Activities on the Absolute Value Table

Bonnie H. Litwiller and
David R. Duncan
Professors of Mathematics
University of Northern Iowa
Cedar Falls, Iowa

Teachers are constantly searching for activities which provide for the maintenance of computational skills. It is a serendipitous occurrence when activities can be found which lend themselves both to the maintenance of skills and the discovery of patterns.

Consider a version of the subtraction table in which the absolute values of the differences are reported. Figure I displays this table.

Figure I
The Absolute Value Table

a \ b	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	0	1	2	3	4	5	6	7	8
2	2	1	0	1	2	3	4	5	6	7
3	3	2	1	0	1	2	3	4	5	6
4	4	3	2	1	0	1	2	3	4	5
5	5	4	3	2	1	0	1	2	3	4
6	6	5	4	3	2	1	0	1	2	3
7	7	6	5	4	3	2	1	0	1	2
8	8	7	6	5	4	3	2	1	0	1
9	9	8	7	6	5	4	3	2	1	0

For example, $|6 - 2| = |4| = 4$; consequently, the entry in the "6-row" and "2-column" is 4. Similarly, $|2 - 6| = |-4| = 4$; the entry in the "2-row" and "6-column" is also 4.

The following are activities which will provide both computational practice and pattern discovery:

I. The table (Figure I) is symmetric about each of its two diagonals. How many symmetries can be found in the addition and multiplication tables?

II. Find the sums of the entries of each row (horizontal) of the table. For instance, the "0-row" sums to 45 since 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45. Table I displays the sums of the entries of the consecutive rows.

TABLE I

Row Name	Sum	Difference (Absolute Value)
0-row	45	
		8
1-row	37	
		6
2-row	31	
		4
3-row	27	
		2
4-row	25	
		0
5-row	25	
		2
6-row	27	
		4
7-row	31	
		6
8-row	37	
		8
9-row	45	

How can the difference pattern be described? Follow the same steps for the columns. Since every row is the same as a corresponding column, the results are the same.

III. Figure II displays the absolute value table with diagonals drawn. Find the sums of the entries on the diagonals. Observe that on any diagonal, the entries are all equal. Table II summarizes the results.

Figure II

|a - b|	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	0	1	2	3	4	5	6	7	8
2	2	1	0	1	2	3	4	5	6	7
3	3	2	1	0	1	2	3	4	5	6
4	4	3	2	1	0	1	2	3	4	5
5	5	4	3	2	1	0	1	2	3	4
6	6	5	4	3	2	1	0	1	2	3
7	7	6	5	4	3	2	1	0	1	2
8	8	7	6	5	4	3	2	1	0	1
9	9	8	7	6	5	4	3	2	1	0

(a)

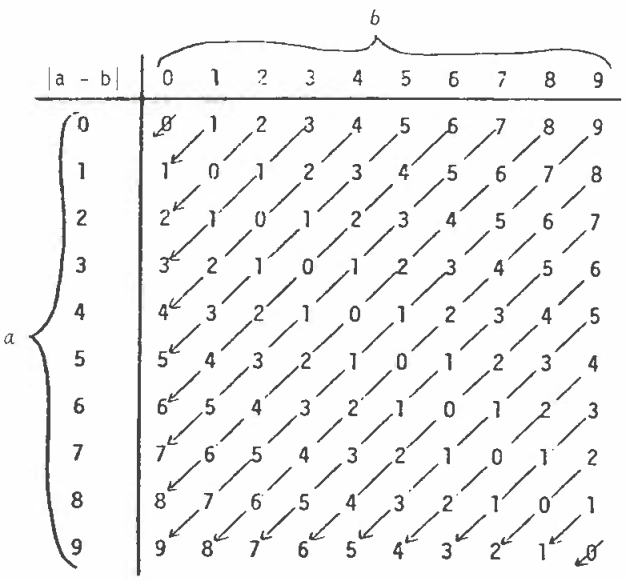
TABLE II

Diagonal Sums	Differences
9	
	7
16	
	5
21	
	3
24	
	1
25	
	1
24	
	3
21	
	5
16	
	7
9	
	9
0	
	9
9	
	7
16	
	5
21	
	3
24	
	1
25	
	1
24	
	3
21	
	5
16	
	7
9	

Describe the patterns found in these sums and differences.

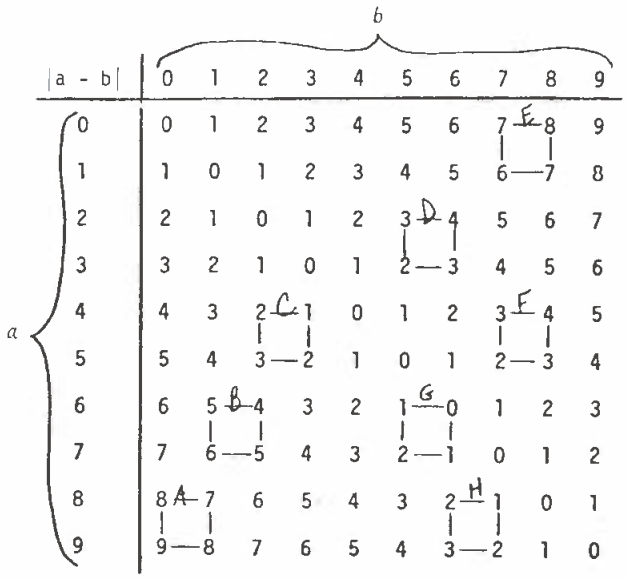
IV. Follow the directions of Activity III using the diagonals as indicated in Figure III. Compare the results with those found in Activity III.

Figure III



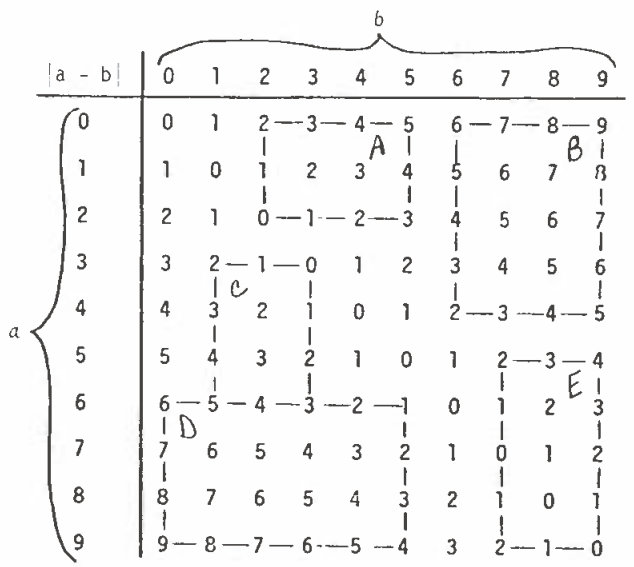
V. Consider squares drawn as shown on Figure IV. (These squares are called 2 by 2 squares since they contain two numbers per side.) Find the products of the opposite pairs of vertices. Did you notice that the products of the opposite pairs of vertices in each square differ by 1?

Figure IV



VI. Figure V is an absolute value table containing rectangles A, B, C, D, and E.

Figure V



For each of the rectangles A,B,C, and D, compute:

1. V, the sum of the 4 vertices.
2. I, the sum of the interior entries.
3. P, the sum of the entries on the perimeter.
4. V/I
5. P/I

In rectangle A: $V = 2 + 5 + 3 + 0 = 10$

$I = 2 + 3 = 5$

$P = 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1 + 0 + 1 = 25$

$V/I = 10/5 = 2/1 = 4/2$

$P/I = 25/5 = 5/1 = 10/2$

In this example there are 4 vertices, 2 interior entries and 10 entries which lie on the perimeter. The ratio, V/I, is the ratio of the number of vertices to the number of interior entries while P/I is the ratio of the number of entries on the perimeter to the number of interior entries. Table III summarizes the results of these computations.

TABLE III

Rectangle	V	I	P	V/I	P/I
A	10	5	25	10/5 = 4/2	25/5 = 10/2
B	22	33	77	22/33 = 4/6	77/33 = 14/6
C	10	5	25	10/5 = 4/2	25/5 = 10/2
D	20	40	80	20/40 = 4/8	80/40 = 16/8

In each of these cases, V/I is the ratio of the number of vertices to the number of interior entries and P/I is the ratio of the number of entries on the perimeter to the number of interior entries. These ratios do *not* hold for rectangle E. Why not?

Draw other rectangles on the absolute value table and compute the ratios. For which of the rectangles will the ratio properties hold? Where are they located on the table?

These are just a few activities and patterns using the absolute value table. Ask your students to find others.