Ideas for the Senior High Class

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The Difference of Two Squares — A Discovery Approach

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Can students be led to discover that $a^2 - b^2$ is equal to $(a+b)(a-b)$? It is my belief that they can. The development of a concept by the use of the discovery method is not always a practical approach for the regular classroom teacher when time is of the essence. However, as a change from the regular lecture-type presentation, the discovery method used at different points in the program can be an enhancing feature.

What I now describe is one of a three-part unit which I used in developing the concept that $a^2 - b^2$ is equal to $(a+b)(a-b)$ with a Grade X class.

I began by writing the following problems on the chalkboard and asking students to supply the answers to them:

$3^2 - 2^2 =$ ____	$7^2 - 6^2 =$ ____
$4^2 - 3^2 =$ ____	$8^2 - 7^2 =$ ____
$5^2 - 4^2 =$ ____	$9^2 - 8^2 =$ ____
$6^2 - 5^2 =$ ____	$10^2 - 9^2 =$ ____

As expected, the solutions given by all the students were the following:

$3^2 - 2^2 = 9 - 4 = 5$	$7^2 - 6^2 = 49 - 36 = 13$
$4^2 - 3^2 = 16 - 9 = 7$	$8^2 - 7^2 = 64 - 49 = 15$
$5^2 - 4^2 = 25 - 16 = 9$	$9^2 - 8^2 = 81 - 64 = 17$
$6^2 - 5^2 = 36 - 25 = 11$	$10^2 - 9^2 = 100 - 81 = 19$

By appropriate questions, the students were led to see that the answers to the above problems can also be obtained in the following way:

$3^2 - 2^2 = 3 + 2 = 5$	$7^2 - 6^2 = 7 + 6 = 13$
$4^2 - 3^2 = 4 + 3 = 7$	$8^2 - 7^2 = 8 + 7 = 15$
$5^2 - 4^2 = 5 + 4 = 9$	$9^2 - 8^2 = 9 + 8 = 17$
$6^2 - 5^2 = 6 + 5 = 11$	$10^2 - 9^2 = 10 + 9 = 19$

At this point, the rule appeared to be $a^2 - b^2 = a + b$.

The following questions were then given:

$4^2 - 2^2 =$ ____ $7^2 - 5^2 =$ ____

$5^2 - 3^2 =$ ____ $8^2 - 6^2 =$ ____

$6^2 - 4^2 =$ ____ $9^2 - 7^2 =$ ____

The students were asked to provide the answers for these, first by using the above rule and then by using the method initially employed with the first set of problems. The results were as follows:

Problem	Answer by Rule	Answer by initial method
$4^2 - 2^2 =$	$4 + 2 = \underline{6}$	$16 - 4 = \underline{12}$
$5^2 - 3^2 =$	$5 + 3 = \underline{8}$	$25 - 9 = \underline{16}$
$6^2 - 4^2 =$	$6 + 4 = \underline{10}$	$36 - 16 = \underline{20}$
$7^2 - 5^2 =$	$7 + 5 = \underline{12}$	$49 - 25 = \underline{24}$
$8^2 - 6^2 =$	$8 + 6 = \underline{14}$	$64 - 36 = \underline{28}$
$9^2 - 7^2 =$	$9 + 7 = \underline{16}$	$81 - 49 = \underline{32}$

The students readily noticed that the answers obtained by the initial method, which were the correct answers, were *twice* the answers obtained by the rule. The rule would yield the correct answers to the second set of problems if it were changed to $a^2 - b^2 = (a + b) \times \underline{2}$.

The students were now asked to compare the initial rule with the modified rule and to look for clues as to why it was necessary to modify the former for the second set of problems. One student quickly pointed out that the initial rule could well be written as $a^2 - b^2 = (a + b) \times 1$, where "1" was the difference between the numbers used in the first set of problems. In the second set of problems, the difference was 2, and so the rule became $a^2 - b^2 = (a + b) \times 2$.

It did not take long after this stage to have the students discover that $a^2 - b^2 = (a + b)(a - b)$.

It's a Puzzlement??

When 4444^{4444} is written in decimal notation, the sum of its digits is A. Let B be the sum of the digits of A. Find the sum of the digits of B. (A, B are written in decimal notation.) ■