An Advisory Exam in Mathematics for Students at the University of Alberta

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In September 1977, the University of Alberta Mathematics Department gave an advisory exam to all students enrolled in an introductory calculus course. (A similar exam was given in 1976 and a report on that exam appeared in the May 1977 issue of *Delta-K*.) The purpose of the exam was:

(a) to give the Math. Dept. an indication of the incoming students' background.

(b) to advise the students of possible areas of weakness in their precalculus knowledge

The exam was divided into two parts. The first 18 questions examined algebra and the last 12 examined trigonometry. The questions are listed below along with the percentages of students answering each question correctly.

Part I

Time: 30 Minutes

1. $\dfrac{2^2 \cdot 3^{1/2}}{3^3 \cdot 2^{1/3}}$ can be simplified to obtain:

a) $2^{4/3} \cdot 3^{3/2}$ b) $\dfrac{2^{4/3}}{3^{5/2}}$ c) $\dfrac{6^{5/2}}{6^{10/3}}$ d) $2^{5/3} \cdot 3^{5/2}$ (44%)

<u>e)</u> $2^{5/3} \cdot 3^{-5/2}$

2. $\frac{2^6 \cdot 3^9}{5^3}$ is equivalent to:

a) $\frac{6^{54}}{5^3}$ b) $\left(\frac{2^2 \cdot 3^3}{5}\right)^3$ c) $\frac{6^{15}}{5^3}$ d) $(\frac{2 \cdot 3}{5})^{12}$

e) $\left(\frac{2^3 \cdot 3^6}{5}\right)^3$ (61%)

3. $\frac{1}{12} - \frac{2}{3} + \frac{5}{6}$ has value:

a) $\frac{1}{3}$ b) $\frac{4}{21}$ c) $\frac{5}{12}$ d) $\frac{1}{4}$ e) none of these (83%)

4. 4! has value:

a) 1 b) 32 c) 24 d) 16 e) 4 (84%)

5. $1 - 3x > 7$ is equivalent to:

a) $x < \frac{1}{2}$ b) $x > \frac{1}{2}$ c) $x < -2$ d) $x > -2$

e) none of these (56%)

6. Which of the following pairs does not satisfy the inequality $|x| - |y| \geq 1$?

a) $x = -3, y = 1$ b) $x = -3, y = 2$

c) $x = -1, y = 0$ d) $x = 0, y = -1$ (89%)

e) $x = 2, y = 0$

7. $(x^2+1)(x^3-x+1)$ is equal to:

a) $x^5 + x^3 + x^2 - 1$ b) $x^5 + 2x^3 - x + 1$

c) $x^5 + x^2 - x + 1$ d) $x^5 - x^3 + 2x^2 - x + 1$ (85%)

e) none of these

8. $x^4 - 1$ is equal to:

a) $(x-1)^2(x+1)^2$ b) $(x-1)^4$

c) $(x-1)^2(x^2+1)$ d) $(x-1)^3(x+1)$ (63%)

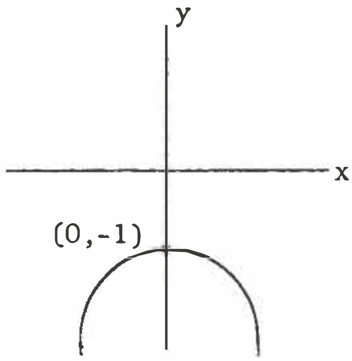
e) $(x-1)(x+1)(x^2+1)$

9. $x^3 + 5x^2 + 13x + 21$ divided by $x + 3$ is $x^2 + 2x + 7$ with remainder:

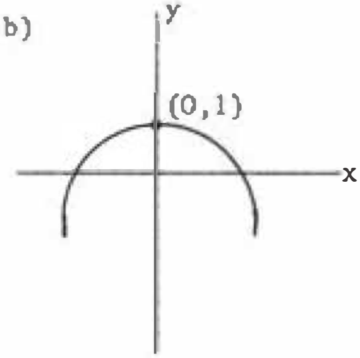
a) 0 b) 21 c) 7 d) $7x + 21$ e) x (89%)

The next two questions involve five possible answers:

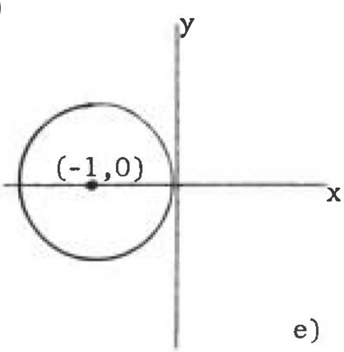
a)



b)



c)



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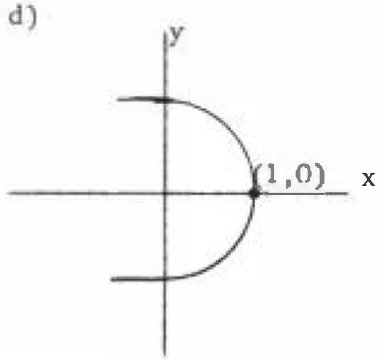

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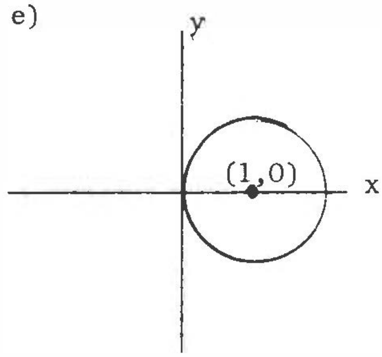


10. The graph of $y = -x^2 + 1$ looks like: (Ans. (b)) (73%)

11. The graph of $y^2 = -(x-1)^2 + 1$ looks like: (Ans. (e)) (46%)

12. If you wish to show that $(1+2+\ldots n) = \frac{n(n+1)}{2}$ for every positive integer value of n, it would be enough to:

a) Show the formula is true for $n = 1$.

b) Show (a) and that $(1+2+\ldots n+n+1) = \frac{(n+1)(n+2)}{2}$ for some value of n. (54%)

c) Show (a) and that if the formula is true for some arbitrary value of n, then it will also be true for the value $n + 1$.

d) Show the formula is true for the first 100 values of n.

e) Show (d) and that $(1+2+\ldots n+1) = \frac{(n+1)(n+2)}{2}$ for some value of n.

13. $\log_{10}(\frac{1}{10})$ is:

a) 0 b) .1 c) 1 d) -1 e) none of these (35%)

14. If $\log_{10} 2 = x$ and $\log_{10} 7 = y$, then $\log_{10} 56 =$

a) x^3y b) $3x + y$ c) $x^3 + y$ d) $3xy$ e) none of these (22%)

15. If $f(x) = \frac{1}{x^2 + 1}$, $f(x+h) =$

a) $\frac{1}{x^2 + h^2 + 1}$ b) $\frac{1}{(x+h)^2}$ c) $\frac{1}{x^2 + h^2} + 1$ (71%)

d) $\frac{1}{x^2h^2 + x + h + 1}$ e) $\frac{1}{x^2 + 2xh + h^2 + 1}$

16. If $x^2 + 2bx + 2c = 0$, then

a) $x = \frac{-b \pm \sqrt{b^2-4c}}{2}$ b) $x = -b \pm 2\sqrt{b^2-2c}$ (42%)

c) $x = -c \pm \sqrt{c^2-4b}$ d) $x = -c \pm 2\sqrt{c^2-2b}$ e) none of these

17. Written in the form $y = a(x-h)^2 + b$, the equation $y = 3x^2 - 6x + 5$ becomes:

a) $y = 3(x-1)^2 + 8$ b) $y = 3(x-1)^2 + 2$ c) $y = 3(x+1)^2 + 2$ (54%)

d) $y = (x-2)^2 + 8$ e) none of these

18. The solution of the equation $\frac{1}{x+3} + \frac{1}{x-2} = \frac{2}{x-1}$ is (51%)

a) $x = 11$ b) $x = \frac{11}{3}$ c) $x = 13$ d) $x = 4$ e) $x = \frac{-11}{3}$

Part II

Time: 20 Minutes

19. $\tan(0)$ is equal to

a) infinity b) 0 c) 1 d) $\frac{1}{\sqrt{2}}$ e) none of these (55%)

20. $\cos(\frac{5}{4}\pi)$ is equal to

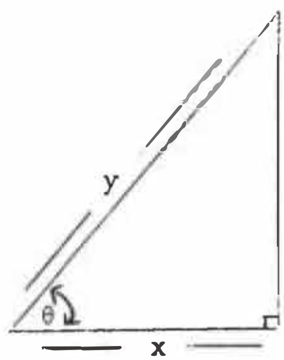
a) $-\frac{1}{\sqrt{2}}$ b) 1 c) $+\frac{1}{\sqrt{2}}$ d) 0 e) none of these (62%)

21. $\sin(\frac{\pi}{3})$ is equal to

a) 0 b) $\frac{1}{\sqrt{2}}$ c) $\frac{1}{2}$ d) $\frac{\sqrt{3}}{2}$ e) $\sqrt{3}$ (62%)

22. In the right triangle shown below, $\cos(\theta) = \frac{1}{3}$.

The value of y is



a) $\frac{1}{3}x$ b) $3x$ c) $\frac{2\sqrt{2}}{3}x$ (43%)

d) 3 e) none of these

23. $\sin(\frac{\pi}{2} - \theta)$ is equal to

a) $-\sin(\theta)$ b) $\cos(\theta)$ c) $\sin(\theta)$ d) $-\cos(\theta)$

e) none of these (45%)

24. The radian measure of each angle of an equilateral triangle is

a) $\frac{\pi}{60}$ b) $\frac{60}{2\pi}$ c) 60 d) $\frac{\pi}{6}$ e) none of these (39%)

25. $\tan(\theta) - \tan(\theta)\sin^2(\theta)$ is equal to

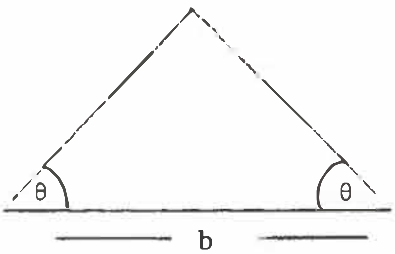
a) $\sin(\theta)\cos(\theta)$ b) $-\frac{\cos^3(\theta)}{\sin(\theta)}$ c) $-\sin(\theta)\cos(\theta)$

d) $\frac{\cos^3(\theta)}{\sin(\theta)}$ e) none of these

26. The function $y = 5\cos(\frac{x}{2})$ has period

a) 5 b) π c) 4π d) 2π e) none of these (38%)

27. The isosceles triangle shown below has area



a) $2b\sin(\theta)$ b) $\frac{1}{4}b^2\tan(\theta)$

c) $\frac{1}{2}b\tan(\theta)$ d) $\frac{1}{4}b^2\sin(\theta)$

e) none of these (27%)

28. $\sin(\theta+\psi)$ is equal to

a) $\sin(\theta)\sin(\psi)$ b) $\sin(\theta) + \sin(\psi)$

c) $\sin(\theta)\cos(\psi) + \cos(\theta)\sin(\psi)$ d) $\frac{1}{2}(\sin(\theta+\psi) + \sin(\theta-\psi))$ (25%)

e) none of these

29. If θ is an acute angle and $\sin(\theta) = \frac{3}{5}$, $\cos(\theta)$ equals:

a) $\frac{3}{4}$ b) $\frac{5}{3}$ c) $\frac{2}{5}$ d) $\frac{4}{5}$ e) none of these (59%)

30. The solution set for the equation $\tan(\theta) = 1$ on the interval $0 \le \theta \le 2\pi$ is

a) $\{\frac{\pi}{4}, \frac{5}{4}\pi\}$ b) $\{\frac{\pi}{4}\}$ c) $\{\frac{3}{4}\pi, \frac{7}{4}\pi\}$

d) $\{\frac{\pi}{4}, \frac{3}{4}\pi, \frac{5}{4}\pi, \frac{7}{4}\pi\}$ e) $\{\frac{\pi}{4}, \frac{3}{4}\pi\}$ (42%)

Results

Distribution of Scores: Separate scores for the two parts of the exam were computed for each student. These scores were obtained by subtracting 1/4 of the number of incorrect answers from the number of correct answers and then rounding up to the next integer. The distributions were as follows:

Algebra Part

Score	0 - 3	4 - 6	7 - 9	10 - 11	13 - 15	16 - 18
% of Students	7.1	13.8	23.6	31.4	14.9	8.7

Trigonometry Part

Score	0 - 2	3 - 4	5 - 6	7 - 8	9 - 10	11 - 12
% of Students	39.9	17.4	20.4	9.3	9.6	3.1

Some Conclusions: As was pointed out in the report on last year's exams, several factors must be taken into account when analyzing the algebra scores. For example, the students had been away from their studies for several months or more. (The enrollment in the introductory courses includes many students who have been away from school and mathematics for several years.) Several of the questions examined material covered long before the last year of high school or in the case of at least one question (#12) material no longer in the high school syllabus. Taking these factors into account, the algebra scores seem quite reasonable.

Trigonometry questions were not included on the 1976 exam and so the large number of low scores on the second part of this year's exam was surprising. Again one must take into account several factors when considering these scores. Trigonometry, more than algebra, involves memorization of formulas that are easily forgotten if not regularly used. As there are two units of measure commonly used for angles, the consistent use of the radian measure on the exam may have placed some students at a disadvantage. Also, the recent increased use of pocket calculators has probably reduced the number of students who have the sine and cosine of common angles at their fingertips.

The Advisory Exam and the distribution of trigonometry scores were discussed at a get-together this winter of representatives of the U. of A. Math. Dept. with some mathematics coordinators from Edmonton high schools. One point that was made at that meeting by some of the high school teachers was that the amount of time devoted to trigonometry in the high school mathematics program has been falling over the years and that this trend has in the past been encouraged by the U. of A. Math. Dept.. It may well be worth considering now whether this trend has gone too far.

During the fall and winter terms, the Math. Dept. offered short remedial programs (about 3 weeks each in duration) in both algebra and trigonometry. Students who did badly on the advisory exam were advised to enroll in one or both of these programs. About 12% of the students in the calculus courses enrolled in the algebra program and about 23% enrolled in the trigonometry program. Comments about the programs were solicited from these students. They generally thought that both programs were beneficial and provided much needed review. We were interested to discover that over 60% of the students attending the algebra classes had been out of school for over a year and that these students were glad to have the opportunity to fill in forgotten areas in their mathematical backgrounds.

We would be happy to hear from any high school teachers about their reaction to the Advisory Exam. As was pointed out in the report on last year's exam, the exam was not formulated with the intention of evaluating or criticizing the teaching of mathematics in the secondary schools. The intention of the exam was rather to help us determine what to expect from our incoming students and to help the students know what is expected of them. ■