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# delta-k

THE ALBERTA TEACHERS' ASSOCIATION
MATHEMATICS COUNCIL

VOLUME XVII, NUMBER 4, MAY 1978


## COMING ...

## MCATA Conference

Capri Motor Hotel, Red Deer

October 12 to 14, 1978

A Position Statement on . . .

# Career Education

In recognition of the prime importance of work in our society and the role that mathematics plays in the lives of all individuals, the National Council of Teachers of Mathematics supports the position that Career Education should be a major goal of all who teach and learn mathematics.

For the purposes of this statement, Career Education is defined as the composite of all learning experiences, classroom and nonclassroom, that promote these goals:

1. The learner's understanding of the values of the work ethic and how these values contribute to his or her personal development
2. The learner's awareness of the nature of various careers and of how mathematics is used in those careers
3. The learner's attainment of mathematical concepts and skills, with the ability to apply that knowledge to the solution of career-related problems

Career Education may be achieved by increasing the emphasis placed by schools on career awareness, exploration, decision-making, and planning. Teachers, parents, and guidance counselors should encourage each student to pursue the study of mathematics to the highest level of his or her ability, making it clear that the value of the knowledge and skills so gained cannot always be judged on the basis of immediate need or use.

The National Council of Teachers of Mathematics, recognizing that the incorporation of concepts and approaches to Career Education into the school curriculum requires that teachers develop special knowledge and skills, recommends that Career Education be given special and immediate attention in the training of mathematics teachers at both preservice and in-service levels.

(September 1977)

National Council of Teachers of Mathematics

The National Council of Teachers of Mathematics has for its object the advancement of mathematics teaching in elementary schools, junior and senior high schools, two-year colleges, and teacher-education colleges. All persons interested in mathematics or the teaching of mathematics are eligible for membership.

Brochures describing services of the NCTM and listings of its current publications are available free on request from the Headquarters Office, 1906 Association Drive, Reston, Virginia 22091 (703/620-9840).

delta-k

THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

VOLUME XVII, NUMBER 4, MAY 1978

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# The Editor's Page

Our new CAG representative is Mr. Bob Robinson, 57 Skyline Drive, Dundas, Ontario L9H 3S3. Bob was a contributor to our February 1978 issue of *Delta-K* by way of introduction. Thank you Joan Routledge for the service you have given for the past three years. We look forward to having you return to Alberta as a guest speaker and/or as a guest participant in the audience in coming years. We are looking forward to having three good years with Mr. Robinson as we know him to be an able leader.

We are looking forward, as well, to our next mini-conference. Our Calgary team is going to Lethbridge ( may have already gone by this time), and a full report will be forthcoming in Vol. XVIII, No. 1, September 1978.

The 1978 ARA referred this resolution for study and report at the 1979 Annual Representative Assembly. Members of the Mathematics Council, ATA, are asked to react to the motion by early fall.

*129L/78 (Directive for Action):*

*BE IT RESOLVED, that The Alberta Teachers' Association request the Department of Education to allow the use of calculators in all physics, chemistry and mathematics examinations issued by the Department of Education.*

Please send your reactions to the editor or to Dr. M. Jampolsky, Assistant Executive Secretary, The Alberta Teachers' Association, 11010 - 142 Street, Edmonton T5N 2R1.

*Ed Carriger*
Editor

# Estimating Roots of Quadratic Equations by Templet and Grid

William J. Bruce

Consider any quadratic function defined by the equation $y = x^2 + ax + b$, and assume that it is known that the graph of this equation is a parabola. In order that our proposed device be operable, it is essential that the given quadratic equation be uniquely determined by knowing the axis of symmetry of the parabola and one point on the Y-axis. This is shown easily.

Let $(0,y_1)$ be the coordinates of a point on the Y-axis and let $x = h$ be the equation of the axis of symmetry of the parabola. Because of this symmetry, $(2h,y_1)$ are the coordinates of another point on the parabola? Substitute the coordinates $(0,y_1)$ into the equation $y = x^2 + ax + b$ and obtain $b = y_1$ so that

$$y = x^2 + ax + y_1.$$

The coordinates $(2h,y_1)$ are substituted next to give

$$y_1 = 4h^2 + 2ah + y_1$$

or

$$4h^2 + 2ah = 0$$

from which we find, since $h \neq 0$, that $a = -2h$. Finally the quadratic equation becomes uniquely

$$y = x^2 - 2hx + y_1.$$

For any quadratic equation $y = x^2 + ax + b$, it is obvious that the point with coordinates $(0,b)$ lies on the graph. Complete the square and obtain

$$y = (x+a/2)^2 + b - a^2/4$$

from which we have that

$$x = -a/2$$

is the equation of the axis of symmetry of the parabola. For example, if $y = x^2 - 5x + 6$, we find that $(0,6)$ are the coordinates of the point on the Y-axis and $x = 5/2$ is the equation of the axis of symmetry.

Since neither horizontal nor vertical translation change the shape of a parabola, all parabolas represented by the equation $y = x^2 + ax + b$ have the same shape. Because $y = x^2$ is the simplest form of this equation, it suffices to cut a templet by using a truncated portion of the graph of this equation. All other parabolas referred to can be drawn by shifting such a templet, without rotation, on a Cartesian grid and tracing along the parabolic boundary. Figure 1 illustrates how to obtain a pattern from which this templet can be cut. Note that equal scale units are used but are not essential. The smaller graph shown is that of $y = x^2$ drawn with considerable accuracy. Included is a larger graph of $y = x^2$ made possible by simply shifting the origin downward on the grid to allow for the plotting of more points. If a higher degree of accuracy is needed, the parabolic plots can be done by computer.

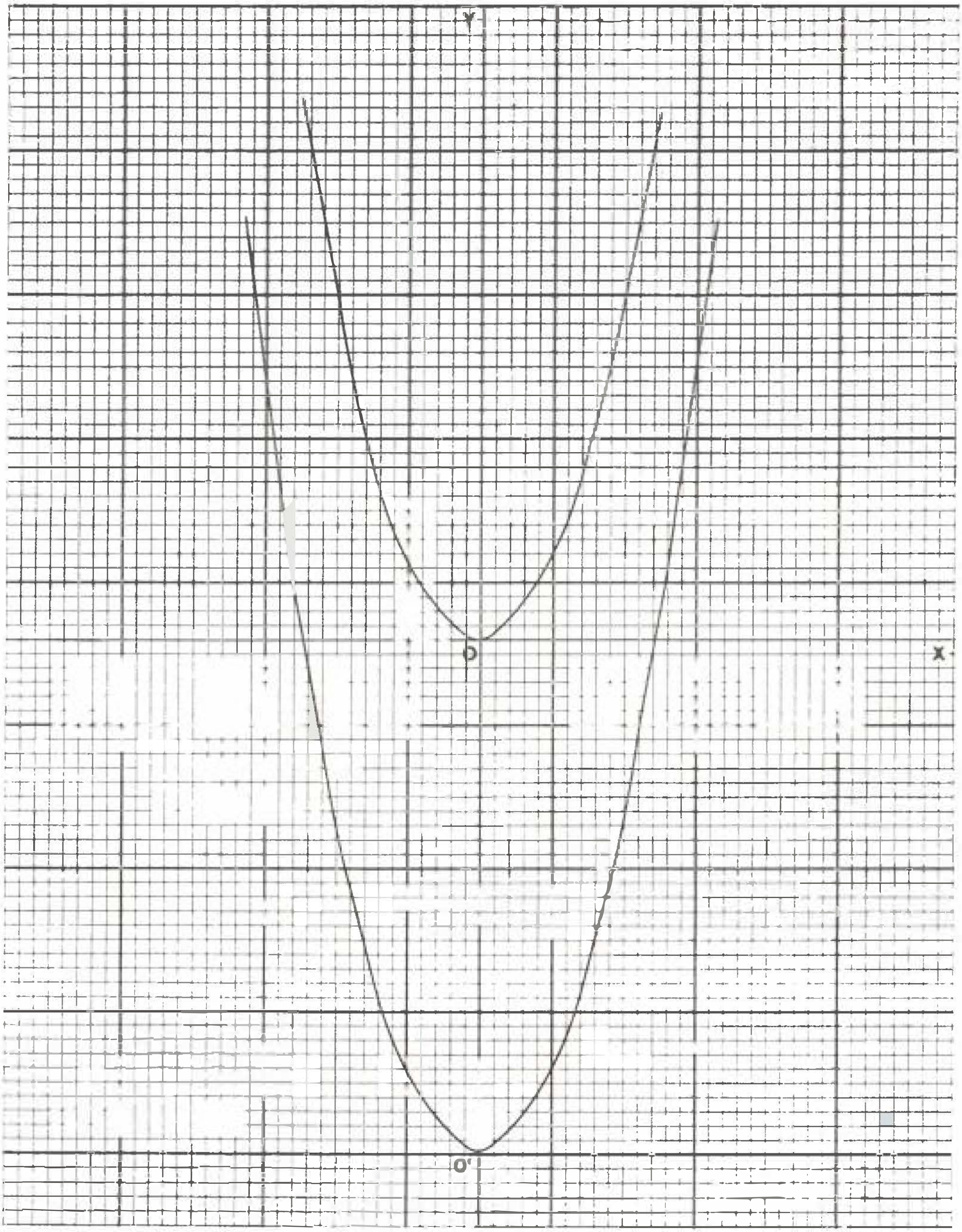



*Figure 1*

A templet can be prepared in the following manner. Place a piece of fairly rigid cardboard behind the graph chosen, and with a long sharp pin with a beaded head carefully punch along the parabola at frequent intervals. One extra punch will be needed to locate the axis of symmetry if this is desired. In using the templet it will be found that it is not absolutely necessary to have the axis of symmetry as long as the vertex is accurately punched and the parabolic region is truncated on a line perpendicular to the axis of symmetry. Carefully cutting along the pin-pricked path, one can produce a reasonably accurate templet.

In order to use a templet to solve quadratic equations of the form $x^2 + ax + b = 0$, we shall need a square grid ruled on a durable sheet of cardboard, plastic or wood. This grid must be prepared so as to match the parabolic templet. Coordinate axes might be pre-marked or could be omitted for greater flexibility and chosen where needed. Some device for marking the axes in the latter case would be necessary. The size of the templet and the grid can be chosen as needed. Standard page size for the grid likely would be adequate for most purposes. Figure 2 shows a grid that might be used.

The following examples illustrate how to use the templet with a grid to estimate the roots of a quadratic equation. At first we shall restrict these examples to equations that have real roots. Figure 2 is used in each case.

(1) Solve $x^2 - 4x + 4 = 0$.

Let $y = x^2 - 4x + 4$. This is of the form $y = x^2 + ax + b$ so that $b = 4$ is the y-intercept while $x = -a/2 = 2$ is the equation of the axis of symmetry. Place the templet on the grid so that its edge cuts the Y-axis at 4 and so that its axis of symmetry lies on the line represented by $x = 2$. In this case the

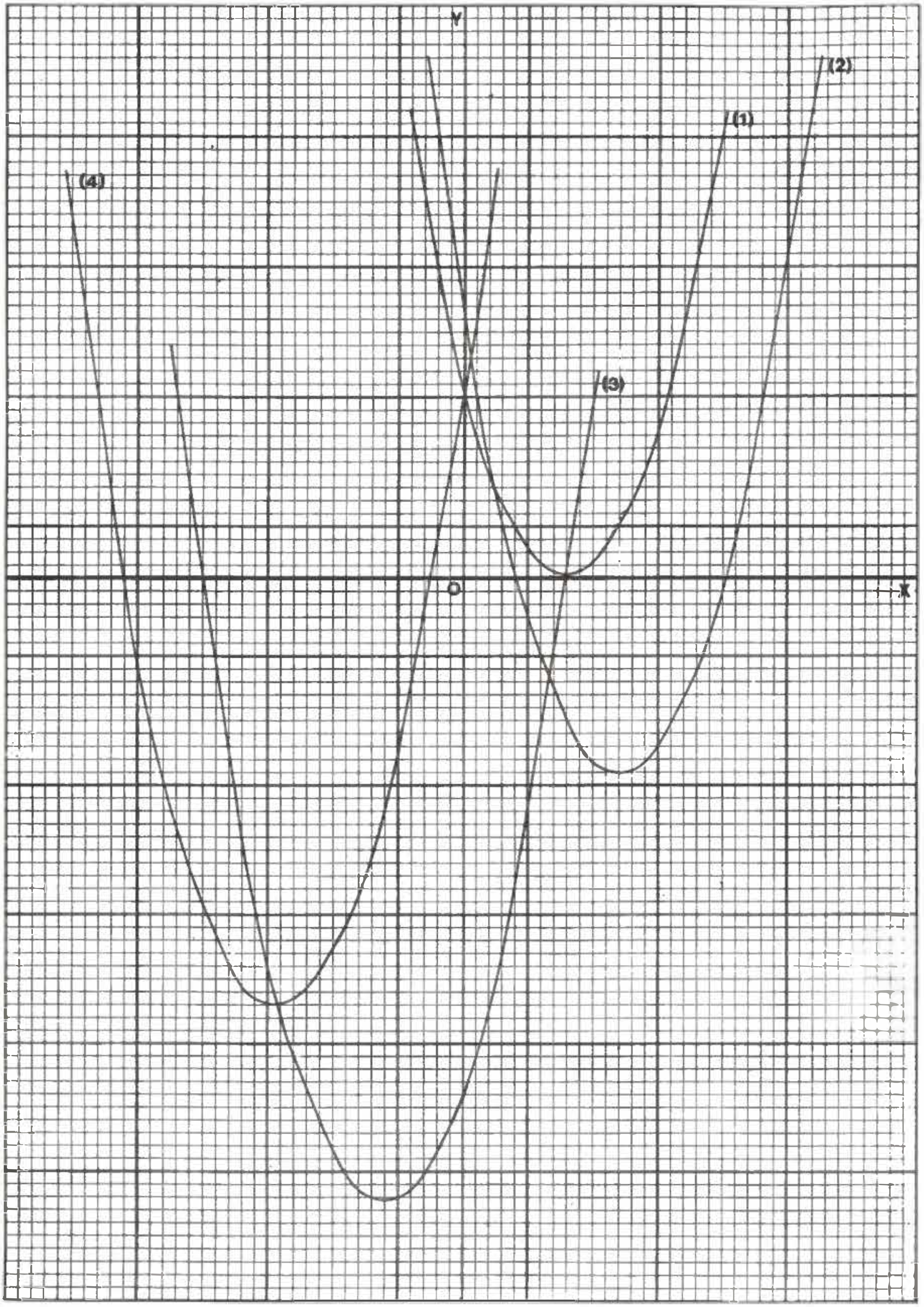


Figure 2

vertex of the parabola touches the X-axis at 2. If $r_1$ and $r_2$ are the roots of the quadratic equation, we have that $r_1 = r_2 = 2$.

(2) Solve $x^2 - 6x + 5 = 0$.

Let $y = x^2 - 6x + 5$. Here $b = 5$ is the y-intercept and $x = 3$ is the equation of the axis of symmetry. Place the templet on the grid carefully and note that its edge crosses the X-axis at 1 and 5. Hence $r_1 = 1$ and $r_2 = 5$ are the roots.

(3) Solve $x^2 + 3x - 10 = 0$.

Let $y = x^2 + 3x - 10$. Here $b = -10$ is the y-intercept and $x = -3/2$ is the equation of the axis of symmetry. Place the templet on the grid so that its edge cuts the Y-axis at -10 and so that its axis of symmetry lies on the graph of $x = -3/2$. The parabola cuts the X-axis at -5 and 2. Hence $r_1 = -5$ and $r_2 = 2$ are the roots.

(4) Solve $x^2 + 7x + 4 = 0$.

Let $y = x^2 + 7x + 4$. Here $b = 4$ is the y-intercept and $x = -7/2$ is the equation of the axis of symmetry. In this case the templet doesn't cut the X-axis at integral values so we have to estimate the roots more carefully than in the previous examples. These appear to be approximately $r_1 = -6.4$ and $r_2 = -0.6$.

For equations with leading coefficients that are not unity, it is sufficient to divide by this coefficient at the outset to get an equivalent equation and then solve the latter as before. If the templet doesn't cut the X-axis, this will indicate that the roots are not real, in which case they cannot be found by the method described above. For such nonreal roots, the method of completing the square can be used to form the well-known quadratic formula.

The presence of a negative discriminant in the latter also indicates that the roots are nonreal.

There are two ways by which one might prepare a broader templet. One of these is to shorten the vertical scale units. For example, the ratio of vertical to horizontal scale units might be taken as 1 to 2. Of course, both the templet and the grid must conform to the same units in all cases. Another way to accomplish this is to retain equal scale units on both scales, but to use a fractional multiplier for the quadratic function. For example, one might produce the templet from the graph of $y = \frac{1}{2}x^2$ and always write the equation in the form $\frac{1}{2}(x^2+ax+b) = 0$ so that the graph would be that of $y = \frac{1}{2}(x^2+ax+b)$ when the templet is shifted about on the grid. The roots of the quadratic equation will not change if variations such as these are used. ■

# 1978 Alberta High School Prize Examination Results

| Prize | Amt. | Student | School |
|---|---|---|---|
| Canadian Mathematics Congress Scholarship | $400 | FISHER, Douglas | Strathcona Composite High<br>Edmonton, Alberta |
| Nickel Foundation Scholarship | $400 | DEWAR, Alan | Sir Winston Churchill High<br>Calgary, Alberta |
| Third Highest | $150 | JENSEN, Lawrence | Queen Elizabeth Jr.-Sr. High<br>Calgary, Alberta |
| Fourth Highest | $150 | WONG, Eric | Ross Sheppard Composite High<br>Edmonton, Alberta |

## Special Provincial Prizes

| | | | |
|---|---|---|---|
| Highest Grade 12 student (below first 4) | $ 75 | LAM, Kay | Harry Ainlay Composite High<br>Edmonton, Alberta |
| Highest Grade 10/11 student (below first 4) | $ 75 | LEUNG, Henry | Bonnie Doon Composite High<br>Edmonton, Alberta |

## District Prizes

| District No. | Amt. | Name | School |
|---|---|---|---|
| 1 | $50 | BANTEL, Darald | Hillside Jr. Sr. High School<br>Valleyview, Alberta |
| 2 | $50 | WU, Willy | Paul Kane High School<br>St. Albert, Alberta |
| 3 | $50 | KOCH, Peter | Salisbury Comp. High School<br>Sherwood Park, Alberta |
| 4 | $50 | NORTON, Duane | Camrose Lutheran College<br>Camrose, Alberta |
| 5 | No Award Made. | | |
| 6 | $50 | RODRIGUES, Ivan | Medicine Hat High School<br>Medicine Hat, Alberta |
| 7 (1) | $50 | BAUDER, Bob | Harry Ainlay Comp. High School<br>Edmonton, Alberta |
| 7 (2) | $50 | TROFIMUK, David | McNally Comp. High School<br>Edmonton, Alberta |
| 8 (1) | $50 | HEWITT, Mark | Sir Winston Churchill High School<br>Calgary, Alberta |
| 8 (2) | $50 | GORDON, Karen | Sir Winston Churchill High School<br>Calgary, Alberta |

356 students from 60 schools in Alberta and the Northwest Territories wrote the 1978 examination. The following students took the first 16 places and are nominated for the Canadian Mathematical Olympiad:

| Student | School |
|---|---|
| BAUDER, Bob | Harry Ainlay Composite High School, Edmonton |
| CHAMBERLAIN, Martin | Harry Ainlay Composite High School, Edmonton |
| DENOTTER, Gordon | M.E. Lazerte Composite High School, Edmonton |
| DEWAR, Alan | Sir Winston Churchill High School, Calgary |
| FISHER, Douglas | Strathcona Composite High School, Edmonton |
| GORDON, Daren | Sir Winston Churchill High School, Calgary |
| HARTWIG, Karen | Jasper Place Composite High School, Edmonton |
| HAYWARD, Geoffrey | Old Scona Academic High School, Edmonton |
| HEWITT, Mark | Sir Winston Churchill High School, Calgary |
| JENSEN, Lawrence | Queen Elizabeth Jr.-Sr. High School, Calgary |
| LAM, Kay | Harry Ainlay Composite High School, Edmonton |
| LAMOUREUX, Mike | Archbishop Macdonald, Edmonton |
| LYNCH, William | Lord Beaverbrook Sr. High School, Calgary |
| TROFIMUK, David | McNally Composite High School, Edmonton |
| WILLIS, Ron | St. Francis Xavier Composite, Edmonton |
| WONG, Eric | Ross Sheppard Composite High School, Edmonton |

The following students placed 17-30:

Henry Baragar (Old Scona Academic High School, Edmonton), Catherine Clelland (Eastglen Composite High School, Edmonton), Wallace Chow (Dr. E.P. Scarlett Sr. High School, Calgary), John Haugen (Jasper Place Composite, Edmonton), Mark Herman (Sir Winston Churchill High School, Calgary), Dean Karlen (Jasper Place Composite, Edmonton), Kok Kwan (Eastglen Composite High School, Edmonton), Agnes Lee (Ross Sheppard Composite, Edmonton), Ming Lee (McNally Composite, Edmonton), Henry Leung (Bonnie Doon Composite High School, Edmonton), Mark Salzyn (Harry Ainlay Composite, Edmonton), Glynn Searl (St. Francis High School, Calgary), Kenneth Tsang (Strathcona-Tweedsmuir School, Okotoks), Fred Woslyng (Harry Ainlay Composite, Edmonton)

The following students placed 31-50:

Janice Bodnarchuk (Austin O'Brien High School, Edmonton), Cathy Brown (Ross Sheppard Composite, Edmonton), John Chmelicek (Strathcona Composite High School, Edmonton), Stephen Crowe (Ernest Manning High School, Calgary), Dane Douglas (Jasper Place Composite, Edmonton), Rick Eykelbosh (Louis St. Laurent, Edmonton), Duane Foote (F.P. Walshe School, Fort Macleod), Guy Fortier (J.H. Picard High School, Edmonton), Lewis Kay (Ross Sheppard Composite, Edmonton), Yeon Kim (Forest Lawn Sr. High School, Calgary), Gordon Lee (Eastglen Composite High School, Edmonton), David Macpherson (Ross Sheppard Composite High School, Edmonton), Simon McClure (Strathcona Composite High School, Edmonton), Bob McCreight (Sir Winston Churchill High School, Calgary), Andrew McIntosh (Bishop Carroll High School, Calgary), Robert Morewood (Medicine Hat High School, Medicine Hat), Duane Norton (Camrose Lutheran College, Camrose), Karl Pierzchajlo (St. Francis Xavier Composite, Edmonton), Ivan Rodrigues (Medicine Hat High School, Medicine Hat), Michael Whitney (Strathcona Composite High School, Edmonton), Paul Yarema (Archbishop Macdonald, Edmonton).

**Part I** **Time: 60 Minutes**

ANSWER SHEET

To be filled in by the Candidate.

PRINT: ______________________________________________

Last Name First Name Initial

______________________________________________

Candidate's Address Town/City

______________________________________________

Name of School

______________________

Grade

ANSWERS:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |

To be completed by the Department of Mathematics, University of Alberta:

| | Points | Points Correct | Number Wrong |
|---|---|---|---|
| 1-20 | 5 | 5 × = | 1 × = |
| | Totals | C = ______ | W = ______ |

SCORE = C - W = ______

*Do all problems. Each problem is worth five points.*

1. Which of the following inequalities are true for all positive numbers $x$?

(A) $x + \frac{1}{x} > 2$ (B) $x + \frac{1}{x} < 2$

(C) $x + \frac{1}{x} \geq 2$ (D) $x + \frac{1}{x} \leq 2$

(E) none of the preceding are true for all positive numbers $x$.

2. A steamer was able to go twenty miles per hour upstream and twenty-five miles per hour downstream. On a return trip the steamer took two hours longer coming upstream than it took coming downstream. The total distance travelled by the steamer was

(A) 100 miles (B) 200 miles

(C) 400 miles (D) 800 miles

(E) 150 miles.

3. If $n$ is a positive integer, then $n^2 + 3n + 1$ is

(A) always a perfect square (B) never a perfect square

(C) sometimes a perfect square (D) sometimes an even integer

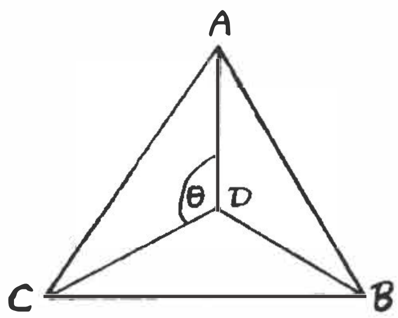
(E) none of the preceding.

4. The solution set of the inequality $x^2(x^2-1) \leq 0$ is

(A) an interval (B) two intervals

(C) a point (D) an interval and a point

(E) all real numbers.

5\. 


In the diagram, $\Delta ABC$ is an equilateral triangle, $\Delta BCD$ is an isosceles triangle, and the angle $\widehat{CDB}$ is a right-angle. Then the angle $\theta$ is

(A) 45° (B) 90° (C) 120° (D) 135°

(E) none of the preceding.

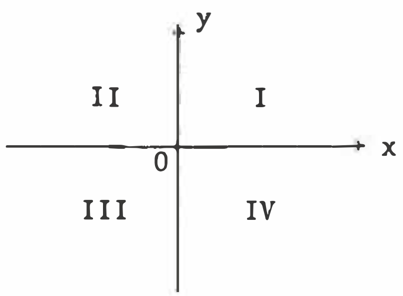
6\. Given the binary operation $*$ between two positive integers m, n such that $m * n = mn + 1$ (mn is the usual multiplication of m and n), which of the following does not hold:

(A) commutative law (B) associative law

(C) $m * n$ is a positive integer (D) $m * n \geq 2$

(E) $m * n$ is odd whenever m is even.

7\. Label the four quadrants of the (x,y)-plane as follows:




Then the solution set of the simultaneous inequalities $x^2 - y < 0$, $x^2 + y^2 < 1$ lies entirely in quadrants

(A) I and II (B) II and III (C) III and IV

(D) IV and I (E) none of the preceding are correct.

8. If $f(n) = n^2$, where $n$ is an integer, then

$$\frac{f\big(f(n+1)\big) - f\big(f(n-1)\big)}{f(n+1) - f(n-1)} \text{ equals}$$

(A) $n^2$ (B) $2n^2 + 2$ (C) $n^2 + 1$ (D) $n^4 + 1$

(E) none of the preceding.

9. Which of the following inequalities hold for all pairs of real numbers $x, y$?

(A) $\sqrt{x^2 + y^2} \le x + y$ (B) $\sqrt{x^2 + y^2} \le x^2 + y^2$

(C) $\sqrt{x^2 + y^2} \le xy$ (D) $\sqrt{x^2 + y^2} \le |x| + |y|$

(E) none of the preceding are true for all real numbers $x, y$.

10. Two similarly proportioned boxes have their surface areas in the ratio 4:1. Their volumes are in the ratio

(A) 9:1 (B) 8:1 (C) 3:1 (D) 2:1

(E) none of the preceding.

11. The roots of the quadratic polynomial $2x^2 + kx + 1$ are $r$ and $s$. Which of the following are impossible?

(A) $r = s$ (B) $r \cdot s = 1$ (C) $r + s = 1$ (D) $r + s = 0$

(E) all of the preceding are possible.

12. A hat contains three slips of paper, of which one bears the name John, one bears the name Diana and the other bears both names. If John and

Diana each draw a slip, the probability that they each draw a slip with their own name on is

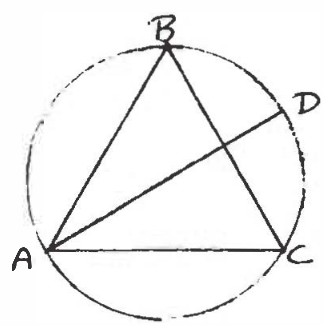
(A) $\frac{1}{9}$ (B) $\frac{1}{6}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) none of the preceding.

13. The value of $k$ such that

$$x^6 - kx^4 + kx^2 - kx + 4k + 6 \text{ is divisible by } x - 2 \text{ is}$$

(A) 1 (B) 5 (C) 7 (D) 11

(E) there is no such value of $k$.

14. 


$\Delta ABC$ is an equilateral triangle inscribed in a circle of diameter 1. If $AD$ is a diameter of the circle, then the length $\overline{BD}$ is

(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) $\frac{1}{2}\sqrt{3}$ (E) $1/\sqrt{2}$ .

15. The equation of the line through the point (1,1) that is perpendicular to the line $y = -2x - 3$ is

(A) $y = \frac{2}{3}x + \frac{1}{3}$ (B) $y = \frac{1}{3}x + \frac{2}{3}$

(C) $y = 2x - 1$ (D) $y = \frac{1}{2}x + \frac{1}{2}$

(E) none of the preceding.

16. If $\log_a b = c$, then $\log_a(b^c) =$

(A) $bc$ (B) $6^c$ (C) $c^c$ (D) $c^2$ (E) $2c$.

17. A circle and a square can never intersect in

(A) one point (B) two points (C) three points

(D) four points (E) all of the preceding are possible.

18. $a_1, a_2, a_3, \cdots$ is a sequence of real numbers such that the sum of the first $n$ of them is $n^2 + n$. Then $a_n$ is equal to

(A) $n$ (B) $2n - 1$ (C) $2n + 1$ (D) 1

(E) none of the preceding.

19. The domain of the function

$$f(x) = \sqrt{1 - \sqrt{1 - x^2}}$$ is

(A) a single point (B) an infinite interval

(C) a finite interval (D) an infinite interval with a point deleted

(E) none of the preceding.

20. A polynomial which passes through the points $(-1,7)$, $(1,0)$, $(2,0)$ is

(A) $x + 8$ (B) $x^2 - 3x + 2$ (C) $x^2 + 9$ (D) $x^3 - x^2 + x - 1$

(E) none of the preceding.

## Part II

**Time: 110 Minutes**

Marks

20 1. Determine all angles $\theta$ with $0 \le \theta \le 2\pi$ such that $\sin^6\theta + \cos^3\theta = 1$.

2. (a) A corner reflector consists of two straight lines, perpendicular to each other, which are assumed capable of reflecting
5 a ray of light which is in the same plane as the lines. If a ray of light reflects successively off each of the lines, prove that the exit ray is parallel in the opposite sense to the entering ray.

(b) [3-dimensional version]. This time the corner reflector consists of three plane mirrors which are mutually perpendicular.
15 If a ray of light reflects successively off each of the three mirrors, in any order, prove that the exit ray is parallel in the opposite sense to the entering ray.

20 3. S is a finite set of positive integers, not necessarily different from each other, such that for any three members $a,b,c$ of $S$, $a+b$ is divisible by $c$. Classify all possible such sets $S$.

20 4. Let $i$ be a square root of $-1$ and for real numbers $x$, $y$, write the complex number $\frac{x+iy+1}{x+iy-1}$ in the form $a+ib$, where $a$, $b$ are real numbers. Find the set of points $(x,y)$ for which $a \le 0$.

20 5. Prove that for any integers $m$, $n$, $mn(m^4-n^4)$ is divisible by $30$.

20 6. For which values of $k$ do the polynomial equations

$$x^6 + x^5 + x^4 + x^3 + x^2 + x - 3k = 0$$
$$x^6 - x^5 + x^4 - x^3 + x^2 - x - k = 0$$

have a common root?

20 7. 36 points are placed inside a square whose sides have length 3. Show that there are 3 points which determine a triangle of area no greater than $\frac{1}{2}$.

20 8. You are given a set of 21 dominoes [a | b] where a and b are integers from 1 to 6, each pair occurring once (note [a | b] is to be considered the same as [b | a]). Any number of dominoes can be joined to form a chain if they have matching numbers at each join. For example, a 3-chain is given by [1 | 1] [1 | 6] [6 | 4]. Show that it is <u>not</u> possible to form a 21-chain.

## *Solutions to Part I*

| C | C | B | A | D | B | A | B | D | B |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

| B | E | C | A | D | D | E | E | C | E |
|---|---|---|---|---|---|---|---|---|---|
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |

## *Solutions to Part II*

1. Since $|\sin\theta| \le 1$ and $|\cos\theta| \le 1$, we have $\sin^6\theta + \cos^3\theta \le \sin^2\theta + \cos^2\theta = 1$, with equality only if $\sin\theta = \pm 1$ and $\cos\theta = 0$, or $\sin\theta = 0$ and $\cos\theta = 1$. For $0 \le \theta \le 2\pi$, this gives $\theta = 0, \pi/2, 3\pi/2, 2\pi$.

2. Using vectors, let $\underline{i}, \underline{j}$ be unit vectors perpendicular to the two mirrors, and let the incident ray have the direction $a\underline{i} + b\underline{j}$. Reflection at the first mirror will cause the ray to have direction $-a\underline{i} + b\underline{j}$, then reflection at the second mirror will change this to $-a\underline{i} - b\underline{j}$, i.e. the exit ray is parallel in the opposite sense to the entering ray. This argument is easily extended to the 3-dimensional situation.

3. If $a,b,c \in S$ with $a \le b < c$ or $a < b \le c$, then $a + b < 2c$. Now $a + b$ is divisible by $c$, and so we must have $a + b = c$. It follows that $S$ cannot have four distinct integers, for if $a,b,c,d \in S$ with $a < b < c < d$, we would have $a + b = c$, $a + b = d$, i.e. $c = d$. So $S$ has at most three <u>distinct</u> integers.

<u>Case (i).</u> $S$ has three distinct integers $a, b, c$, with $a < b < c$. If $d$ is any other member of $S$, it must be equal to one of $a, b, c$. But $d = a$ implies $d + a = b = c$; $d = b$ implies $a + d = b$, i.e. $a = 0$; $d = c$ implies $b + c = d$, i.e. $b = 0$. All of these are impossible and so there can be no other member of $S$.
Now $a < b < c$ implies $a + b = c$, and $b < a + c = 2a + b < 3b$. Since $a + c$ is divisible by $b$, we must have $a + c = 2b$. $a + b = c$ and $a + c = 2b$ imply that $c = a + b = 2b - a$, i.e. $b = 2a$, $c = a + b = 3a$. So the only sets $S$ with three distinct integers are of the form $S = \{a, 2a, 3a\}$.

Case (ii). S has two distinct integers a, b with $a < b$. If c is any third member of S, we have $a = c < b$ or $a < c = b$, and so $a + c = b$. So c must equal a, and the only sets S with two distinct integers are of the form $S = \{a,a,\cdots,a,2a\}$.

Case (iii). Finally, S may be of the form $\{a,a,\cdots,a\}$.

4. Note that $\frac{x + iy + 1}{x + iy - 1}$ is undefined for $x = 1$, $y = 0$.

$$\frac{x + iy + 1}{x + iy - 1} = \left(\frac{x + iy + 1}{x + iy - 1}\right)\left(\frac{x - iy - 1}{x - iy - 1}\right)$$

$$= \frac{x^2 + y^2 - 1 - 2iy}{(x-1)^2 + y^2} = a + ib, \quad \text{where}$$

$$a = \frac{x^2 + y^2 - 1}{(x-1)^2 + y^2}, \quad b = \frac{-2y}{(x-1)^2 + y^2}.$$

$a \le 0 \Leftrightarrow x^2 + y^2 \le 1$ and $(x,y) \neq (1,0)$, i.e. the set of points $(x,y)$ for which $a \le 0$ is the circle, center $(0,0)$, radius 1, with the point $(1,0)$ removed.

5. $mn(m^4-n^4) = mn(m^2-n^2)(m^2+n^2) = mn(m-n)(m+n)(m^2+n^2)$. To show divisibility by 30, we need to show divisibility by each of the prime factors 2, 3, 5.

Divisibility by 2: Either one of m, n is divisible by 2 or their sum is divisible by 2.

Divisibility by 3: Either one of m, n, m+n is divisible by 3, or two of them leave the same remainder on dividing by 3. In the latter case, the difference of these two integers will be divisible by 3, which implies that one of the integers m, n, m-n is divisible by 3.

Divisibility by 5: If neither m nor n is divisible by 5, then each of them has a remainder 1, 2, 3 or 4 when divided by 5, and their squares will have a remainder 1 or 4. These remainders are either the same, in which case $m^2 - n^2$ is divisible by 5, or they add up to 5, in which case $m^2 + n^2$ is divisible by 5.

6. $x^6 + x^5 + x^4 + x^3 + x^2 + x = 3k$ — (1)

$x^6 - x^5 + x^4 - x^3 + x^2 - x = k$ — (2).

Subtracting (2) from (1) gives $2x^5 + 2x^3 + 2x = 2k$, i.e.

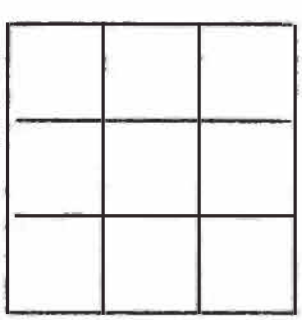
$$k = x(x^4+x^2+1) \quad \text{— (3).}$$

Multiplying (2) by (3) and subtracting (1) gives $2x^6 - 4x^5 + 2x^4 - 4x^3 + 2x^2 - 4x = 0$, which simplifies first to $2x^2(x^4+x^2+1) - 4x(x^4+x^2+1) = 0$ and then to

$$x(x-2)(x^4+x^2+1) = 0, \text{ i.e.}$$

$$x = 0 \text{ or } 2, \quad \text{or} \quad x^4 + x^2 + 1 = 0.$$

The first and third of these options give $k = 0$ (using (3)), the second gives $k = 42$.

7. 

Subdivide the large square into 9 squares of side 1. Since there are 36 points, at least one of the small squares will contain 4 (or more) points. These 4 points determine at least 2 non-overlapping triangles lying inside the small square. Since the small square has area 1, one of the triangles must have area no greater than 1/2.

8. Except at the extreme ends of a dominoe chain, each time an integer is introduced into the chain, it must be "paired" by the next dominoe placed in the chain. Therefore each of the integers 1 through 6 must be used an even number of times, except possibly those occurring at the ends of the chain. However, each integer occurs seven times in the complete set of 21 dominoes and so at least four integers cannot be used up in any chain, which means that at least two dominoes will not be used. Thus a 19-chain is the longest possible (and this can be achieved). ■

# *An Advisory Exam in Mathematics for Students at the University of Alberta*

*Bruce Allison*
Department of Mathematics
University of Alberta
Edmonton

In September 1977, the University of Alberta Mathematics Department gave an advisory exam to all students enrolled in an introductory calculus course. (A similar exam was given in 1976 and a report on that exam appeared in the May 1977 issue of *Delta-K*.) The purpose of the exam was:

(a) to give the Math. Dept. an indication of the incoming students' background.

(b) to advise the students of possible areas of weakness in their precalculus knowledge

The exam was divided into two parts. The first 18 questions examined algebra and the last 12 examined trigonometry. The questions are listed below along with the percentages of students answering each question correctly.

## *Part I*     *Time: 30 Minutes*

1. $\dfrac{2^2 \cdot 3^{1/2}}{3^3 \cdot 2^{1/3}}$ can be simplified to obtain:

a) $2^{4/3} \cdot 3^{3/2}$    b) $\dfrac{2^{4/3}}{3^{5/2}}$    c) $\dfrac{6^{5/2}}{6^{10/3}}$    d) $2^{5/3} \cdot 3^{5/2}$    (44%)

e) $2^{5/3} \cdot 3^{-5/2}$

2. $\frac{2^6 \cdot 3^9}{5^3}$ is equivalent to:

a) $\frac{6^{54}}{5^3}$   b) $\left(\frac{2^2 \cdot 3^3}{5}\right)^3$   c) $\frac{6^{15}}{5^3}$   d) $(\frac{2 \cdot 3}{5})^{12}$

e) $\left(\frac{2^3 \cdot 3^6}{5}\right)^3$ (61%)

3. $\frac{1}{12} - \frac{2}{3} + \frac{5}{6}$ has value:

a) $\frac{1}{3}$   b) $\frac{4}{21}$   c) $\frac{5}{12}$   d) $\frac{1}{4}$   e) none of these (83%)

4. 4! has value:

a) 1   b) 32   c) 24   d) 16   e) 4 (84%)

5. $1 - 3x > 7$ is equivalent to:

a) $x < \frac{1}{2}$   b) $x > \frac{1}{2}$   c) $x < -2$   d) $x > -2$

e) none of these (56%)

6. Which of the following pairs does not satisfy the inequality $|x| - |y| \geq 1$ ?

a) $x = -3$, $y = 1$   b) $x = -3$, $y = 2$

c) $x = -1$, $y = 0$   d) $x = 0$, $y = -1$ (89%)

e) $x = 2$, $y = 0$

7. $(x^2+1)(x^3-x+1)$ is equal to:

a) $x^5 + x^3 + x^2 - 1$   b) $x^5 + 2x^3 - x + 1$

c) $x^5 + x^2 - x + 1$   d) $x^5 - x^3 + 2x^2 - x + 1$ (85%)

e) none of these

8. $x^4 - 1$ is equal to:

a) $(x-1)^2(x+1)^2$ b) $(x-1)^4$

c) $(x-1)^2(x^2+1)$ d) $(x-1)^3(x+1)$ (63%)

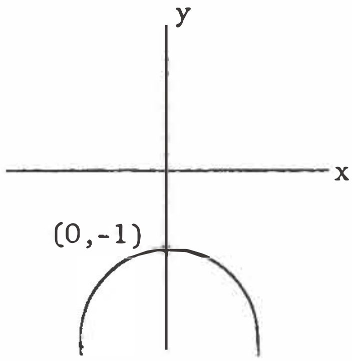
<u>e)</u> $(x-1)(x+1)(x^2+1)$

9. $x^3 + 5x^2 + 13x + 21$ divided by $x + 3$ is $x^2 + 2x + 7$ with remainder:

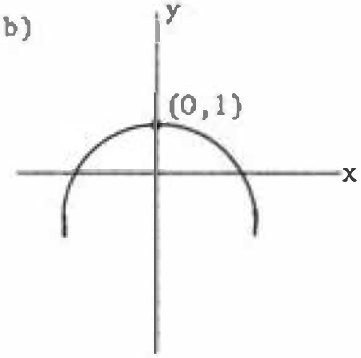
<u>a)</u> 0 b) 21 c) 7 d) $7x + 21$ e) $x$ (89%)

The next two questions involve five possible answers:

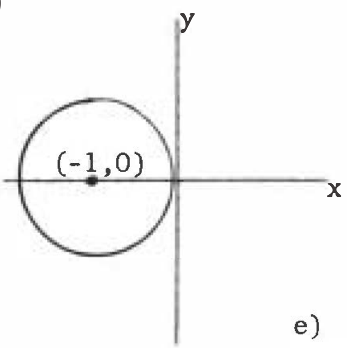
a)



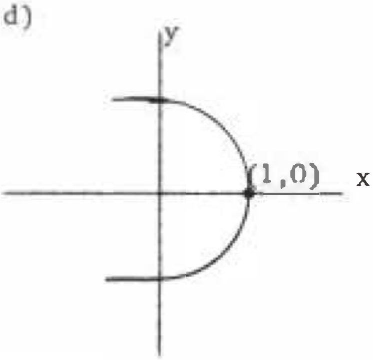

b)



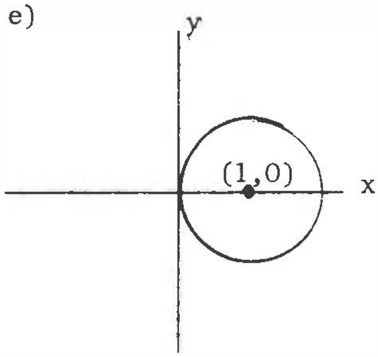

c)




d)




e)




10. The graph of $y = -x^2 + 1$ looks like: (Ans. (b)) (73%)

11. The graph of $y^2 = -(x-1)^2 + 1$ looks like: (Ans. (e)) (46%)

12. If you wish to show that $(1+2+\ldots n) = \frac{n(n+1)}{2}$ for every positive integer value of $n$, it would be enough to:

a) Show the formula is true for $n = 1$.

b) Show (a) and that $(1+2+\ldots n+n+1) = \frac{(n+1)(n+2)}{2}$ for some value of $n$. (54%)

c) Show (a) and that if the formula is true for some arbitrary value of $n$, then it will also be true for the value $n + 1$.

d) Show the formula is true for the first 100 values of $n$.

e) Show (d) and that $(1+2+\ldots n+1) = \frac{(n+1)(n+2)}{2}$ for some value of $n$.

13. $\log_{10}(\frac{1}{10})$ is:

a) 0 b) .1 c) 1 d) -1 e) none of these (35%)

14. If $\log_{10} 2 = x$ and $\log_{10} 7 = y$, then $\log_{10} 56 =$

a) $x^3y$ b) $3x + y$ c) $x^3 + y$ d) $3xy$ e) none of these (22%)

15. If $f(x) = \frac{1}{x^2 + 1}$, $f(x+h) =$

a) $\frac{1}{x^2 + h^2 + 1}$ b) $\frac{1}{(x+h)^2}$ c) $\frac{1}{x^2 + h^2} + 1$ (71%)

d) $\frac{1}{x^2h^2 + x + h + 1}$ e) $\frac{1}{x^2 + 2xh + h^2 + 1}$

16. If $x^2 + 2bx + 2c = 0$ , then

a) $x = \frac{-b \pm \sqrt{b^2-4c}}{2}$ b) $x = -b \pm 2\sqrt{b^2-2c}$

(42%)

c) $x = -c \pm \sqrt{c^2-4b}$ d) $x = -c \pm 2\sqrt{c^2-2b}$ e) none of these

17. Written in the form $y = a(x-h)^2 + b$, the equation $y = 3x^2 - 6x + 5$ becomes:

a) $y = 3(x-1)^2 + 8$ b) $y = 3(x-1)^2 + 2$ c) $y = 3(x+1)^2 + 2$

(54%)

d) $y = (x-2)^2 + 8$ e) none of these

18. The solution of the equation $\frac{1}{x+3} + \frac{1}{x-2} = \frac{2}{x-1}$ is

(51%)

a) $x = 11$ b) $x = \frac{11}{3}$ c) $x = 13$ d) $x = 4$ e) $x = \frac{-11}{3}$

## Part II

***Time: 20 Minutes***

19. tan(0) is equal to

a) infinity b) 0 c) 1 d) $\frac{1}{\sqrt{2}}$ e) none of these (55%)

20. $\cos(\frac{5}{4}\pi)$ is equal to

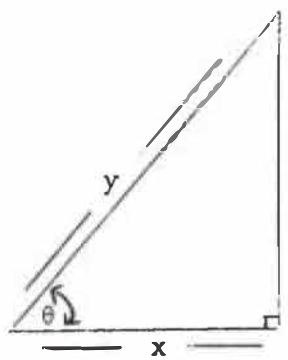
a) $-\frac{1}{\sqrt{2}}$ b) 1 c) $+\frac{1}{\sqrt{2}}$ d) 0 e) none of these (62%)

21. $\sin(\frac{\pi}{3})$ is equal to

a) 0 b) $\frac{1}{\sqrt{2}}$ c) $\frac{1}{2}$ d) $\frac{\sqrt{3}}{2}$ e) $\sqrt{3}$ (62%)

22. In the right triangle shown below, $\cos(\theta) = \frac{1}{3}$.

The value of $y$ is




a) $\frac{1}{3}x$ b) $3x$ c) $\frac{2\sqrt{2}}{3}x$ (43%)

d) 3 e) none of these

23. $\sin(\frac{\pi}{2} - \theta)$ is equal to

a) $-\sin(\theta)$ b) $\cos(\theta)$ c) $\sin(\theta)$ d) $-\cos(\theta)$

e) none of these (45%)

24. The radian measure of each angle of an equilateral triangle is

a) $\frac{\pi}{60}$ b) $\frac{60}{2\pi}$ c) 60 d) $\frac{\pi}{6}$ e) none of these (39%)

25. $\tan(\theta) - \tan(\theta)\sin^2(\theta)$ is equal to

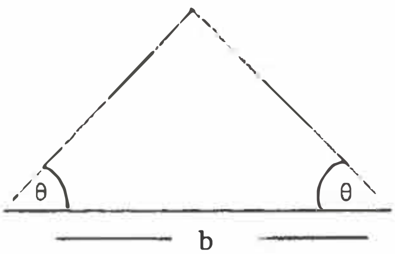
a) $\sin(\theta)\cos(\theta)$ b) $-\frac{\cos^3(\theta)}{\sin(\theta)}$ c) $-\sin(\theta)\cos(\theta)$

d) $\frac{\cos^3(\theta)}{\sin(\theta)}$ e) none of these

26. The function $y = 5\cos(\frac{x}{2})$ has period

a) 5 b) $\pi$ <u>c)</u> $4\pi$ d) $2\pi$ e) none of these (38%)

27. The isosceles triangle shown below has area




a) $2b\sin(\theta)$ <u>b)</u> $\frac{1}{4}b^2\tan(\theta)$

c) $\frac{1}{2}b\tan(\theta)$ d) $\frac{1}{4}b^2\sin(\theta)$

e) none of these (27%)

28. $\sin(\theta+\psi)$ is equal to

a) $\sin(\theta)\sin(\psi)$ b) $\sin(\theta) + \sin(\psi)$

<u>c)</u> $\sin(\theta)\cos(\psi) + \cos(\theta)\sin(\psi)$ d) $\frac{1}{2}(\sin(\theta+\psi) + \sin(\theta-\psi))$ (25%)

e) none of these

29. If $\theta$ is an acute angle and $\sin(\theta) = \frac{3}{5}$, $\cos(\theta)$ equals:

a) $\frac{3}{4}$ b) $\frac{5}{3}$ c) $\frac{2}{5}$ <u>d)</u> $\frac{4}{5}$ e) none of these (59%)

30. The solution set for the equation $\tan(\theta) = 1$ on the interval $0 \le \theta \le 2\pi$ is

<u>a)</u> $\{\frac{\pi}{4}, \frac{5}{4}\pi\}$ b) $\{\frac{\pi}{4}\}$ c) $\{\frac{3}{4}\pi, \frac{7}{4}\pi\}$

d) $\{\frac{\pi}{4}, \frac{3}{4}\pi, \frac{5}{4}\pi, \frac{7}{4}\pi\}$ e) $\{\frac{\pi}{4}, \frac{3}{4}\pi\}$ (42%)

## *Results*

Distribution of Scores: Separate scores for the two parts of the exam were computed for each student. These scores were obtained by subtracting 1/4 of the number of incorrect answers from the number of correct answers and then rounding up to the next integer. The distributions were as follows:

Algebra Part

| Score | 0 - 3 | 4 - 6 | 7 - 9 | 10 - 11 | 13 - 15 | 16 - 18 |
|---|---|---|---|---|---|---|
| % of Students | 7.1 | 13.8 | 23.6 | 31.4 | 14.9 | 8.7 |

Trigonometry Part

| Score | 0 - 2 | 3 - 4 | 5 - 6 | 7 - 8 | 9 - 10 | 11 - 12 |
|---|---|---|---|---|---|---|
| % of Students | 39.9 | 17.4 | 20.4 | 9.3 | 9.6 | 3.1 |

Some Conclusions: As was pointed out in the report on last year's exams, several factors must be taken into account when analyzing the algebra scores. For example, the students had been away from their studies for several months or more. (The enrollment in the introductory courses includes many students who have been away from school and mathematics for several years.) Several of the questions examined material covered long before the last year of high school or in the case of at least one question (#12) material no longer in the high school syllabus. Taking these factors into account, the algebra scores seem quite reasonable.

Trigonometry questions were not included on the 1976 exam and so the large number of low scores on the second part of this year's exam was surprising. Again one must take into account several factors when considering these scores. Trigonometry, more than algebra, involves memorization of formulas that are easily forgotten if not regularly used. As there are two units of measure commonly used for angles, the consistent use of the radian measure on the exam may have placed some students at a disadvantage. Also, the recent increased use of pocket calculators has probably reduced the number of students who have the sine and cosine of common angles at their fingertips.

The Advisory Exam and the distribution of trigonometry scores were discussed at a get-together this winter of representatives of the U. of A. Math. Dept. with some mathematics coordinators from Edmonton high schools. One point that was made at that meeting by some of the high school teachers was that the amount of time devoted to trigonometry in the high school mathematics program has been falling over the years and that this trend has in the past been encouraged by the U. of A. Math. Dept.. It may well be worth considering now whether this trend has gone too far.

During the fall and winter terms, the Math. Dept. offered short remedial programs (about 3 weeks each in duration) in both algebra and trigonometry. Students who did badly on the advisory exam were advised to enroll in one or both of these programs. About 12% of the students in the calculus courses enrolled in the algebra program and about 23% enrolled in the trigonometry program. Comments about the programs were solicited from these students. They generally thought that both programs were beneficial and provided much needed review. We were interested to discover that over 60% of the students attending the algebra classes had been out of school for over a year and that these students were glad to have the opportunity to fill in forgotten areas in their mathematical backgrounds.

We would be happy to hear from any high school teachers about their reaction to the Advisory Exam. As was pointed out in the report on last year's exam, the exam was not formulated with the intention of evaluating or criticizing the teaching of mathematics in the secondary schools. The intention of the exam was rather to help us determine what to expect from our incoming students and to help the students know what is expected of them. ■

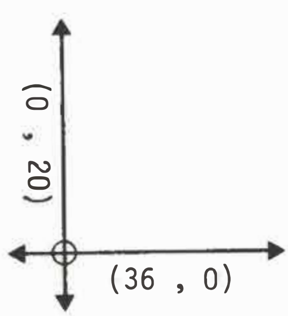
# Graph of the Month

*Harold Marenus*
9th Grade
Van Nuys Junior High
Van Nuys, California

*Reprinted from* THE CALCULATOR, *Volume 17, Number 7, April 1977*

## Contemplate Decimalized Measurements

Connect, in order, the following coordinate points. Use 1/4-inch graph paper, horizontally.




1. ( 9,14)
2. ( 7,14)
3. ( 7, 9)
4. ( 5, 9)
5. ( 5,14)
6. ( 3,14)
7. ( 3,16)
8. ( 9,16)
9. ( 9, 9)
10. (11, 9)
11. (11,12)
12. (13,12)
13. (13, 9)
14. (19, 9)
15. (19,12)
16. (21, 9)
17. (25, 9)
18. (25,12)
19. (27, 9)
20. (29, 9)
21. (27,12)
22. (29,16)
23. (27,16)
24. (25,13)
25. (25,16)
26. (21,16)
27. (21,13)
28. (19,16)
29. (13,16)
30. (13,14)
31. (11,14)
32. (11,16)
33. ( 9,16)

Lift pencil ---
begin again at:
34. (15,16)
35. (15, 9)

Lift pencil ---
begin again at:
36. (17,16)
37. (17, 9)

Lift pencil ---
begin again at:
38. (23,16)
39. (23, 9)

Lift pencil ---
begin again at:
40. ( 7, 8)
41. ( 5, 8)
42. ( 4, 6)
43. ( 3, 8)
44. ( 1, 8)
45. ( 1, 1)
46. ( 3, 1)
47. ( 3, 5)
48. ( 4, 3)
49. ( 5, 5)
50. ( 5, 1)
51. (12, 1)
52. (12, 3)
53. ( 9, 3)
54. ( 9, 3½)
55. (11, 3½)
56. (11, 5½)
57. ( 9, 5½)
58. ( 9, 6)
59. (14, 6)
60. (14, 1)
61. (16, 1)
62. (16, 6)
63. (18, 6)
64. (18, 1)
65. (20, 1)
66. (20, 3)
67. (22, 1)
68. (24, 1)
69. (22, 3)
70. (23, 3)
71. (24, 4)
72. (24, 1)
73. (26, 1)
74. (26, 2)
75. (27, 1)
76. (30, 1)
77. (31, 2)
78. (31, 4)
79. (29, 4)
80. (29, 3)
81. (28, 3)
82. (28, 6)
83. (29, 6)
84. (29, 5)
85. (31, 5)
86. (31, 7)
87. (30, 8)
88. (27, 8)
89. (26, 7)
90. (26, 8)
91. (24, 8)

Lift pencil ---
begin again at:
92. (24, 7)
93. (23, 8)
94. ( 7, 8)

Lift pencil ---
begin again at:
95. ( 7, 8)
96. ( 7, 1)

Lift pencil ---
begin again at:
97. (12, 8)
98. (12, 6)

Lift pencil ---
begin again at:
99. (18, 8)
100. (18, 6)

Lift pencil ---
begin again at:
101. (24, 7)
102. (24, 4)

Lift pencil ---
begin again at:
103. (26, 7)
104. (26, 2)

Lift pencil ---
begin again at:
105. (20, 6)
106. (20, 5)
107. (22, 5)
108. (22, 6)
109. (20, 6)

--- THE END ---

# *Ideas for the Primary Class*

*Reprinted from* The Manitoba Mathematics Teacher, *Volume IV, No. 4, June 1976*

Children's gift wrap is a relatively inexpensive source of identical pictures for making pictographs.

Staples used on materials (cardboard or paper) can be used on the magnaboard rather than using magnetic strips.

- *H. Ward*
Lynn Lake, Manitoba

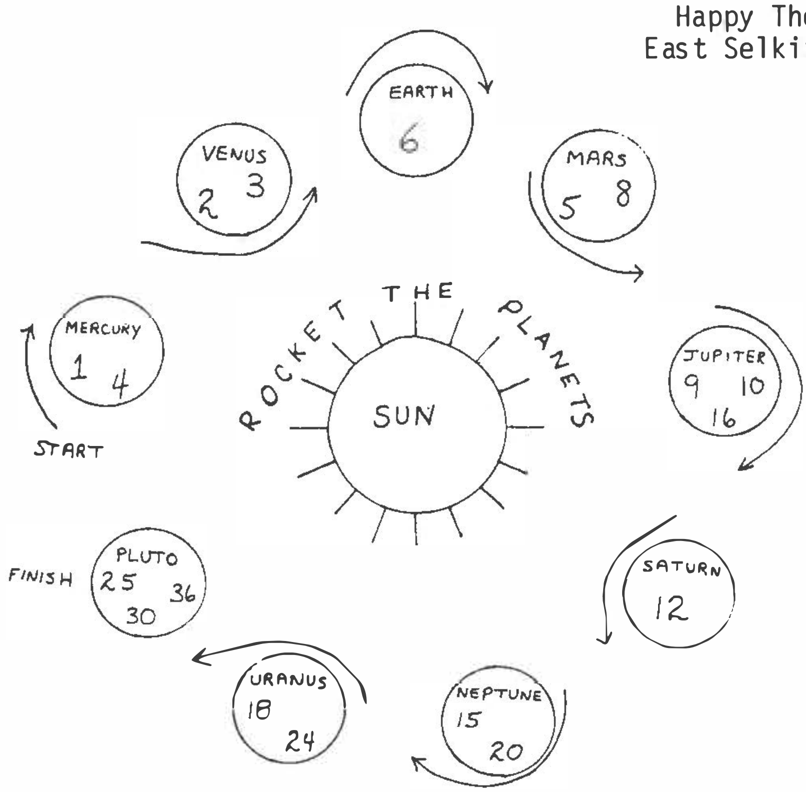
## *Travel Around the Planets*

Objective: Travel around the planets. The first one to reach Pluto wins.

Concept: Multiplication (or any other operation)

First person rolls dice. He multiplies the two numerals; if the answer is on "Mercury," he may place his counter on it. If not, he does not get on "Mercury." Continue.

- *Colleen Yakielashek*
Grade III
Happy Thought School
East Selkirk, Manitoba




- two dice
- each child has marker (2 - 5 players)

## Multiplication

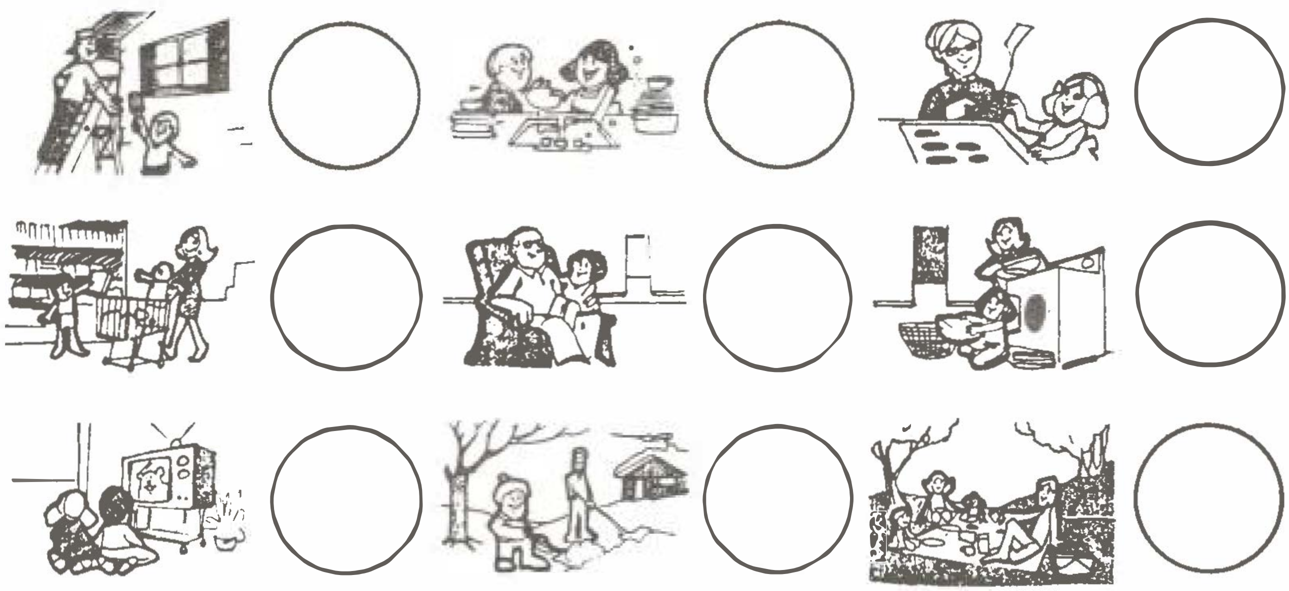
Take an egg carton and put the numbers from 1 to 12 in the egg holders. Vary the numbers, for example,

| | |
|---|---|
| 1 | 7 |
| 12 | 3 |

and so on. Put two coins in the carton. It can be used for multiplication, subtraction, or addition. Shake the container. Multiply, add, or subtract whatever two numbers the coins fall in.

## Happy or Sad Face?

**If you were one of the people in the picture, would you be happy or sad? Draw a face in the circle to show how you would feel.**



**Circle the number of happy faces. 1 2 3 4 5 6 7 8 9**

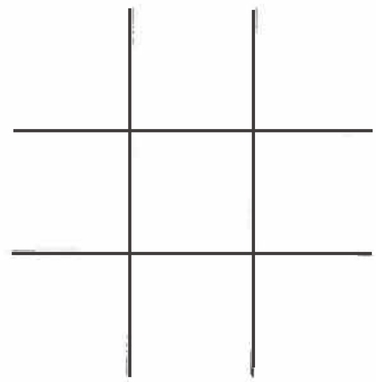
**Circle the number of sad faces. 1 2 3 4 5 6 7 8 9** ■

# *Ideas for the Intermediate Class*

*Reprinted from* The Manitoba Mathematics Teacher, *Volume IV, No. 4, June 1976*

## *tic-tac-toe*

The activities can be adapted to various grade levels, and can be expanded to include the other three basic operations of subtraction, multiplication, and division, as well as fractions. I hope that at least one of these activities is enjoyed by you and your class.



Equipment:

1. strips of masking tape on the floor to make a tic-tac-toe game with spaces large enough for a pupil to stand in.

2. a pile of flashcards, placed face down, with appropriate equations:

   (17 + 8 = ___; 9 + ___ = 18, and so on)

3. a way of marking five pupils as "O" and five pupils as "X" (cards pinned to shirt or hat; "O" pupils stand, "X" pupils sit, and so on)

Directions:

1. Decide which team starts.

2. First member of starting team picks top flashcard, answers questions, and, if answer is correct, the pupil chooses a spot on the tic-tac-toe game. If the answer is incorrect, the pupil does not choose a spot, and he goes to the end of his team. This pupil could keep the flashcard to memorize.

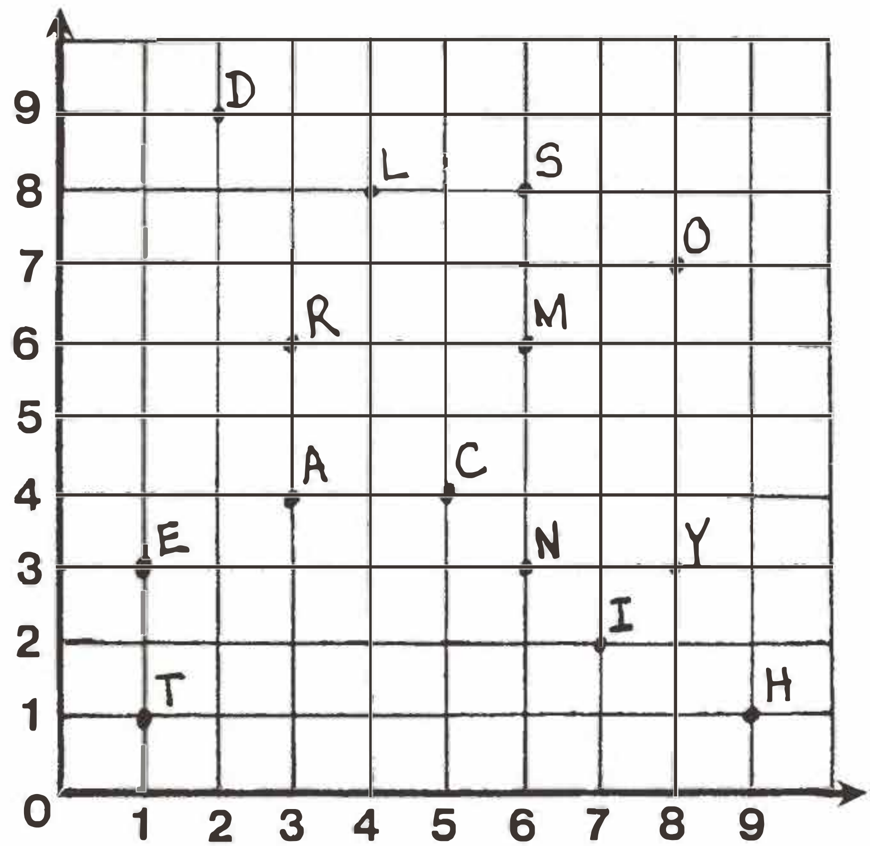
3. First member of the second team does the same as described in number 2.

4. Game continues until a team forms a straight line of three "O's" or three "X's" as in a regular game of tic-tac-toe.

Careful:

Make sure the pupils know how to play tic-tac-toe.

## *A Backwards Graph*




This is a backwards graph.
Once you find a word,
turn it around.

Example: M A E T

T E A M

(8,3) (4,8) (6,3) (8,7)   (6,8) (6,3) (3,4) (7,2) (1,1) (3,6) (3,4) (6,6)

(6,3) (3,4) (5,4)   (2,9) (3,4) (1,3) (3,6)   (6,8) (7,2) (9,1) (1,1).

Answer: Only Martians can read this.

## *Rounding to the Nearest Hundred*

If "hundred's" place is:

1 - color the space red
2 - color blue
3 - color green
4 - color brown
5 - color yellow
6 - color orange
7 - color black
8 - color purple
0 - don't color

# A Teacher's Opinion

*Reprinted from* The Manitoba Mathematics Teacher, *Volume IV, No. 4, June 1976*

*Assignment: Give a short description of what you would do to change mathematics and mathematics instruction in Grades VII to XII.*

This short paper will consist of ideas in three parts: province-wide changes, divisional or school thoughts, and course ideas for mathematics.

First, let's deal with the province of Manitoba as a whole. A key concept which needs to be expressed regards the division of mathematics from Grade IX up into three disjoint areas or domains. These include social mathematics, technical mathematics, and academic mathematics. The social domain would include all mathematics important to personal being and citizenship in society and would cover such skills as reading, use of symbols, arithmetic calculations, measurement, ratio, estimation, graphing, data interpretation, intuitive geometry, metrication and logical thinking. Preparation for leisure time should also be part of social mathematics. Technical mathematics would include the use and calculations necessary for various skilled jobs and professions. Topics might include the ratio approach to trigonometry, scale drawings and measurement, access to computer programs and some on-the-job training. Academic mathematics would consider mathematics as a formal system to be studied in and of itself. This program would be for those students capable of and interested in a theoretical study of mathematics.

A basic standardized curriculum across Canada with provisions for provincial and divisionally instituted options at each grade level would satisfy the traditional and the modern teacher.

The public school system needs regulations regarding attendance and promotion. An aid to a well-disciplined school system would be a governmental lowering of the compulsory attendance school age to fourteen. Expulsion should be rare, but a student who does nothing and who is not interested in any aspect of academic learning should terminate his relations with the public schools as a favor to himself and society. The school and society should then arrange a job for this student.

During the final year of high school, each student could be awarded a subject grade by his school. He should also have the option of writing a final examination, set and marked by a chosen committee. The committee should consist of representatives from Red River Community College, Manitoba's three universities, and a number of representatives from various divisions. An attempt should be made to keep the committee together for a three-year period.

Questions which ought to be considered on a provincial basis might include:

1. Should "new" techniques be tested under controlled conditions?

2. Is a stable period of three or four years with few content changes necessary?

3. Are renewable five-year contracts or compulsory moves from one division to another after seven years useful?

4. Should sabbatical leaves for all teachers be awarded for the purpose of full-time study in education? This would necessitate the elimination of all summer and evening programs for a Master of Education degree. Courses could still be taken in summer in order to change grant ratings or to improve expertise in a certain field.

5. Should the budget for books and materials to aid individualization be increased?

Next, let us consider some needed changes at the divisional and school levels. There should be a removal of unsatisfactory teachers and administrators from the system, although several pertinent questions need to be answered here. Who is unsatisfactory? Why are they unsuitable? Who decides?

A student should have free choice as to which course he will attempt, but in order to remain in that level he should have to maintain a "C" or 60 percent grade.

Evaluation should be based on knowledge, manipulation, the understanding of concepts and processes, the ability to solve mathematical problems, sound reasoning, the use and appreciation of mathematics. What is the value of any subject which cannot be used or at the least cannot be appreciated for some intrinsic value?

Other concerns at the divisional level are:

1. the possibility of some form of streaming in junior high (why does everything happen in Grade X?);
2. more guidance for those teachers who need or want it;
3. a reintroduction of the work ethic for both students and staff;
4. the use of inspection teams to evaluate staff and school;
5. some stress on retention and drill;
6. an end to continuous promotion after Grade VII;
7. an increased interest in mathematics contests and puzzles; and
8. discussion with mathematics teachers both above and below one's own level.

Finally, let us turn our attention toward the mathemactics course content. This has been dealt with briefly already but other changes which could be included are:

1. two courses at the Grade IX level (these courses would be in the areas of social and technical mathematics);
2. academic courses would begin in Grade X;
3. geometry taught in one full course and as an option;
4. changes in the Algebra 100 course content;
5. a full course in trigonometry; and
6. the same 24 percent time allotment for Math 300 as for English 300.

The conservative reactionary feelings seemingly expressed in this paper may not be really mine. The devil made me do it! (Author's name withheld.)

Note: *The foregoing is a response provided to the above assignment by a student in course 81.701 Seminar in Mathematics Education. A response would be welcome.* - *A.M. MacPherson* ■

# ANNUAL NCTM REGIONAL CONFERENCE
## SEPTEMBER 28-30, 1978
## CHEYENNE, WYOMING

## Name-of-Site Convention Presents A Cure for Mathemyopia

*Walter Rader* and *Christine Ivey*
Publicity
1978 NCTM Regional Conference

The 1978 National Council of Teachers of Mathematics (NCTM) Regional Conference to be held September 28 to 30 in Cheyenne, Wyoming promises to be a very special one.

INTERFACE is the key word for this conference. Reading, science, and math are all brought together in over 118 workshops and sectionals in a true interdisciplinary effort. Hosting parts of this conference, in addition to the NCTM, are the National Science Teachers Association (NSTA), and the International Reading Association.

There is no danger that this conference will be a grim affair. With Dr. Harold Jacobs, the featured speaker from Van Nuys, California, there is a promise of a good share of insight and entertainment. Dr. Jacobs will be speaking on such topics as "The Clock That Had Raisins in It" and "Of Ostrich Eggs, Rotating Pyramids, and Hershey Bar Graphs." Dr. Jacobs is a contributing author to Freeman Press and has written, among many things, a book *Mathematics: A Human Endeavor*. The book is a sure cure for those of you who are "sick and tired" of math.

Sectionals and workshops will cover Grades K-14 in math/reading, math/science, and science/reading as well as each area alone. A few of the concerns to be covered are: "Math Among Other Things"; "Why Debbie Can't Visualize"; "What Students Say About Reading Science and Math"; "Teaching Reluctant Readers in the Secondary School"; "Appetizers For Learning." The offerings are chock-full of humor, hard facts, and new ideas to stimulate interest in topics that may have grown dry and difficult to student and teacher.

Presiding will be Dr. Dorothy Strickland (IRA), Dr. Shirley Hill (NCTM), and Mr. John Akey (NSTA). As many as 1500 educators are expected from the Rocky Mountain area to attend this '78 conference in Cheyenne, Wyoming. Mark your calendar for this fall extravaganza, September 28-30. Join in! This conference is a winner!

# *Ideas for the Junior High Class*

*Reprinted from* The Manitoba Mathematics Teacher, *Volume IV, No. 4, June 1976*

## *Outdoor Metrics*

Now that the nicer weather is here, you might want to take your classes out for some metric fun. This can be done in the schoolyard, but, more impressive from the students' viewpoint, is a trip to a larger park. The workshop should involve linear measurement only and, with 25 to 30 students, will take an entire morning or afternoon. The students should work in pairs to check out each other's findings. Some classroom work in the metric system, and especially conversion from one metric unit to another, is a must before such a workshop is attempted.

The necessary equipment can easily be made in classrooms in some cases, or the larger pieces can be borrowed from resource centers or other schools. For each workshop involving 25 students, the following are necessary:

1. small metric tapes made in the classroom from seam binding, and so forth, about 1 to 2 metres in length;
2. 20 to 30 cm rulers, either manufactured or made from lino strips and clearly marked in the classroom;
3. about 4 trundle wheels for long distances;
4. longer tapes - 50, 60 metre size from your phys. ed. department;
5. clinometers which can easily be made in the classroom;
6. metre sticks;
7. stopwatches for running events, which add to the fun; and
8. centimetre graph paper.

If the workshop activities are divided into three sections, the scramble for equipment is eliminated. Keep the activities involving the limited pieces in one section so that only a few students will need them at one time. Here are a few suggestions for activities, most of which came from the Department of Education through Bob Mak, Area 3's consultant in Winnipeg Division #1.

Math in the Out-of-Doors

1. (a) Practice your metric measures! By placing stakes in the ground, estimate the following distances; then check by measuring with a metric tape or trundle wheel. Record your results in the chart.

| Estimated Distance | Actual Distance |
|---|---|
| 1 Metre | |
| 10 Metres | |
| 100 Metres | |
| 1000 Metres | |

(b) Count the number of paces it takes you to walk 100 metres. Calculate the length of each pace.

2. How far do you think you can walk in one minute? Estimate, walk, then measure (stopwatch needed).

Estimate: ________________________ metres per minute

Actual: ________________________ metres per minute

3. Choose three pairs of objects within your view. Estimate the distance between each pair. Measure to check. Record your data.

| Objects | Estimated Distance Apart | Actual Distance Apart |
|---|---|---|
| | | |
| | | |

4. How many blades of grass is it necessary to stack together to be the thickness of a centimetre? Estimate first, then measure. Calculate the average thickness of one blade of grass.

5. Choose three different trees. Measure their circumference and diameters. Record data. Calculate the ratios. C/D for each. What mathematical relationship have you discovered?

| | Circumference C | Diameter D | Ratio C/D |
|---|---|---|---|
| Tree #1 | | | |
| Tree #2 | | | |
| Tree #3 | | | |

6. Mark out an area of 1 square metre ($1m^2$) using stakes.

7. Mark out with stakes each of the following:

(a) a square of area $16m^2$. What is its perimeter?

(b) a rectangle of area $16m^2$. State the dimensions of three other rectangles which would each have area of $16m^2$. Find the perimeter of each.

(c) a triangle of area $16m^2$. Find the perimeter.

8. Using grid paper ($cm^2$), count the number of squares covered by each of five different objects found in the environment. Record data.

| Object | Area ($cm^2$) |
|---|---|
| | |
| | |

9. Make a list of things that you can find in nature or in your immediate environment (out-of-doors) which have geometric shapes or properties such as triangle, rectangle, circle, sphere, spiral, polygon, parallel lines, perpendicular lines, planes, symmetry.

| Object | Shape or Property |
|---|---|
| | |
| | |
| | |

10. Find height of objects:

(a) Have a person whose height is known stand by the object to be measured. This person is the "unit of measurement."

Hold a stick (or pencil) at arm's length. Sight over the top of the stick to the head of the "unit of measurement." Place the thumbnail on the stick where the line of sight meets the foot of the person.

Determine the number of "units of measurement" fitting onto the object being measured by moving the stick upwards a unit at a time.

(b) The Isosceles Right Triangle Method:

Back away from the object (for example, tree) until an imaginary line from your eye to the top of the tree forms a 45 degree angle to a horizontal line from your eyes to the tree. Use a clinometer (vertical protractor) to help determine the correct angle.

In a 45 degree (isosceles right) triangle, the two sides are equal in length; hence distance A is equal to distance B.

*To find the height of the object, measure the distance from you to the object and add this amount to the height of your eyes from the ground.

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## *A Desperate Student's Prayer*

Now I sit me down to cram,
The night before my Math 10 exam.
With notes in hand and books on the floor
My mind's a blank; my eyes are sore.
The formulas and proofs I can hardly recall;
How will I ever remember them all!
"A triangle has 160 degrees ..."
No, that's not right! Lord, help me PLEASE!
The midnight hour is drawing near;
The dreaded day is almost here!
I'm panicky, I'm sick, I'm full of remorse;
Oh Lord, why did I ever take this course?
So, tomorrow, Lord, help me survive.
All I want is a 65.

- *Beverlee* and *Geraldine Hill*
New York

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■

# Ideas for the Senior High Class

*Reprinted from* The Manitoba Mathematics Teacher, *Volume IV, No. 4, June 1976*

## The Difference of Two Squares — A Discovery Approach

*Cecil Grant*

Can students be led to discover that $a^2 - b^2$ is equal to $(a+b)(a-b)$? It is my belief that they can. The development of a concept by the use of the discovery method is not always a practical approach for the regular classroom teacher when time is of the essence. However, as a change from the regular lecture-type presentation, the discovery method used at different points in the program can be an enhancing feature.

What I now describe is one of a three-part unit which I used in developing the concept that $a^2 - b^2$ is equal to $(a+b)(a-b)$ with a Grade X class.

I began by writing the following problems on the chalkboard and asking students to supply the answers to them:

| | |
|---|---|
| $3^2 - 2^2 =$ ____ | $7^2 - 6^2 =$ ____ |
| $4^2 - 3^2 =$ ____ | $8^2 - 7^2 =$ ____ |
| $5^2 - 4^2 =$ ____ | $9^2 - 8^2 =$ ____ |
| $6^2 - 5^2 =$ ____ | $10^2 - 9^2 =$ ____ |

As expected, the solutions given by all the students were the following:

| | |
|---|---|
| $3^2 - 2^2 = 9 - 4 = 5$ | $7^2 - 6^2 = 49 - 36 = 13$ |
| $4^2 - 3^2 = 16 - 9 = 7$ | $8^2 - 7^2 = 64 - 49 = 15$ |
| $5^2 - 4^2 = 25 - 16 = 9$ | $9^2 - 8^2 = 81 - 64 = 17$ |
| $6^2 - 5^2 = 36 - 25 = 11$ | $10^2 - 9^2 = 100 - 81 = 19$ |

By appropriate questions, the students were led to see that the answers to the above problems can also be obtained in the following way:

| | |
|---|---|
| $3^2 - 2^2 = 3 + 2 = 5$ | $7^2 - 6^2 = 7 + 6 = 13$ |
| $4^2 - 3^2 = 4 + 3 = 7$ | $8^2 - 7^2 = 8 + 7 = 15$ |
| $5^2 - 4^2 = 5 + 4 = 9$ | $9^2 - 8^2 = 9 + 8 = 17$ |
| $6^2 - 5^2 = 6 + 5 = 11$ | $10^2 - 9^2 = 10 + 9 = 19$ |

At this point, the rule appeared to be $a^2 - b^2 = a + b$.

The following questions were then given:

| | |
|---|---|
| $4^2 - 2^2 =$ ____ | $7^2 - 5^2 =$ ____ |
| $5^2 - 3^2 =$ ____ | $8^2 - 6^2 =$ ____ |
| $6^2 - 4^2 =$ ____ | $9^2 - 7^2 =$ ____ |

The students were asked to provide the answers for these, first by using the above rule and then by using the method initially employed with the first set of problems. The results were as follows:

| Problem | Answer by Rule | Answer by initial method |
|---|---|---|
| $4^2 - 2^2 =$ | $4 + 2 = \underline{6}$ | $16 - 4 = \underline{12}$ |
| $5^2 - 3^2 =$ | $5 + 3 = \underline{8}$ | $25 - 9 = \underline{16}$ |
| $6^2 - 4^2 =$ | $6 + 4 = \underline{10}$ | $36 - 16 = \underline{20}$ |
| $7^2 - 5^2 =$ | $7 + 5 = \underline{12}$ | $49 - 25 = \underline{24}$ |
| $8^2 - 6^2 =$ | $8 + 6 = \underline{14}$ | $64 - 36 = \underline{28}$ |
| $9^2 - 7^2 =$ | $9 + 7 = \underline{16}$ | $81 - 49 = \underline{32}$ |

The students readily noticed that the answers obtained by the initial method, which were the correct answers, were *twice* the answers obtained by the rule. The rule would yield the correct answers to the second set of problems if it were changed to $a^2 - b^2 = (a + b) \times \underline{2}$.

The students were now asked to compare the initial rule with the modified rule and to look for clues as to why it was necessary to modify the former for the second set of problems. One student quickly pointed out that the initial rule could well be written as $a^2 - b^2 = (a + b) \times 1$, where "1" was the difference between the numbers used in the first set of problems. In the second set of problems, the difference was 2, and so the rule became $a^2 - b^2 = (a + b) \times 2$.

It did not take long after this stage to have the students discover that $a^2 - b^2 = (a + b)(a - b)$.

## *It's a Puzzlement??*

When $4444^{4444}$ is written in decimal notation, the sum of its digits is A. Let B be the sum of the digits of A. Find the sum of the digits of B. (A, B are written in decimal notation.) ■

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