The Role of Mathematics in the Development of Science

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Introduction

Mathematics and science have interacted through the ages. Mathematics, as a means of articulation and theorization in science, now spans the universe all the way from the largest galaxy to the smallest elementary particle. The present-day relationship between mathematics and science is by no means static. It has evolved from the past and will continue to evolve in the years ahead. Out of past associations new ones emerge, then with a further change in the intellectual and cultural climate, new interactions develop.

Today's mathematical involvement in the physical and social sciences can be traced to its historical routes. By doing so, one gains an overview of the various roles of mathematics in the development of science. From such a perspective one can understand the ways in which mathematics interacts with science today; but more, one can bring greater clarity into speculations of future relationships.

Mathematics itself is not a part of the realm of science. Both have different subject matters, differences which cannot be bridged by anything but superficial similarities. Basically, natural science deals with objects and events in the "external" world while mathematics concerns itself with the objects in its own "aesthetic" perception. These objects are internally conceived and inwardly structured.

Through the ages and, indeed, today, people have had diverse ideas regarding the relationship between mathematics and science. Some would have us believe that mathematics is the "queen of the sciences," while others insist that it is simply a servant to science:

Pythagoreans (ca.530 B.C.) maintained that in nature "all is number."

Aristotle (ca.350 B.C.) defined mathematics as an abstraction from nature.

Kepler (ca.1600 A.D.) thought that just as ears are made for sound and eyes for color, the mind of man is meant "to consider quantity."

Galileo (ca.1600 A.D.) said that the book of the universe was written in mathematical language and its alphabet consisted of geometrical figures.

Bacon (ca.1600 A.D.) regarded mathematics merely as a servant to physics, and actually complained of the dominion which it was beginning to exercise in science.

Feynman (1965) writes, "mathematics can help physics, but they are two quite different activities - mathematics deals with the abstract world, and physics deals with the real world."

Eric Rogers (1960) suggests that we might describe mathematics as a "master architect designing the building in which science can grow at its best."

Just as there is a danger to science in over-glorifying mathematics and attempting to subordinate all of science to it, so is there a danger in calling mathematics the "handmaiden of science." Accelerations and retardations in the development of science can be traced, in large measure, to such attitudes.

This paper, in investigating the role of mathematics in the rise of science, chronologically examines various epochs of our past.

Pre-Greek and Greek Era

The mathematics of Egypt and Babylon preponderantly served a practical function. Pre-Greek mathematics was integral to astronomy, taxation, and the construction of moats and temples.

The most successful product of the Greek mind was the deductive quality of geometry. The first Greeks to grasp this possibility of abstraction in geometry were probably Thales (600-550 B.C.) and Pythagorus. Then around 300 B.C. one of the most famous masters of geometry, Euclid, set out to collect the theorems of his predecessors and to arrange them as a single self-contained work entitled *Elements*.

The next century produced two or more gifted mathematicians. Apollonius (ca.200 B.C.) discovered the so-called conic sections which later contributed directly to astronomy. In addition there was Archimedes (ca.250 B.C.) whose brilliance at mathematics was matched by his genius for mechanics.

In Greek astronomy, the first mathematically conceived system was that of Eudoxus (ca.370 B.C.). Spurred on by Plato's notion of reality, Eudoxus reduced the irregular movements of celestial bodies to uniform circular motion. His system consisted of 27 concentric spheres, one inside the other. The whole system was a purely geometrical hypothesis, calculated to represent the apparent paths of the planets. Later, Ptolemy (90-168 A.D.) wrote his famous astronomical treatise *De Algamest* in which he strove for a mathematical model of the universe, known today as the Ptolemaic system.

Greek physics did in no way bequeath a book comparable to the mathematical works of Euclid, Archimedes or Apollonius. Yet, Greek physics culminated in the system of Aristotle and held the stage of physics for almost 2000 years.

The Greeks created a general concept of mathematics and a general concept of physics. Yet, if one assesses the pragmatic outcomes of Greek mathematics and physics, and if one confronts the mathematics and physics of Greek antiquity with 20th century knowledge, then one may find that Greek physics as a whole *never* developed into a mathematical system. The system of Greek mathematics had severe limitations and shortcomings. For example, they were incapable of "founding" functions. Their symbolization did not advance beyond an elementary stage, the stage of abstraction from "direct actuality." (Full scale symbolization involves abstraction from abstraction.) These inadequacies made Greek mathematics unsuited for promoting the rise of theoretical physics as we know it today. In other words, these shortcomings prevented a type of scientific thinking from developing. Perhaps the Greeks did not introduce mathematics as a technique for mastering problems that arose in man's mind. Although Greek astronomy was mathematical and its mathematization made it successful, the Greeks never had the insight (necessary in pursuing mechanics, physics and other sciences) to articulate qualitative attributes by quantitative magnitudes. They never realized that writing physical laws as mathematical formulae and applying mathematical procedures to the formulae can sometimes lead to further explications and developments. The Greeks never arrived at such an insight in spite of the dramatic first steps taken by Archimedes. There was no determination to solve "scientific puzzles" through mathematical manipulation and empirical verification. This was so fateful that eventually even Greek mathematics tended to wither away.

Nevertheless, the Greeks did have areas of knowledge in which science and mathematics overlapped. Their astronomy was clearly mathematical in Eudoxus and Ptolemy. Pythagoreans envisioned some sort of mathematization of physics, although the extent and depth of their insights is not easy to appraise. Again, the laws of Archimedes on balancing the lever and on floating bodies clearly pertain to mathematical physics and were the first of their kind. And yet they did not have the effect of initiating a mathematical physics at that time. Even atomists like Epicurus and Democritus did not show any tendency to initiate a mechanics of the stature of the 17th or 18th century.

The Renaissance

The Greeks wrote hundreds of books on mathematics, treating it for the first time as worthy of study for its own sake. In the centuries of darkness that followed, much of this mathematical treasury was lost. But enough remained so that the scholars of the late Middle Ages once again launched a search for knowledge, giving the study of mathematics an impetus that kept on accelerating.

Men in the 16th and 17th centuries were already looking beyond arithmetic into the vistas of algebra. It was the French mathematician, Rene Descartes (1596-1650 A.D.) who first started writing algebraic equations. In his ambition to "remake the world" he developed a new branch of mathematics, analytic geometry, a technique for visualizing numbers as points on a graph, equations as geometric shapes, and shapes as equations. Trigonometry and logarithms also emerged from the Cartesian system of Descartes.

Then in 1665, England's Isaac Newton produced calculus, which for the first time permitted the mathematical analysis of all movement and change. Meanwhile, a German mathematician, Leibnitz (1646-1716), independently invented his version of calculus and in 1684 published his account of it. (Today the symbols derived by Leibnitz: $\frac{d}{dx}$, $\frac{d}{dy}$, $\frac{d}{dz}$, are more generally used than those derived by Newton: $\dot{x}$, $\dot{y}$, $\dot{z}$.)

In Rene Descartes' work, we meet a system of thought much more intensive, concentrated and intricately interlocked than the Greek system. In his system everything was to be accounted for mathematically - by configuration or by number. He regarded physics as reducible to mechanism, and even considered the human body as being analogous to a machine. The mechanization of his highly concentrated deductive system became the template for the structure of physical science.

Men of the 17th century were extremely conscious of the importance of mathematics to scientific development. Therefore, it is not surprising that the development of science after 1600 A.D. began with the establishment of this so-called "rational mechanics" which held the stage of science during the 17th, 18th and 19th centuries. Mechanics was a problem which only became manageable when, in a certain sense, it had been "geometrized." Motion became envisaged as occurring in the emptiness of Euclidean space. (The Aristotelian system had discouraged the idea of the composition of motion, and was uncongenial to any mathematical treatment of it.) Although Galileo was one of the first persons to treat motion quantitatively, he failed to achieve the perfect formulation of the modern law of inertia because he could not imagine a purely geometrical body sailing off into an utterly empty and directionless Euclidean space. The law of inertia had to wait for Descartes.

The 17th century produced the greatest single statement on the relation between mathematics and physics. It was Galileo's dictum that mathematics is a language of science. He went so far as to say that the mind was to be constantly directed only to those things, and to apply itself to only those problems which were amenable to measurement and calculation. Descartes, Torricelli, Kepler, Huygens, Newton, and others who succeeded Galileo, clarified this scientific value by their geometrizing problems concerning natural phenomena.

The mechanized study of motion may well have been the high point of 17th century science. The century witnessed one attempt after another not only to explain motion and other natural phenomena, but to interpret all changes of the physical universe in terms of a purely mechanistic universe. Kepler inaugurated the scientist's quest for a mechanistic universe.
To generations of astronomers, the basic celestial figure was a circle. Kepler broke away from this supposition by introducing ellipses. He did not find his ellipses in the tables of Tycho Brahe or the writings of Copernicus. He found them by searching untiringly in the work of Apollonius. Guided by astronomical observations, he was the first to grasp the true meaning of foci of conics. Conics and their theory were in no way Kepler's private mathematical invention. They had existed for nearly 2,000 years for anybody to find and use.

The determination to formulate all explanations in mechanistic terms had important effects upon the biological sciences. Harvey (1567-1650), in his enquiries into the circulation of the blood, had a purely mechanical approach. G.A. Borelli (1608-1679), in his book *The Motion of Animals*, wrote a chapter on the "Mechanical Propositions Useful for the More Exact Determination of the Motive Power of Muscles." This tendency to glorify mere mechanization led to the ubiquitous view that the animal body was nothing more than a piece of clockwork.

The effects of the new mechanistic outlook are vividly illustrated in the works of Robert Boyle (1627-1691). He is quoted to have said that he did not expect to "see any principles proposed more comprehensible and intelligible than the corpuscularian." This philosophical position is often called "mechanical philosophy," since it tends to give a mechanical explanation of the physical universe. One of

Boyle's works includes a discourse on the mechanical origin of heat and magnetism. According to him, chemistry itself could be reduced to micromechanics.

Later in the 17th century, a culminating event took place when Newton and Leibnitz introduced "derivatives" and laid the foundation for calculus and mechanics. The ultimate triumph came in publishing the *Principia*, in 1686. The *Principia* is important not so much because of its laws, definitions, concepts of time, space, and gravitational force, but because Newton constructed and deduced, by mathematical reasoning, what Kepler had only divined and postulated.

Outwardly, in the *Principia* there is hardly any mathematics invoked or presupposed which should not have been quickly accessible to Archimedes and Apollonius. No attempt had been made in the *Principia* to introduce Descartes' innovation of analytically using symbols and functions. Newton was quite skilled in the use of symbols and functions as he was very familiar with Descartes' work. In fact, Newton mastered the method of Descartes much better than Descartes himself. However, in the *Principia*, Newton's definitions of limit and derivative ("ultimate ratio") seem to be such that a personal disciple of Archimedes should have been able to compose them in principle. Why did this not occur? Greek thinking, in general, did not formulate such logical abstractions as: a relation of a relation, a property of properties, an aggregate of aggregates (for example, a rate of change of a rate of change - acceleration). Second derivatives were at the center of Newton's mechanics. It was this kind of limitation to Greek rationality that separated Archimedes from Newton; a hiatus which Archimedes could never succeed in crossing.

Inwardly, there is a difference between the Euclidean space that underlies the *Principia* and the Euclidean space that underlies Greek mathematics and physics. The Euclidean space of the *Principia* continues to emphasize Greek congruencies and similarities between figures. However, it does something new. Several significant physical entities found in the *Principia*, velocities, momenta and forces, are vectors. Vectorial composition and decomposition of these entities constitutes an innermost scheme of the entire theory. In the course of the 18th century, the vectorial statements of Newton and others were gradually transferred and reinterpreted into analytical statements. The 20th century widened the concept of a vector into the broader concept of a tensor.

To summarize the events in the 17th century, one could say that there was considerable scientific development where geometrical and mathematical methods could be easily and directly applied. Therefore, not only did the sciences make a remarkable development in the 17th century, but mathematics also progressed to a great extent. This is because the sciences, especially physics and dynamics, were pressing upon the frontiers of mathematics all the time. The sciences created a need for mathematics, and therefore mathematics flourished. The relationship between science and mathematics has never quite been the same since then. Today, both tend to go their separate ways and draw upon one another when the need arises. But it was not so in the 17th century. Science depended on mathematics and mathematics depended on science. Without the achievements of mathematics the scientific revolution, as we know it, would have been impossible.

18th-19th Centuries

Let us first look at the achievements of some of the great mathematicians and scientists in the 18th and

19th centuries. Then we shall analyze the role mathematics played in the rise of science at this point in history.

Because of Newton's success, mathematical theorists of the 18th and 19th centuries held fast to a philosophy of "mechanistic determination." The French mathematician, Pierre Simon de Laplace (1749-1827), perfected Newtonian analysis of the solar system in a great work entitled *Mechanique Celeste*. He also used calculus to explore and advance probability theory. The most celebrated partial differential equation was devised by Laplace.

$$\frac{\partial^2 v}{\partial^2 x} + \frac{\partial^2 v}{\partial^2 y} + \frac{\partial^2 v}{\partial^2 z} = 0$$

(This equation has been used to describe the stability of the solar system, the electric field around a charge, a steady distribution of heat, and many other phenomena.)

Euler (1707-1783), of Swiss fame, created a host of new uses for calculus as it applies to curves and surfaces. He has been called the father of modern topology.

Lagrange's (1716-1813) hallmarks were his famous works *Mechanique Analytique* and *Theorie des Fonctions Analytiques* - master textbooks in its subject. But by far his greatest achievement was the space of "generalized coordinates" of our mechanics of today.

The genius who dominated 19th century mathematics and physics was Carl Friedrick Gauss (1776-1855). He gave direction to the new movement toward generality in mathematics by imposing on it his own stern standard - a demand for absolute rigorous thinking. As a 17-year-old, he audaciously questioned certain rules of Euclid's geometry that generations of mathematicians had taken for granted, pointing out that many of them did not hold true on curved surfaces.

But it remained for Gauss's pupil, Riemann (1827-1866), to shatter the boundaries of traditional geometry by postulating not only curved spaces of three dimensions, but spaces made up of four and more dimensions. Fifty years later, the physicist, Albert Einstein, brought the process to a stunning climax by borrowing these abstractions and using them in his theory of relativity to describe the real universe.

Out of the 17th and into the 18th century, preference continued to be given to rational mechanics (mathematical analysis of everything possible). In this context several so-called 'principles of mechanics' were produced. Meanwhile, the theories of light, heat, electricity, and magnetism were not forgotten but they advanced at a slower pace until their turn for full attention came in the 19th century.

The mechanics of the 18th century and the first decades of the 19th century was virtually inseparable from the mathematics. Almost all the leading architects of the various parts of mechanics were eminent mathematicians: James Bernoulli (1667-1748), d'Alembert (1717-1783), Euler (1707-1783), Cauchy (1789-1857), Lagrange (1716-1813), Poisson (1791-1840), Laplace (1749-1827), Gauss (1776-1855), and Jacobi (1804-1851), to name a few. Correspondingly, most of their theorizing emanated from "pure thinking" with very marginal entanglements in direct experimentation.

The need for 17th century mathematics continued to grow. Scientific progress was now even more strongly dependent on mathematics than it was before. In this period, the amount of mathematics which was created for, and because of, mechanics (theoretical and applied) was enormous - especially in the area of analysis. The calculus of variation was instigated largely by mechanics of particles (finite systems), while other mathematical theories were instigated largely by mechanics of continua (hydro-dynamics, acoustics, general theory of elasticity). Virtually all of partial differential equations were created this way. Indeed, the mathematical theory of waves, which eventually became the hallmark of theoretical physics in all its parts, emerged from mechanics of continua. Fourier analysis was the result of the mechanics of continua and the theory of heat. The concept of potential energy originated in the Lagrangian theory of finite particles. Finally, it appears that the mechanics of continua had a share in the emergence of tensor theory.

Beginning with the 19th century, the relationship between mathematics and mechanics changed. Mathematics became more or less independent of mechanics and physics. It assumed a philosophical nature and began to develop for its own sake. Yet another kind of relationship between mathematics and theoretical physics developed. It was a rapport built more on parallelisms of pursuits rather than on identities of aims. Mathematical formulations were no longer created for a particular purpose.

From time to time in this century, theoretical physics was able to seize upon an unfamiliar ready-made piece of mathematics and use it instantly. It would have appeared as if the mathematics had been prefabricated especially for the theoretical physicist. For example, in the second half of the 19th century, statistical mechanics of the kinetic theory of matter was able to draw upon the mathematical theory of probability (initiated in the Renaissance age and developed by Laplace). Another example is the theory of relativity. It utilized the non-Euclidean geometry of the 19th century.

For all of physics, and gradually for other sciences as well, mechanics became a model of mathematization in the 19th century. Most of the development in electricity, magnetism, optics, and heat conduction was mathematically modeled on paradigms from mechanics of continua. Therefore, in many parts of physics the mathematics was uniformly the same, not only in technique but in the manner in which mathematical and physical conceptions were correlated with each other. However, there was one part of physics which did not conform to this general pattern - the theory of thermodynamics. It was mathematically linked to a novel kind of mechanics - statistical mechanics.

It would seem that the relationship between mathematics and science from the Greek times to the 19th century took a full circle. Generally speaking the Greeks regarded mathematics and science as two separate entities. Scientists in the Renaissance had a totally mechanistic outlook toward all knowledge. This resulted in a remarkable development of mathematics. In the 18th century, the scientific community's need for new mathematics continued to grow. The amount of mathematics that was created for and because of mechanics was enormous. Then, in the 19th century, the relationship between mathematics and science changed. Mathematics began to be independent of science. There developed a limited collaboration between physicists and mathematicians that remains unbroken today.

20th Century

It is intriguing that every so often it is possible to apply an almost forgotten mathematical development of yesterday to a scientific problem of today. The 20th century has some fine examples of this.

The power of mathematics has rarely been proven more effective than in relativity theory - a brilliant application of the geometry of curved surfaces to the treatment of space, time and motion. In his theory of general relativity, Einstein applied the 19th century ideas of Gauss and Reimann in suggesting the existence of a curved universe of four dimensions.

Another example of science drawing upon prefabricated mathematics is Boolean Algebra. Contemporary studies in network and information theory, mechanical and human, had to fall back on the work of George Boole (1815-1864). Boole developed symbolic logic to clarify difficult Aristotelian logic. Today, his sytem is widely used as a tool to augment sound reasoning and has practical uses in designing parts of telephone circuits and electronic computers.

In quantum physics, it happened that a scientific setting was fashioned out of a mathematics created 20 years previously. The original disparate quantum physics versions of Heisenberg and Schrodinger were merged into one by Schrodinger. The union of the two was mathematically brought about in the precincts of so-called Hilbert space. Since entering physics, this theory of operators has developed the concept of an operator from a tool in physics to a reality in nature, and it has raised the mathematization of physics to new levels. There is hardly a purely mathematical statement on operators in Hilbert space which some physicist would not interpret as an event, or as a property of an event in nature. In fact, it has become a general belief that mathematics and science have correspondence rules: if a purely unexpected mathematical formula arises, then a corresponding unknown occurrence in nature exists. Maxwell's prediction that light is an electromagnetic wave is a good example.

Meanwhile, pure mathematicians are climbing to new levels of abstraction. How their work will relate to future scientific knowledge no one really knows. It may be decades before science gets a chance to draw upon the mathematics of today.

Conclusion

We have made a cursory historical review of the role of mathematics in the rise of science. Some general observations are suggested.

(1) As seen from history, any area of inquiry capable of mathematization developed the earliest and fastest. This is why physics developed before chemistry, chemistry before biology, and biology before any of the social sciences. The characteristics of physical science are such that a vast range of phenomena can be handled by linear algebra or differential equations. On the other hand, the inexact sciences are less amenable to mathematical treatment and, therefore, have not developed so fast. Apparently, the mathematization of a science affects the role and nature of revolutions that may and do occur in it.

(2) Much of the newly created mathematics has, at the time of its creation, no overt bearing on applied science or even on theoretical science. There is today a warehouse of mathematical knowledge of which scientists have not yet taken advantage.

(3) It has become a part of the celebrated scientific methodology that, if a purely unexpected mathematical conclusion arises, then a corresponding unknown occurrence in nature should be detectable.

(4) Mathematical formulation of scientific statements bestows a peculiar kind of lucidity and precision upon them and establishes logical and cognitive relations among them. It also introduces challenging analogies and unifications. For instance, we have seen that most wave propagation phenomena, whether in acoustics, electricity or optics, are assumed to be governed by virtually the same set of differential equations.

Mathematics is not part of or subordinate to science. Mathematics is a unique realm of knowledge from which science borrows in order to develop a) a set of tools for inquiry into natural phenomena, and b) a language for the articulation of subsequent explanations.

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