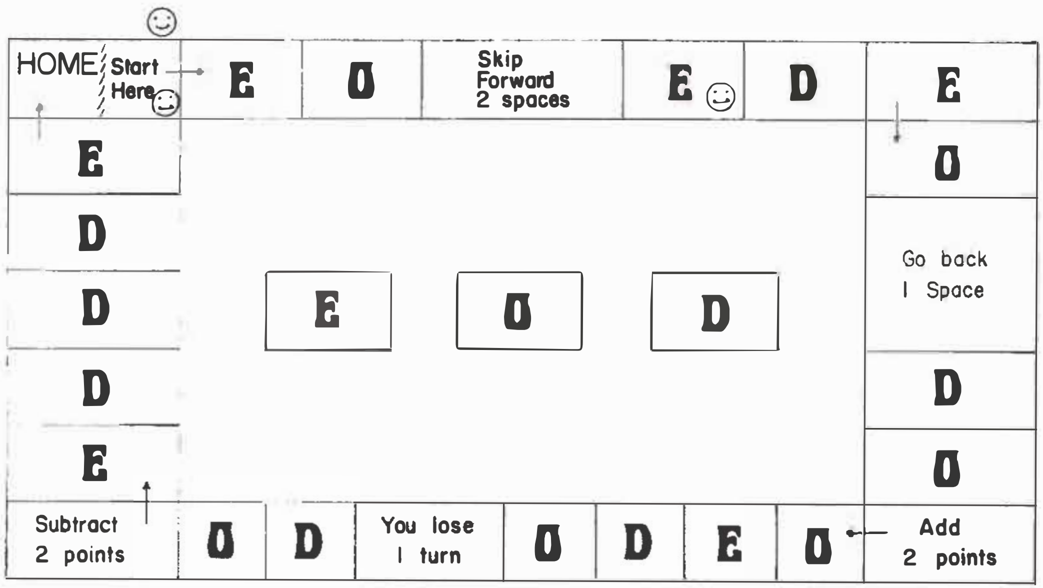

Fig. 2

Provide immediate feedback

By placing the answer on the back of the task card and appointing a student leader, the teacher who developed this game built into the game an important characteristic, immediate knowledge of the results of each player's performance. In most cases this feedback feature can be built into a game—by using the back of task cards, by creating an answer deck, or by using a student leader whose level of performance would permit him to judge the adequacy of other students' performance in a reliable manner. Feedback is one of the key features of an instructional game because it has motivational as well as instructional impact.

Have students record diagnostic information. The student record card is an important feature of the game. The cards help the teacher to judge when the difficulty of the task card should be altered and which players should play together in a game, and to designate student leaders for succeeding games. The card also provides the player with a record that shows his scores and motivates him to improve.

This evaluative feature can be built into most games by using an individual record card, by having the student leader pile cards yielding right answers in one pile and cards with wrong answers in another pile, or by having the student leader record the results of each play on a class record sheet.

Build in some suspense

Experience has shown that games enjoyed by students contain some element of risk or chance. In this particular game a player gets a task card based upon the roll of the die. He also has the possibility of being skipped forward or skipped back spaces, or of losing his turn. Skipping back builds in the possibility of getting additional opportunities to score points; this feature helps low-scoring students catch up. Skipping forward cuts the number of opportunities a high-scoring player has to accumulate points. The possibility of adding or subtracting points also helps create some suspense. These suspense-creating features help make the game what the students call "a fun game."

Student's name ______________________ Date __________

Card number	Answer given	Number of points
E_3	2 kg.	2
E_2	1 kg.	1

Fig. 3

Create the materials to allow variation

A game that has the potential for variation with minor modifications of the rules or materials has at least two advantages. First, it allows a new game to be created without a large time investment on the part of the teacher. Second, it keeps the game from becoming stale because the students know all the answers. For instance, the exemplar game can be quickly changed by making new task cards that require that students estimate and measure the area of common surfaces found in the classroom such as a desk or table tops. By combining the two decks mixed practice could be provided.

Evaluate the game

Try the game and variations with a small group of students and observe their actions. Use the first-round record cards as a pretest. Keep the succeeding record cards for each student in correct order. By comparing the last-round record cards with the first-round record cards for a specific student, you can keep track of the progress a particular student is making. Filing the cards by student names will provide a longitudinal record of a student's progress for a given skill as well as diagnostic information for future instruction.

Finally, decide whether the students enjoy the game. The best way is to use a self-report form containing several single questions like the following, which can be answered in an interview or in writing:

1. Would you recommend the game to someone else in the class? __Yes __No

2. Which face indicates how you felt when you were playing the game?

3. What part of the game did you like best?

4. How would you improve the game?

Concluding remarks

The procedure just illustrated can be generalized to other topics in science and mathematics. The following list provides some suggested topics.

1. Classifying objects measured in metric units by weight and shape
2. Measuring volume and weight with metric instruments
3. Measuring length and area with metric instruments
4. Classifying objects measured in metric units by size and shape
5. Comparing the weight of a liquid to its volume
6. Comparing the weight of a liquid with the weight of an equal volume of water
7. Predicting what will happen to a block on an inclined plane
8. Comparing the weights of different metals of equal volume

Why don't you try and create some games for each of these topics? Then share the results with your colleagues. Additional examples developed by the authors are available in "Metric Games and Bulletin Boards" in *The Instructor Handbook Series* No. 319 (Dansville, New York, 1973). ■

The four operations: wrapping it up

RICHARD J. TEMPLE
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Mathematical games and puzzles have gained tremendous popularity and support in today's "hands-on" approach to classroom instruction. Manipulatives can be useful both as learning devices and as a means to drill. However, many middle school students feel the need for written drill work. The trend is to let the student do drill work on his own. Too often, such drill is never checked or perhaps never even done.

In an effort to develop meaningful drill work that reinforces several skills in the process, I came up with the idea described here. It requires a bare minimum of materials, namely pencil and paper. At first students should be provided with paper ruled as shown in figure 1. As the format becomes more familiar, the students can rule their own paper. The only other thing needed is two numbers, one to be placed to the left, the other to the right of the

$$\begin{bmatrix} + \\ - \\ \times \\ \div \end{bmatrix}$$

at the top of the drill sheet.

The following skills can be included in this drill work:

(1) writing fractions as equivalent decimals;
(2) writing decimals as equivalent fractions;
(3) addition, subtraction, multiplication, and division of
  (*a*) two whole numbers,
  (*b*) two fractions,
  (*c*) two mixed numbers,
  (*d*) two decimals,
  (*e*) two mixed decimals.

$\begin{pmatrix} + \\ - \\ \times \\ \div \end{pmatrix}$	NAME
FRACTIONS	DECIMALS
(+)	(+)
(−)	(−)
(×)	(×)
(÷)	(÷)

Fig. 1

The addition of fractions and mixed numbers can be with or without common denominators. The subtraction of fractions and mixed numbers can be with or without common denominators and with or without regrouping. The multiplication of fractions and mixed numbers can be with or without cancellation. (See fig. 2.) The degree of difficulty of the drill depends on the numbers given. Repeating decimals and their equivalent fractions, however, should be avoided.

The ideas involved in this drill sheet are by no means new. I think its value lies in its organization; it provides the student with a systematic means of checking his work and a clear means of comparing the algorithms of the four basic operations. As the student becomes more familiar with the exercise, he tends to rely less on the teacher and more on himself for success. Since no "answer key" is necessary, the student is able to proceed from operation to operation with growing confidence. There seems to be, somehow, a built-in motivational force that impels the student to complete the exercise successfully. As one eighth grade student put it: "This sheet kinda wraps it all up!"

$2\frac{1}{2}$ $\left(\begin{smallmatrix}+\\-\\\times\\\div\end{smallmatrix}\right)$ 1.75	Name ______
Fractions	**Decimals**
(+) $2\frac{1}{2} = 2\frac{2}{4}$ $1\frac{3}{4} = 1\frac{3}{4}$ $3\frac{5}{4} = 4\frac{1}{4}$	(+) $\begin{array}{r} 2.5 \\ 1.75 \\ \hline 4.25 \end{array}$
Check $4\frac{1}{4} = 4\frac{25}{100} = 4.25$	
(−) $2\frac{1}{2} = 2\frac{2}{4} = 1\frac{6}{4}$ $-1\frac{3}{4} = 1\frac{3}{4} = 1\frac{3}{4}$ $\frac{3}{4}$	(−) $\begin{array}{r} 2.5 \\ -1.75 \\ \hline .75 \end{array}$
Check $\frac{3}{4} = \frac{75}{100} = .75$	
(×) $2\frac{1}{2} \times 1\frac{3}{4} = \frac{5}{2} \times \frac{7}{4}$ $= \frac{35}{8}$ $= 4\frac{3}{8}$	(×) $\begin{array}{r} 1.75 \\ 2.5 \\ \hline 875 \\ 350 \\ \hline 4.375 \end{array}$
Check $4\frac{3}{8} = 4\frac{375}{1000} = 4.375$	
(÷) $2\frac{1}{2} \div 1\frac{3}{4} = \frac{5}{2} \div \frac{7}{4}$ $= \frac{5}{\cancel{2}_1} \times \frac{\cancel{4}^2}{7}$ $= \frac{10}{7}$ $= 1\frac{3}{7}$	(÷) $1.75\overline{)2.50000000}$ = $1.\overline{428571}$ $\begin{array}{r} 175 \\ \hline 750 \\ 700 \\ \hline 500 \\ 350 \\ \hline 1500 \\ 1400 \\ \hline 1000 \\ 875 \\ \hline 1250 \\ 1225 \\ \hline 250 \\ 175 \\ \hline 75 \end{array}$
Check $\frac{3}{7} = .\overline{428571}$	

Fig. 2. Sample work sheet

GIVE SNOOPY A HAND TO FIND THE SECRET TREASURE

Submitted by Bob Holt, Edmonton Public Schools, Edmonton, Alberta.

WHERE'S THE TREASURE?

Join the dots. Where the lines cross you'll find the treasure.
Complete the number sentence; the answer will tell you the first number in the next number sentence.

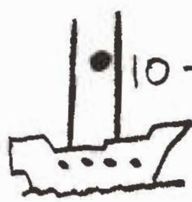
Snoopy did the first one to help you!

START HERE

1+1= 2

7+3=

6+1=



10-2=

4+2=

2+3=

45-23 =

8+3=

1+3=

22 + 6 =

5-4=

11 + 4 =

31

5×9=

30÷6

15- 6=

14+3=

17-2=

49-18=

36-6=

9+7=

16-2=

15÷5=

28÷4=

3×8=

7×7=

6×6=

24÷4=

WHAT'S THE TREASURE?

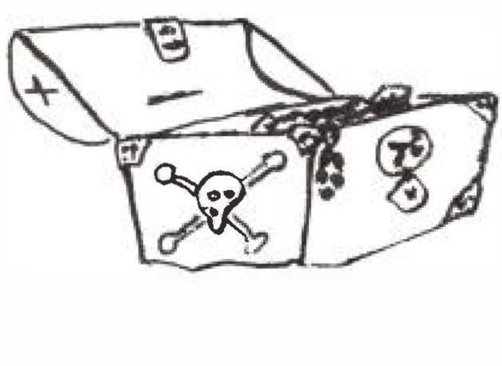
The answers to these questions will give you some scrambled words. The words will complete the sentence on the clue page.
The clues are on the next page.

$$\begin{array}{r} 556 \\ 32 \\ +\ 301 \\ \hline \end{array} \qquad \begin{array}{r} 355 \\ 466 \\ +\ 22 \\ \hline \end{array} \qquad \begin{array}{r} 27 \\ 46 \\ +82 \\ \hline \end{array} \qquad \begin{array}{r} 410 \\ 30 \\ +840 \\ \hline \end{array}$$

$$\begin{array}{r} 3066 \\ +2325 \\ \hline \end{array} \qquad \begin{array}{r} 4658 \\ +3767 \\ \hline \end{array} \qquad \begin{array}{r} 350 \\ +206 \\ \hline \end{array} \qquad \begin{array}{r} 425 \\ +250 \\ \hline \end{array}$$

$$\begin{array}{r} 327 \\ -163 \\ \hline \end{array} \qquad \begin{array}{r} 448 \\ -325 \\ \hline \end{array} \qquad \begin{array}{r} 667 \\ -377 \\ \hline \end{array} \qquad \begin{array}{r} 575 \\ -486 \\ \hline \end{array}$$

$$\begin{array}{r} 300 \\ -\ 28 \\ \hline \end{array} \qquad \begin{array}{r} 400 \\ -333 \\ \hline \end{array} \qquad \begin{array}{r} 500 \\ -226 \\ \hline \end{array} \qquad \begin{array}{r} 600 \\ -199 \\ \hline \end{array}$$

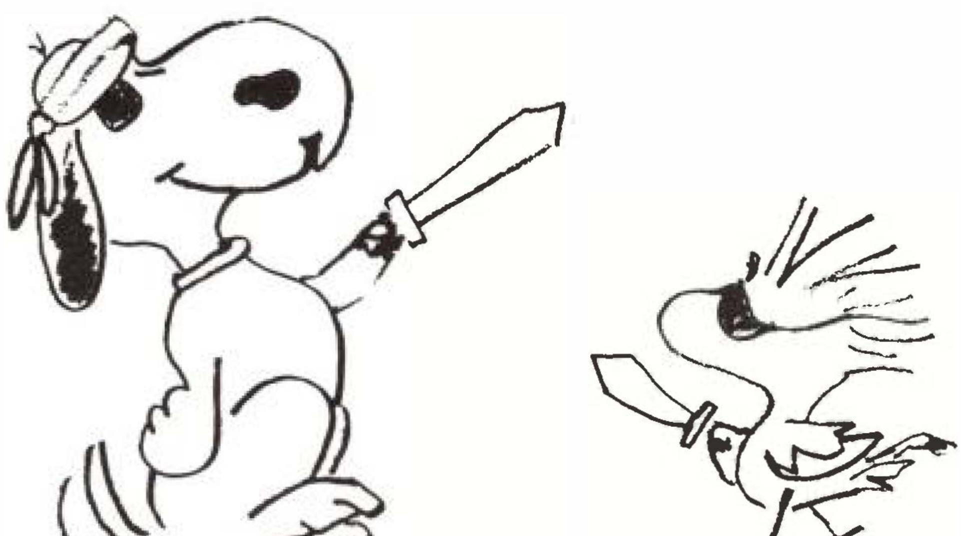


CLUES

DIAMONDS	880	SEED	556
JEWELS	1295	CANDY	295
PEARLS	5300	DOG	123
BONES	675	MONEY	90
DIRT	3420	BULLION	65
ROCKS	160	BIRD	274
AND	155	TO	400

THE TREASURE IS ______________

______________ .



If you are finished, color the pictures and write a story on the back of this page telling about the treasure hunt.

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