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delta-k

THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

VOLUME XVIII, NUMBER 1, SEPTEMBER 1978

Mathematics Council, ATA 18th Annual Conference

Friday evening, October 13
and
Saturday, October 14, 1978

Capri Motor Hotel, Red Deer

THE "WHOLE" STUDENT
NEEDS "WHOLE" MATH

GOING BEYOND THE
SKILLS AND CONCEPTS

delta-k

THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

VOLUME XVIII, NUMBER 1, **SEPTEMBER 1978**

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EDPRESS

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The Editor's Page

Welcome back to school! Are you in need of some inspiration and/or fresh ideas? Hear what Dr. Jesse Rudnick, NCTM director and professor at Temple University, Philadelphia, has to offer in this realm. He will speak on "Beyond the Skills and Concepts" in a keynote address at our annual meeting, October 13/14, at the Capri Motor Hotel in Red Deer. Then, on Saturday, he will join Bob Robinson, NCTM Council of Affiliated Groups representative for Canada, and several outstanding Albertans in 12 sessions and 12 workshops.

Our luncheon speaker, Richard Guy, professor at the University of Calgary, is taking on "Numbers and Games." Can you include games to help develop skills, concepts and application? Let professor Guy assist you in adding games to your repertoire of techniques for improving learning, no matter what level you are teaching from K to 12.

At the business session, be prepared to make known your suggestions for ways we can improve existing services, and your ideas for new activities and services that we may be overlooking at present.

On Saturday morning we are having a buffet breakfast. In order to attend it will be necessary to get your ticket Friday night at the registration desk. The cost will be $4.95.

Remember we are planning a joint "name-of-site" meeting with the National Council of Teachers of Mathematics for 1979. Details will be announced as plans progress through the coming year.

Ed Carriger
Editor

Junior High Mathematics Contests

Edmonton

The first annual Edmonton City Junior High Mathematics Contest was held May 9, 1978. Five hundred and forty-four Grade IX students from Edmonton Public and Edmonton Separate schools wrote the contest exams. The purpose of the contest was to challenge these mathematically talented students and to recognize their achievements. An awards banquet was held May 31, 1978 at Barnett House, ATA headquarters. Awards were given to the top five teams and the top 58 students. (A team consisted of the top three students from a school.) The winners were:

Team Awards

DICKINSFIELD -
Katherine Chan
Barry Laiss
Doug van Uffelen

BALDWIN -
Todd Duquette
Jim Kenyon
Loretta Lee

AVALON -
Quinton Hackman
Dean Roehl
Ted Yoo

McKERNAN -
Don Driver
Michael Markowski
Donna Yurko

ST. THOMAS MORE -
Andrea Goerres
Michele Gunderson
David Wyrstiuk

Individual Awards

Kathy Ayer
Arthur Baragar
Katherine Chan
Todd Duquette
David Durand
Ron Ewoniak
Susan Fenske
Freddy Findling
Duncan Foster
Barton Frauenfeld
Jim Gleeson
Blaine Gregg
Michele Gunderson
Quinton Hackman
Michael Hrynchyshyn
Tommy Huh
Dennis Idler
Donald Jessop
Jim Kenyon
Susan Kim
Don Koziak
Bill Krys
Janis Krywiak
Barry Laiss
Darryl Lamoureux
Michael Lee
William Lee
Jeremy Leung
David Leibovitz
Tina Lui
Trevor MacLean
Michael Markowski
John Mellon
Heidi Miehe
Donald Murray
Bruce Peterson
Martin Ray
Dean Roehl
Walter Romaniuk
David Salopek
Brade Saville
Monica Sawchyn
Ursula Schmidt
Irene Sharagovich
Keith Shillington
Danny Stephen
Heather Strachan
Martin Tanner
Anthony Vader
Doug van Uffelen
Denis Vincent
Stella Walsh
Dave Warner
Alphonse Weber
Elizabeth Willekes
David Wrystiuk
Ted Yoo
Donna Yurko

Calgary

In the Calgary Mathematics Association Junior High Mathematics Examination contest, the winners were:

First: William Graham (Queen Elizabeth)
Second: Sheila Stewart (Branton)
Third: Trevor Prior (Simon Fraser)
John O'Leary (St. James)
Irene Kim (Simon Fraser)
Fourth: Kyle Maschmeyer (Ernest Morrow)
Fifth: Debbie Douglas (St. Margaret)

Other individual and team contest winners (not in order) were as follows:

ST. MARGARET -
Linda Chow
Eric Geppert
Darlene Iozzi
Charles Roks
Debbie Douglas
Judy Fairburn
Kismet Fung
Alison Li

JOHN WARE -
Cheryl Appelhof
Steve Dietz
Robert Geddes

ROBERT WARREN -
Stephen Bell

JOHN G. DIEFENBAKER
Michael Chan
William Chee

ERNEST MORROW -
Robert Hornung
Terry Martin
Kyle Maschmeyer

SIMON FRASER -
Meijer Drees
Mike Flegel
Diana Kim
Irene Kim
Trevor Prior

BISHOP PINKHAM -
Bruce Wright

SUNALTA -
Michael Lam

ST. HELENA -
Denise Fleigham
John Turk

ST. STEPHEN -
Kathy Brocklebank
Audrey Unfug

F.E. OSBORNE -
John Braun

ST. JAMES -
Tom Kloepfer
Mark McKenna
John O'Leary
Antony Tobias

COLONEL IRVINE -
Alan Abraham
Rob Winstanley

SENATOR PATRICK BURNS -
Charlene Kolla
Florence Mah
Kevan Notter

HAROLD PANABAKER -
Scott Craig

BRANTON -
Sheila Stewart
Robert Straker

ST. MARTHA -
Antony Hoong

ST. MARY'S -
Dale Tardiff

CALGARY HEBREW SCHOOL -
Melaine Cohen

NICKLE -
Brad Dick
Stan Ebel

MONTGOMERY -
Mariane Hertzsprung

ST. JOSEPH -
Marrimo Seremia

QUEEN ELIZABETH -
William Graham
Derek Wooher

Activities on the Absolute Value Table

Bonnie H. Litwiller and
David R. Duncan
Professors of Mathematics
University of Northern Iowa
Cedar Falls, Iowa

Teachers are constantly searching for activities which provide for the maintenance of computational skills. It is a serendipitous occurrence when activities can be found which lend themselves both to the maintenance of skills and the discovery of patterns.

Consider a version of the subtraction table in which the absolute values of the differences are reported. Figure I displays this table.

Figure I
The Absolute Value Table

$\|a - b\|$ (a ↓, b →)	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	0	1	2	3	4	5	6	7	8
2	2	1	0	1	2	3	4	5	6	7
3	3	2	1	0	1	2	3	4	5	6
4	4	3	2	1	0	1	2	3	4	5
5	5	4	3	2	1	0	1	2	3	4
6	6	5	4	3	2	1	0	1	2	3
7	7	6	5	4	3	2	1	0	1	2
8	8	7	6	5	4	3	2	1	0	1
9	9	8	7	6	5	4	3	2	1	0

For example, $|6 - 2| = |4| = 4$; consequently, the entry in the "6-row" and "2-column" is 4. Similarly, $|2 - 6| = |-4| = 4$; the entry in the "2-row" and "6-column" is also 4.

The following are activities which will provide both computational practice and pattern discovery:

I. The table (Figure I) is symmetric about each of its two diagonals. How many symmetries can be found in the addition and multiplication tables?

II. Find the sums of the entries of each row (horizontal) of the table. For instance, the "0-row" sums to 45 since 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45. Table I displays the sums of the entries of the consecutive rows.

TABLE I

Row Name	Sum	Difference (Absolute Value)
0-row	45	
		8
1-row	37	
		6
2-row	31	
		4
3-row	27	
		2
4-row	25	
		0
5-row	25	
		2
6-row	27	
		4
7-row	31	
		6
8-row	37	
		8
9-row	45	

How can the difference pattern be described? Follow the same steps for the columns. Since every row is the same as a corresponding column, the results are the same.

III. Figure II displays the absolute value table with diagonals drawn. Find the sums of the entries on the diagonals. Observe that on any diagonal, the entries are all equal. Table II summarizes the results.

Figure II

|a - b| (a ↓, b →)	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	0	1	2	3	4	5	6	7	8
2	2	1	0	1	2	3	4	5	6	7
3	3	2	1	0	1	2	3	4	5	6
4	4	3	2	1	0	1	2	3	4	5
5	5	4	3	2	1	0	1	2	3	4
6	6	5	4	3	2	1	0	1	2	3
7	7	6	5	4	3	2	1	0	1	2
8	8	7	6	5	4	3	2	1	0	1
9	9	8	7	6	5	4	3	2	1	0

TABLE II

Diagonal Sums	Differences
9	
	7
16	
	5
21	
	3
24	
	1
25	
	1
24	
	3
21	
	5
16	
	7
9	
	9
0	
	9
9	
	7
16	
	5
21	
	3
24	
	1
25	
	1
24	
	3
21	
	5
16	
	7
9	

Describe the patterns found in these sums and differences.

IV. Follow the directions of Activity III using the diagonals as indicated in Figure III. Compare the results with those found in Activity III.

Figure III

$\lvert a - b \rvert$ (a \ b)	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	0	1	2	3	4	5	6	7	8
2	2	1	0	1	2	3	4	5	6	7
3	3	2	1	0	1	2	3	4	5	6
4	4	3	2	1	0	1	2	3	4	5
5	5	4	3	2	1	0	1	2	3	4
6	6	5	4	3	2	1	0	1	2	3
7	7	6	5	4	3	2	1	0	1	2
8	8	7	6	5	4	3	2	1	0	1
9	9	8	7	6	5	4	3	2	1	0

V. Consider squares drawn as shown on Figure IV. (These squares are called 2 by 2 squares since they contain two numbers per side.) Find the products of the opposite pairs of vertices. Did you notice that the products of the opposite pairs of vertices in each square differ by 1?

Figure IV

$\lvert a - b \rvert$ (a \ b)	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	0	1	2	3	4	5	6	7	8
2	2	1	0	1	2	3	4	5	6	7
3	3	2	1	0	1	2	3	4	5	6
4	4	3	2	1	0	1	2	3	4	5
5	5	4	3	2	1	0	1	2	3	4
6	6	5	4	3	2	1	0	1	2	3
7	7	6	5	4	3	2	1	0	1	2
8	8	7	6	5	4	3	2	1	0	1
9	9	8	7	6	5	4	3	2	1	0

VI. Figure V is an absolute value table containing rectangles A, B, C, D, and E.

Figure V

$\lvert a - b \rvert$ (a \ b)	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	0	1	2	3	4	5	6	7	8
2	2	1	0	1	2	3	4	5	6	7
3	3	2	1	0	1	2	3	4	5	6
4	4	3	2	1	0	1	2	3	4	5
5	5	4	3	2	1	0	1	2	3	4
6	6	5	4	3	2	1	0	1	2	3
7	7	6	5	4	3	2	1	0	1	2
8	8	7	6	5	4	3	2	1	0	1
9	9	8	7	6	5	4	3	2	1	0

For each of the rectangles A,B,C, and D, compute:

1. V, the sum of the 4 vertices.
2. I, the sum of the interior entries.
3. P, the sum of the entries on the perimeter.
4. V/I
5. P/I

In rectangle A:

$$V = 2 + 5 + 3 + 0 = 10$$
$$I = 2 + 3 = 5$$
$$P = 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1 + 0 + 1 = 25$$
$$V/I = 10/5 = 2/1 = 4/2$$
$$P/I = 25/5 = 5/1 = 10/2$$

In this example there are 4 vertices, 2 interior entries and 10 entries which lie on the perimeter. The ratio, V/I, is the ratio of the number of vertices to the number of interior entries while P/I is the ratio of the number of entries on the perimeter to the number of interior entries. Table III summarizes the results of these computations.

TABLE III

Rectangle	V	I	P	V/I	P/I
A	10	5	25	10/5 = 4/2	25/5 = 10/2
B	22	33	77	22/33 = 4/6	77/33 = 14/6
C	10	5	25	10/5 = 4/2	25/5 = 10/2
D	20	40	80	20/40 = 4/8	80/40 = 16/8

In each of these cases, V/I is the ratio of the number of vertices to the number of interior entries and P/I is the ratio of the number of entries on the perimeter to the number of interior entries. These ratios do *not* hold for rectangle E. Why not?

Draw other rectangles on the absolute value table and compute the ratios. For which of the rectangles will the ratio properties hold? Where are they located on the table?

These are just a few activities and patterns using the absolute value table. Ask your students to find others.

Report on NCTM Conference in San Diego, April 12-16, 1978

A.O. Jorgensen

Participation in the Delegate Assembly proved to be an interesting experience. I found, as did several of the delegates, that many of the resolutions were poorly worded, narrow in scope or very local, and, as a result, failed to pass.

An interesting resolution that did pass was one asking NCTM to encourage more students at the college level to choose a concentration in mathematics courses because of a shortage of mathematics teachers in various parts of the U.S.A. Another resolution approved was one asking the NCTM to inform the public of the value of converting to the metric system. Apparently, there is considerable hostility toward the conversion. The Assembly was opposed to any resolution asking for standardized competency exams. There was a request for more articles in NCTM publications and at conferences related to teaching the mathematically gifted student.

Something which surprised me was that the Delegate Assembly of the NCTM functioned primarily in an advisory role rather than in a policy-making capacity. This is in contrast to other Delegate Assemblies in which I have participated.

Concerns which were expressed at the Council of Affiliated Groups (CAG) sessions were:

1. how better to provide services to the large affiliates,
2. the role of the NCTM representative,
3. regional boundaries,
4. membership,
5. leadership training.

At the Canadian Caucus a discussion took place with regard to the relative merits of B.C. and possibly Alberta joining a Northwest area involving Wyoming, Idaho, Washington, Oregon, and Alaska. The B.C. members felt that they would benefit from such an organization. They wished to also remain part of the Canadian group.

I attended a number of sessions which, in quality, ranged from mediocre to excellent. As a result of listening to the sessions and discussing the general state of mathematics education with various people, my conviction that we still have a long way to go was further confirmed. The need for improved programs at the teacher training level seemed to be repeatedly voiced.

As usual, the displays were extensive and varied in quality.

I would like to take this opportunity to express my sincere appreciation to the Mathematics Council of The Alberta Teachers' Association for honoring me with this trip.

* * *

Mini Calculator - another name for a midget mathematician.

* * *

Philosophy of Education and The Mathematics Curriculum

Marlow Edigar
Northeast Missouri State University
Kirksville, Missouri

Reprinted from Mathematics in Michigan, *Vol. XVII, No. 4, March 1978.*

Teachers and supervisors need to study, appraise, and ultimately implement what is deemed worthwhile of ideas from diverse educational philosophies. These strands of thought provide for individual differences, but in diverse ways. How do each of these philosophies provide for pupils of diverse achievement levels as well as of different styles of learning?

Experimentalism in the Mathematics Curriculum

Experimentalists emphasize the importance of learners perceiving a need for learning selected content. In addition, the following generalizations also apply to experimentalism:

1. pupils identifying and solving real problems;

2. interest in learning provides for effort on the part of pupils in ongoing learning activities;

3. experiences are the heart or core of the mathematics curriculum;

4. pupils are actively involved in learning and not passive individuals;

5. the whole pupil is involved in learning. Thus, the social, intellectual, and emotional facets of a learner's development are important!

Experimentalism would not emphasize -

1. pupils, for example, working page 55 in sequence from a basal mathematics textbook because the previous pages have been completed in logical order.

2. the teacher largely selecting objectives, learning experiences, and appraisal techniques in the mathematics curriculum.

3. learners engaging in rote learning and drill to achieve new objectives in the area of mathematics.

4. the teacher emphasizing explanations and lectures as methods of teaching to passive pupils in the class setting.

5. pupils attaining measurable objectives sequentially in the mathematics curriculum.

A felt need for pupils to solve realistic, lifelike problems is important to experimentalists!

The Basics in the Curriculum

Educators in the school setting emphasizing the basics in the curriculum place high values on reading, writing, and arithmetic (the three r's). Teachers and supervisors stressing the basics would generally -

1. advocate exact standards of achievement for pupils in each lesson pertaining to mathematics,

2. stress a no nonsense learning environment (a quiet learning environment, no doubt, would then be in evidence),

3. de-emphasize social promotion of pupils (pupils would need to master definite content in mathematics before moving on to the next grade level),

4. follow a basal textbook series sequentially in terms of learning activities for pupils,

5. emphasize passive learners in gaining ideas from explanations, lectures, and textbooks.

Reasons for advocating the basics in the curriculum would include -

1. the lay public feeling that pupils are not achieving well in arithmetic as well as other curriculum areas,

2. a lack of firmness being in evidence in disciplining pupils in the class setting,

3. a lack of achievement in pupils' test scores on standardized achievement tests.

Existentialism and the Curriculum

Extentialists emphasize the importance of individuals choosing what to learn and the means of learning. Thus, learning centers in the school-class setting would harmonize well with the thinking of existentialists. There would need to be more tasks or learning activities than pupils can possibly complete so that decision-making is truly involved in terms of experiences selected. The following centers, as an example, could be inherent in the school-class setting:

1. Line, bar, circle, and picture graphs.

2. The metric system of measurement.

3. Using a pocket calculator.

4. Working from a textbook.

5. Working with fractions.

6. Utilizing decimals.

7. The mathematics laboratory.

8. Geometry for everyday use.

9. Addition, subtraction, multiplication, and division.

The teachers may write tasks (learning activities) on cards for each of these centers. Pupils may then choose sequentially which tasks to complete. Creative endeavors are highly recommended by existentialists as learning experiences for pupils.

The following are also emphasized thoroughly by existentialists:

1. Pupils have ample opportunities to engage in sessions devoted to values clarification.

2. Clearcut answers to questions and problems are not of major importance. Relevant questions and problems require creative solutions.

Behaviorism and the Curriculum

Behaviorists have made strong inroads in the curriculum during the past decade in particular. Behaviorism in the mathematics curriculum would stress the following:

1. It is definitely possible to determine what pupils are to learn (objectives) as well as measure the amount of learning after instruction.

2. Learning activities for pupils are to guide in achieving these measurable objectives. If objectives are attained by learners without harmful side effects, the learning activities are then considered suitable.

3. The objectives, learning activities and their sequence, as well as measurement procedures are basically teacher determined.

4. It is good teaching procedure to pretest pupils before initiating a new unit in the mathematics curriculum.

Thus, each pupil may be placed in instruction within the new unit in terms of his or her present achievement level.

The following objectives may well represent teacher determined ends which are measurable:

1. The pupil will multiply correctly in nine out of ten problems.

2. The learner will solve 95 percent of the division problems on page fifty involving a five place dividend and a two place divisor.

Related to the use of behaviorally stated objectives in the mathematics curriculum, numerous states in the United States have implemented accountability plans. Accountability laws, among other generalizations, stress the following:

1. specific objectives for pupils to achieve,

2. teachers showing evidence to interested persons what children have learned,

3. responsibility for what pupils have learned being rather completely in the teacher's domain.

In Closing

Teachers and supervisors need to study and analyze diverse philosophies of education pertaining to the teaching of mathematics. Ultimately, a recommended philosophy needs to be implemented. It may well be an eclectic philosophy in which selected strands are chosen from diverse schools of thought. Whichever philosophy or philosophies are chosen, the following principles of learning need emphasizing:

1. Objectives should be adjusted to the present achievement levels of each learner.

2. Learning activities to achieve desired ends should be meaningful.

3. Pupil interests must be obtained in ongoing units of study.

4. Learnings obtained need to be sequential from the point of view of each pupil's own unique perception.

5. Pupils need to experience ample success in each unit of study in the mathematics curriculum.

6. Teacher-pupil planning should receive adequate emphasis.

7. Positive attitudes should be acquired by each pupil.

8. Balance in objectives (understandings, skills, and attitudes) need to be stressed in the area of mathematics.

9. Problem-solving needs to be stressed adequately in each unit of study.

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Theoretical Properties of the Fourth Dimension

Bradley Gorzitza
Grade XI, 1977-78
Winston Churchill High School
Lethbridge

Although the existence of universes with either fewer (1 or 2) or more (4+) dimensions than our own 3-d universe can't at present be proven, (and, indeed, possibly never will be) neither can their existence be disproven. Therefore, I will hypothesize that they exist, and, furthermore, assume that the latter (4+) are governed by physical laws similar to those of our own universe. If these two hypotheses are true, then the following may also be taken as true.

I will start off with three borrowed concepts. First of all, we know that the first dimension goes in one direction, say the horizontal line passing through point X in diagram A (see page 16). The second dimension would be represented by the vertical line passing through point X at right angles to the first. The third dimension, while represented by the dotted line, would actually be a line sticking straight up and down out of the paper, also passing through point X, and at right angles to both the first and second dimensions. A fourth dimension would also pass through point X and be perpendicular to all of the first three.

Secondly, there is the theory that an object transferred from its native universe into one that is one dimension higher, given a half rotation, and then returned to its original world, would be an exact reversal, a mirror image, of what it once was. In diagram B, a line segment in a 1-d world, which is dotted on top and solid on the bottom, can only move up and down. There is no way for the dots to get on the other side of the solid line. But take it into a 2-d world, give it a half turn, and stick it back into its 1-d world, and it is completely reversed, down to its basic subatomic particles.

The same with 2-d object in diagram C. Left in its planer universe (C-1) it can only rotate in a clockwise or counter-clockwise direction, (as well as being able to move forward, backward, left and right, of course). But take it out into a 3-d universe (C-2), turn it half way around and then put it back, and it too becomes its own reflection (C-3). Again the reversal would be complete, down to the smallest detail. Similarly, a 3-d object, taken into a 4-d world and given a four dimensional half twist and then returned to its original world, would be completely reversed.

Thirdly, there is the differing of distances among the worlds. For instance, the 1-d world represented by the curved line AD in diagram D is straight, from the point of view of any 1-d creatures living in it. In this 1-d world, point B is closest to A, point C is next, and point D is farthest. Furthermore, point D appears twice as far from point A as B is. But from a 2-d point of view, C is closest to A with points B and D equi-distant from A. The same type of relationship exists when a 2-d world is folded over three dimensionally (diagram E). B is closer than C to A, two dimensionally, but the reverse is true three dimensionally.

Now for my own work. The simplest 1-d structure possible is a line. The simplest 2-d structure possible is a triangle. The simplest 3-d structure possible is a pyramid with a triangular base, diagram F. The line needs two points (at least), the triangle needs three (joined by lines), and the pyramid needs four. The simplest 4-d structure possible then, must consist of 5 points joined by lines. The 2-d triangle is made up of three 1-d lines. The 3-d pyramid is made up of four 2-d triangles. The 4-d structure, therefore, should be composed of five 3-d triangular based pyramids.

A line has two sides (its no dimensional ends, diagram G), a square four sides, and a cube six. A 4-d "cube," therefore, should have eight sides.

1-d objects have length. 2-d objects have length (and width), and area. 3-d objects have length (width and height), area (surface) and volume. 4-d objects should then have length, area, volume, and something more again.

Another concept is "unlimited vision." This means that everything in a world of X-dimensions is visible to beings in a universe one or more dimension(s) greater. For instance, the middle point in diagram H is invisible to the two line beings X and Y because the two endpoints block their vision. But the mid point is perfectly visible to the two square beings A and B in their 2-d world, as are all parts of X and Y. In turn, creatures A and B can't see the dot inside circle L because from their 2-d viewpoint it is completely surrounded. But we can see it, as well as all parts of A and B. In turn, something totally enclosed by a box in the 3-d world would be perfectly visible to a 4-d world. Note that 2-d creatures can look inside only a 1-d world, while we 3-d creatures can see inside both 1- and 2-d worlds. A 4-d creature should, therefore, be able to see into 1-, 2- and our 3-d world.

Then we come to the intersection of lines and planes. In a 1-d world, the intersection of either lines or planes is impossible. In a 2-d world (diagram I), two lines extending out to infinity in both directions, can be either parallel or intersecting. Two planes cannot. In a 3-d world, two planes, extending out to infinity along two dimensions, can be either intersecting or parallel (diagram J) as can two lines; two infinite cubes (cubes extending out to infinity in all three dimensions) cannot.

In a 4-d world therefore, two infinite cubes should be able to be either parallel or intersecting. When two lines intersect, their intersection is no dimensional (a point). When two planes intersect, their intersection is 1-dimensional (a line). Therefore, when two infinite cubes intersect in a 4-d world, their intersection should be 2-dimensional, a plane.

Also, in the 3-d world 1-d lines can intersect, be parallel or skew (one goes over the other) (diagram K). Therefore, in a 4-d world, two planes should be able to "skew."

This brings us to the next concept - the intersection of an X-dimensioned object by an X-minus-one dimensioned world. For example, if a 2-d circle passes through a 1-d world, (diagram L) it would first start out as a point (line 1) appearing out of nowhere from a 1-d point of view, expand into a line (line 2) which would get longer until its length matched that of the circle's diameter (line 3). Then it would get shorter again (line 4) until it was once more a point, which would then "vanish," again from a 1-d viewpoint. A 3-d object passing through a 2-d world would follow a similar pattern. It would also apparently come out of nowhere, starting as a point (diagram M), grow into a circle as the sphere progressed and get larger and larger until the circle matched the sphere's diameter (as the line matched the circle's diameter before). The circle then would get smaller until it was back to a point, which would then "vanish."

A similar pattern should be followed if a 4-d sphere passes through a 3-d world. It would start out as a point appearing out of "nowhere," grown into a 3-d sphere which would become larger and larger until it matched the 4-d sphere in diameter. It would then shrink until it was once again a point which would go as the 4-d sphere passed out of the 3-d world.

If an X-dimensioned object were to leave its respective X-dimensioned world and move into an X-1 dimensioned world, how would it go? All at once? Or would it leave a little at a time as though going through a doorway? To answer this question two things must be taken into account. Was the X-dimensioned object moving or stationary in its respective world, and was its world straight, or bent from an X+1 dimensional point of view? As an example let's take a 1-d object in a 1-d world which is bent at one point two dimensionally (diagram N) although it appears straight from a 1-d point of view. If it is moving in the direction indicated, it would be possible for it to leave its world a little at a time, but only if it was moving and only if its world is "bent." The same is true of the circle in its 2-d world (diagram O).

On the other hand, if one or both of these conditions is not met, then the object could only go all at once; there one instant, gone the next. For example, the line in diagram P has no width whatsoever so it can't be depicted in the circle, half in and half out of its world. It's there or it isn't. The same with the circle in diagram Q. So a 3-d object leaving this world should also follow this pattern and vanish either all at once or into a crook in the universe.

However, one could say that the object would curve with its respective universe (diagram R) since it is straight from the object's point of view. However, while this is undoubtedly true for most cases (and probably true for all), there might be exceptions. For instance, if the object were travelling fast enough, it might "break out," much like a car rounding a corner too quickly. Of course, the car has mass and thus inertia, but, while it is impossible for 1- and 2-d objects to have mass, 3-d objects do. Perhaps that is why, supposedly, nothing can travel faster than light in our universe; going any faster might cause us to "run off the road."

Also an object might leave its respective universe if it were pushed from outside. For example, a 1-d object could be pushed out into the 2-d world by a 2-d object (diagram S) or the 2-d object could be pushed into the 3-d world by a 3-d object (diagram T). Thus a 3-d object might leave this universe if given a boost from the fourth dimension.

Conclusion

In conclusion, I will list the properties of the 4th dimension. It is at right angles to the first three. A 3-d object taken into a 4-d world and given a half rotation would be completely reversed. Distances in the 4-d world differ from distances in the 3-d world. The simplest 4-d structure consists of five points joined by lines and has in it five 3-d triangular pyramids. A 4-d cube should have eight sides. 4-d objects have length, area, volume, and something else. Everything in a 3-d world is visible from a 4-d point of view. Two planes may skew in a 4-d world. Two infinite cubes may intersect. If a 4-d sphere is intersected by a 3-d world, it would start as a point, grow into a 3-d sphere and back again. A 3-d object going into a 4-d world could leave either all at once or into a corner in the universe. Our universe might be curved from a 4-d point of view, and travelling faster than light might cause us to break out. A 3-d object might be pushed out of this universe from outside. There are, undoubtedly, more things that could be concluded if one took the time to think about it.

Diagram A

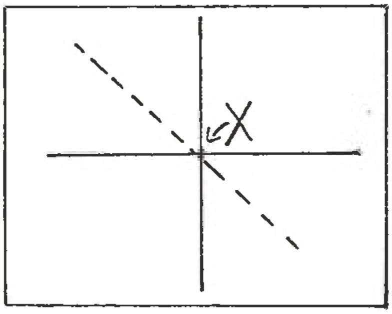


Diagram B

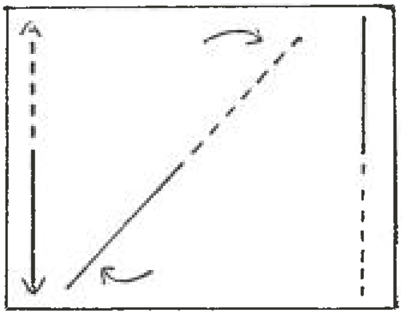


Diagram C

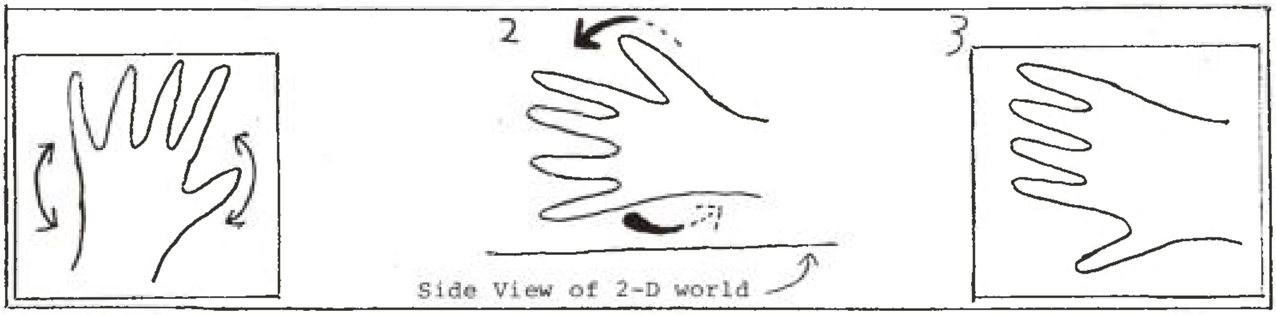


Diagram D

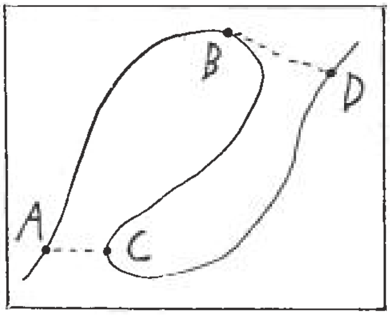


Diagram E

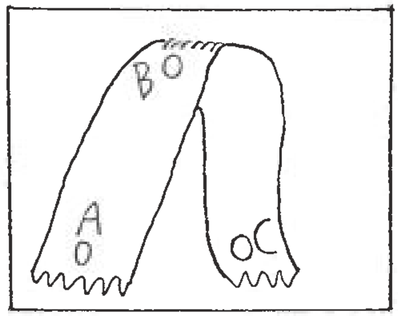


Diagram F

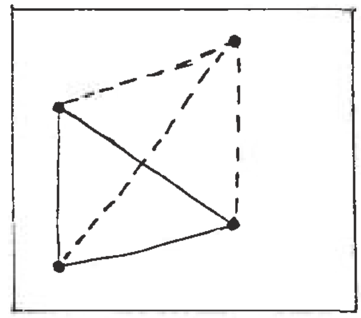


Diagram G

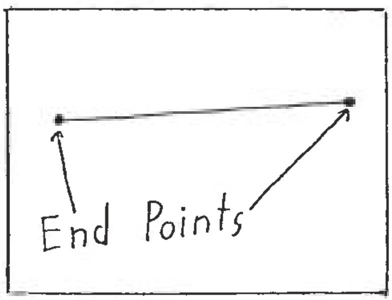


Diagram H

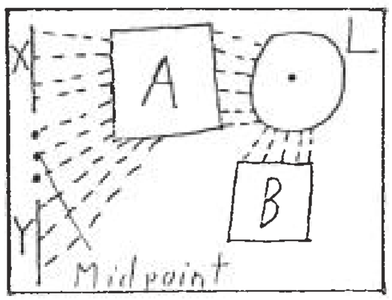


Diagram I

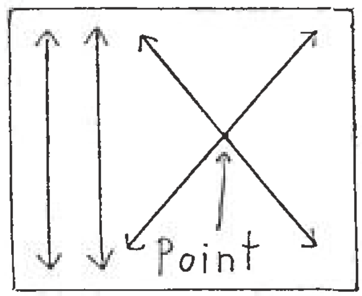


Diagram J

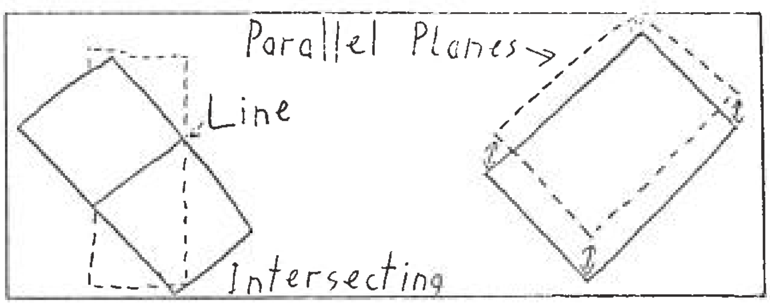


Diagram K

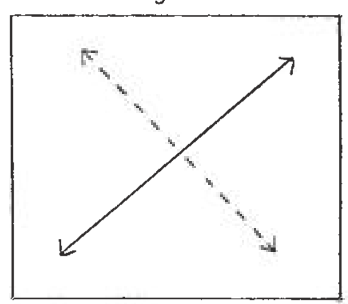


Diagram L

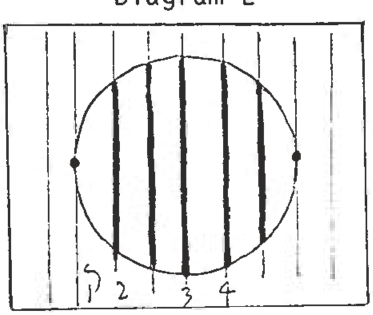


Diagram M

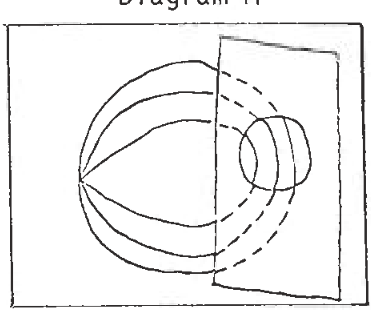


Diagram N

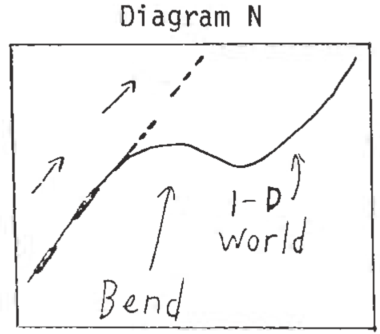


Diagram O

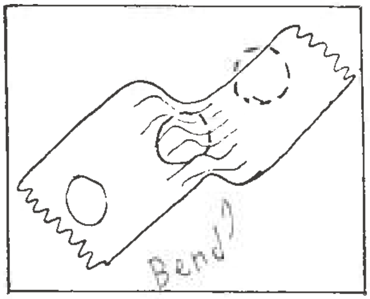


Diagram P

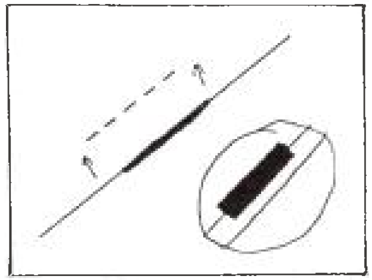


Diagram Q

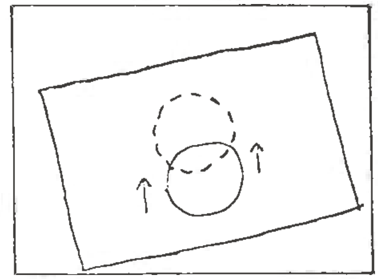


Diagram R

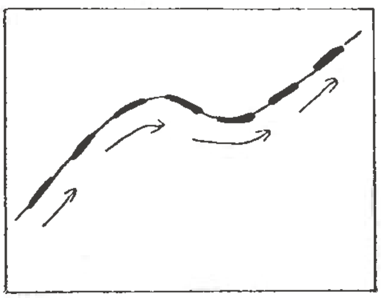


Diagram S

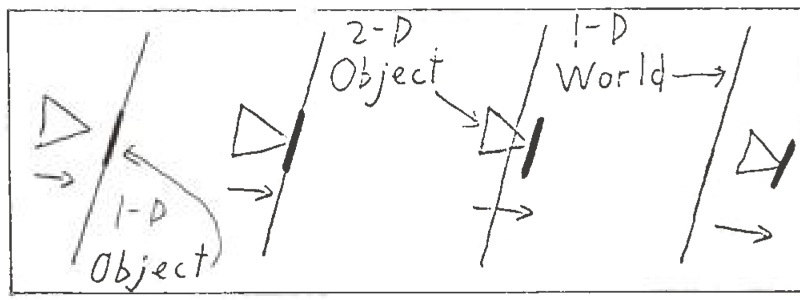
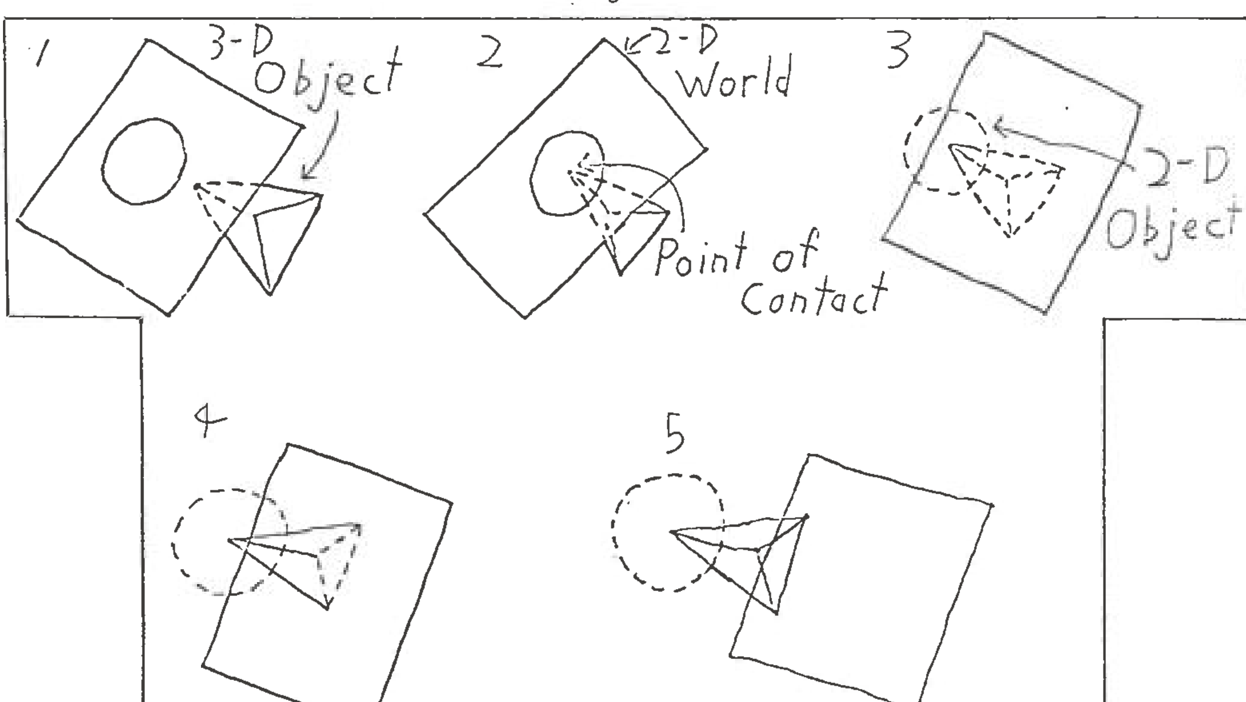


Diagram T



Graph of the Month

Cynthia Olaso
Grade 4
Mayberry Street School
Los Angeles, California

Reprinted from The Calculator, *Volume 18, Number 8, May 1978.*

1. (22, 8)
2. (8, 8)
3. (8, 7)
4. (7, 6)
5. (6, 6)
6. (5, 7)
7. (5, 8)
8. (4, 8)
9. (3, 9)
10. (3,11)
11. (4,12)
12. (6,12)
13. (8,15)
14. (21,15)
15. (24,12)
16. (27,12)
17. (28,11)
18. (28, 9)
19. (27, 8)
20. (25, 8)
21. (25, 7)
22. (24, 6)
23. (23, 6)
24. (22, 7)
25. (22, 8)

Lift pencil ---
and begin with:

26. (22, 8)
27. (23, 9)
28. (24, 9)
29. (25, 8)

Lift pencil ---
and begin with:

30. (5, 8)
31. (6, 9)
32. (7, 9)
33. (8, 8)

Lift pencil ---
and begin with:

34. (4,10)
35. (8,10)
36. (10,13)
37. (8,15)

Lift pencil ---
and begin with:

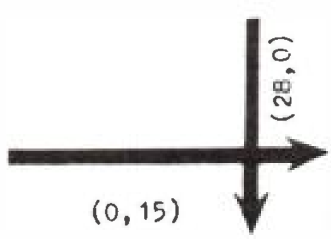
38. (11, 8)
39. (11,11)
40. (12,13)
41. (21,13)
42. (22,11)
43. (17,11)
44. (17, 8)

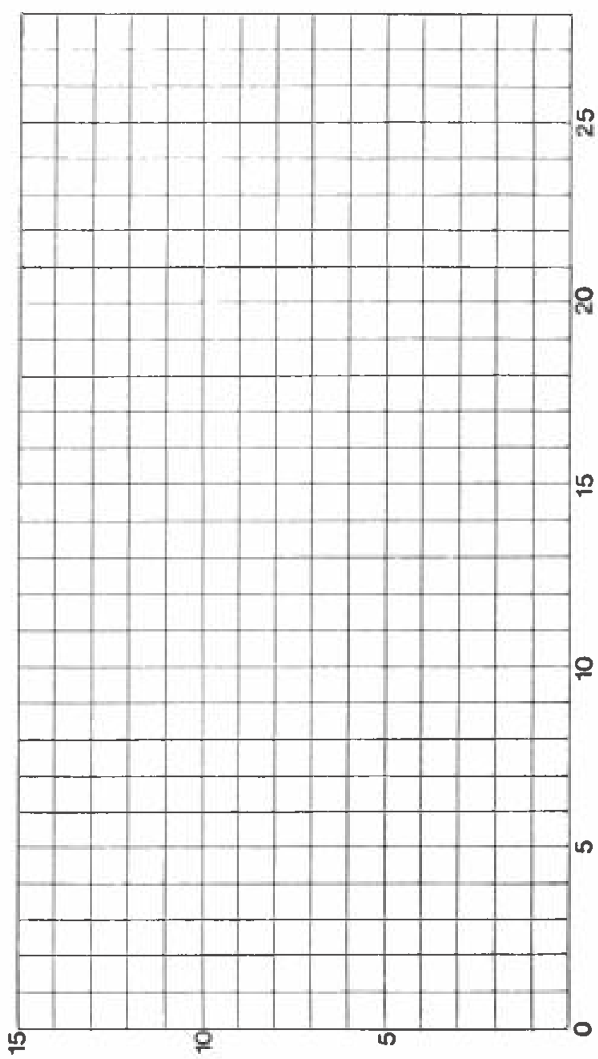
Lift pencil ---
and begin with:

45. (11,11)
46. (17,11)
47. (17,13)

Lift pencil ---
and connect:

48. (15,10) to (16,10)
    Stop
49. (4,12) to (4, 8)
    Stop
50. (6,12) to (8,10)
    Stop
51. (8,12) to (10,13)
    Stop
52. (27, 8) to (27,12)
    The End



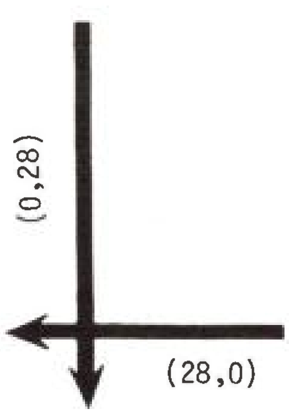


Design Graphitti

by *Linda Silvey*
Van Nuys Junior High School
Van Nuys, California

Reprinted from The Calculator, *Volume 18, Number 8, May 1978.*

Connect, in order, the following coordinate points. Use 1/4 inch graph paper, vertically. Use a ruler for a neat design. Color the design to form a pattern.



1. (10½,16)
2. (14 ,26)
3. (17½,16)
4. (24½, 8)
5. (14 ,10)
6. (3½, 8)
7. (10½,16)
8. (17½,16)
9. (14 ,10)
10. (10½,16)

Start again at:

11. (12 ,17½)
12. (20 ,24½)
13. (18 ,14)
14. (20 , 3½)
15. (12 ,10½)
16. (2 ,14)
17. (12 ,17½)
18. (18 ,14)
19. (12 ,10½)
20. (12 ,17½)

Start again at:

21. (14 ,18)
22. (24½,20)
23. (17½,12)
24. (14 , 2)
25. (10½,12)
26. (3½,20)
27. (14 ,18)
28. (17½,12)
29. (10½,12)
30. (14 ,18)

Start again at:

31. (16 ,17½)
32. (26 ,14)
33. (16 ,10½)
34. (8 , 3½)
35. (10 ,14)
36. (8 ,24½)
37. (16 ,17½)
38. (16 ,10½)
39. (10 ,14)
40. (16 ,17½)

The end

Facts In A Flash

by *Betty Iehl*

Reprinted from The Calculator, *Volume 18, Number 8, May 1978.*

MAKE YOUR OWN FLASH CARDS --- A BIT DIFFERENTLY!

Use 3" x 5" cards or word cards (3" x 9").

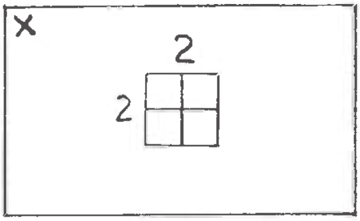
Front Side

1. Indicate the operation of the upper left-hand corner.
2. Cut out squares and glue down on card to represent the problem.
3. Label sides of squares.

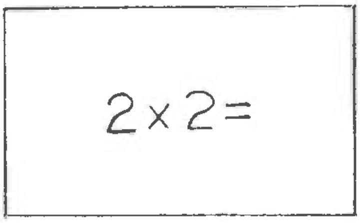
Back Side

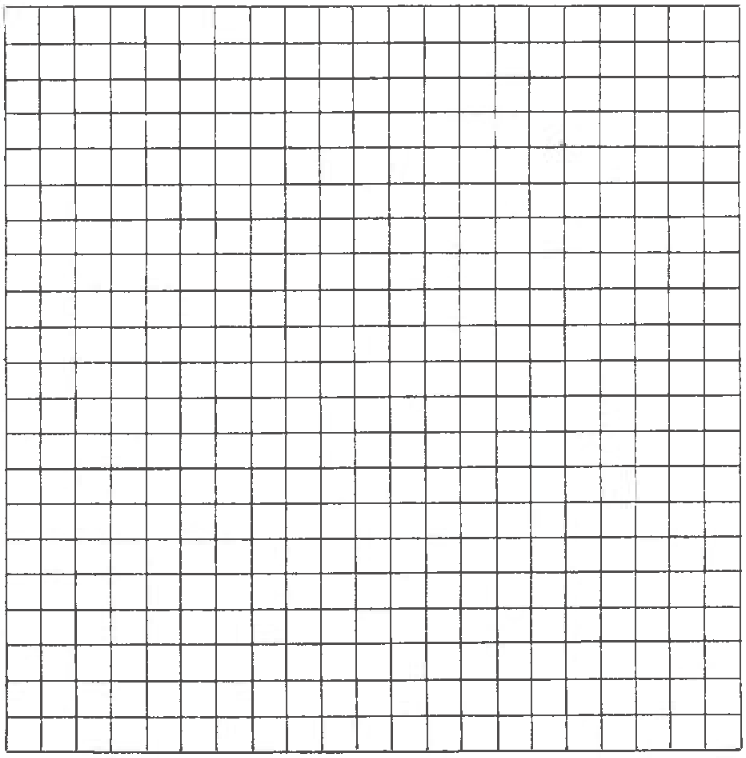
4. Write the problem.
5. Work with a friend!

Front



Back





Some Ideas

Ideas contributed
by *Evelyn Minor*

What is the percent savings when you compare the sale price to the regular price of each item?

Translation:
"walls might be nice to have."

Make a table listing the items, their prices, percent discounts, dollars and/or cents saved. Rank the items in an order of your choosing.

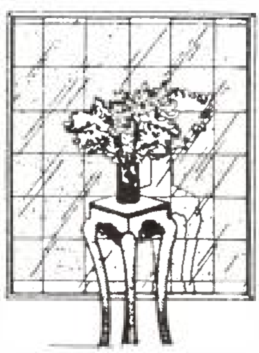
What is the average percent discount?

What is the average saved in dollars and cents?

Which item is the best bargain?

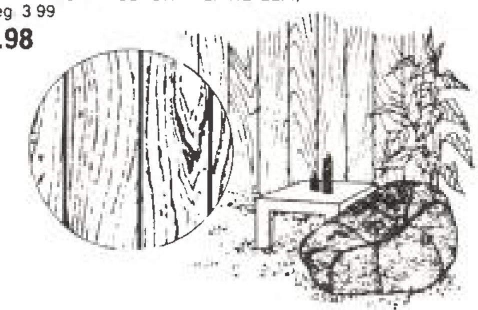
Golden ripples on silvered squares

Create an elegant wall with this distinguished antique gold vein mirror tile. Makes a small room look larger and airier. Easy to apply. 12" x 12". Golden Vein or Smoked Gold Vein
MIRROR TILES
Reg. 99c
68c



Paneling at its peak

From the highest Alps came the inspiration for Permaneer's Alpine Elm panel. Bring the feeling of a far-off mountainside into your home with this beautiful panel. Particle board with a tough vinyl overlay. 4' x 8' x 5/32".
PERMANEER PASSPORT ALPINE ELM.
Reg. 3.99
2.98

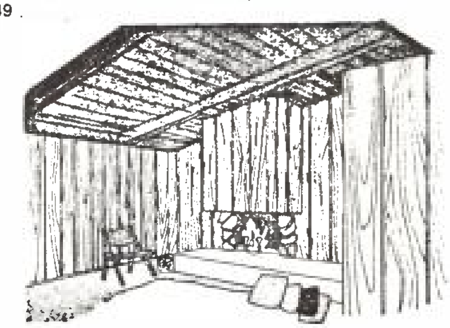


Cork is a show stopper

No one can ignore the dramatic effect of rich, dark cork walls. More than just looking good, they also help to absorb noise and insulate. In a pack of 3. 12" x 12" x 10mm panels.
CORK PANELS, 3-PACK.
Reg. 1.19
88c

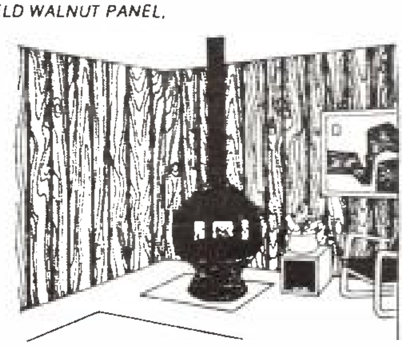
Good enough to eat

This panel is simply scrumptious—creamy white with carmel-tone wood grain. Gives an expansive feeling to a small room, an elegant feeling to any room. Printed luan panel from U.S. Plywood. 4'x8'x5/32".
CANDYLAND BUTTERSCOTCH PANEL.
Reg. 7.49
5.88



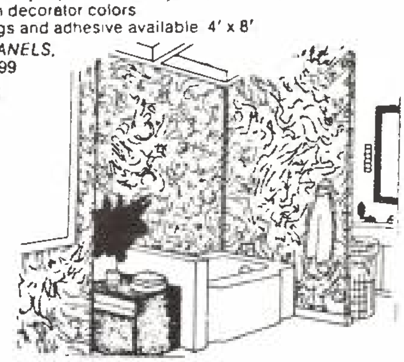
Panel of experts

Be an expert at installing Springfield Walnut paneling. It's that easy. And these rich, warm panels have luan-back and printed finish to turn rooms into showplaces, at big savings. 4'x8'x5/32".
SPRINGFIELD WALNUT PANEL.
Reg. 5.49
4.28



Waterproof your walls

Give your bath a fresh, new waterproof look with bath panels. Great for the kitchen too because they wipe clean easily. Marble effect panels in decorator colors. Mouldings and adhesive available. 4' x 8'
BATH PANELS.
Reg. 13.99
10.88



Bad, Bad One Hundred

Dr. Carol F. Novillis, University of Arizona

Reprinted from AATM Newsletter, *Volume XIV, Winter 1978*

This card game, based on the game Twenty-One, was designed by Anthony Johnson while he was an undergraduate elementary education major at Florida International University in Miami, Florida. It provides practice with adding upper decade facts.

Rules and Play:

1. For two to four players and using one or two regular decks of cards with the tens and face cards removed from the deck.
2. The dealer shuffles the deck and any player cuts the deck. Two cards are dealt (1 down and 1 up) to each player. The cards have the numerical value presented on the cards with the ace equaling one, except each player secretly chooses one card to represent its face value times 10. This is his tens card.
3. The object of the game is to have two or more cards total 99 or as close to 99 as possible without exceeding 99. After the first two cards are dealt, each player may either pass or be dealt, face up, an additional card. A player may continue to receive additional cards as long as his total is less than 99 or he has not previously passed. If a player has a hand with a total of more than 99, he may be able to stay in the game by changing his tens cards so that his new total is less than 100. After everyone has passed, each player identifies his tens card and then his total so that the player with the largest total less than 100 can be declared the winner.
4. The winner of a round scores 10 points. In case of a tie, each person scores 5 points.
5. At the conclusion of each round, the cards are reassembled in a pack and the deal passes clockwise around the table. The winner of the game is the first person to score 100.

Sample Game:	**Player 1**	**Player 2**
Original Deal	9 3 9 is selected to be the tens card. Total is 93.	8 7 8 is selected to be the tens card. Total is 87.
Next Deal	8 Total us 101. He's over, so he changes the tens card to 8. New total is 92.	6 Total is 93.
Next Deal	6 Total is 98.	7 Total is 100. He's over, so he changes his tens card to 7. New total is 91.
Next Deal	PASS	5 Total is 96.
Next Deal		PASS

Player 1 wins with a total of 98.

The Role of Mathematics in the Development of Science

by *Glen S. Aikenhead*
Department of Curriculum Studies
University of Saskatchewan, Saskatoon

and

Ranjana Kiran
Teacher at St. Clements School, Toronto

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Introduction

Mathematics and science have interacted through the ages. Mathematics, as a means of articulation and theorization in science, now spans the universe all the way from the largest galaxy to the smallest elementary particle. The present-day relationship between mathematics and science is by no means static. It has evolved from the past and will continue to evolve in the years ahead. Out of past associations new ones emerge, then with a further change in the intellectual and cultural climate, new interactions develop.

Today's mathematical involvement in the physical and social sciences can be traced to its historical routes. By doing so, one gains an overview of the various roles of mathematics in the development of science. From such a perspective one can understand the ways in which mathematics interacts with science today; but more, one can bring greater clarity into speculations of future relationships.

Mathematics itself is not a part of the realm of science. Both have different subject matters, differences which cannot be bridged by anything but superficial similarities. Basically, natural science deals with objects and events in the "external" world while mathematics concerns itself with the objects in its own "aesthetic" perception. These objects are internally conceived and inwardly structured.

Through the ages and, indeed, today, people have had diverse ideas regarding the relationship between mathematics and science. Some would have us believe that mathematics is the "queen of the sciences," while others insist that it is simply a servant to science:

Pythagoreans (ca.530 B.C.) maintained that in nature "all is number."

Aristotle (ca.350 B.C.) defined mathematics as an abstraction from nature.

Kepler (ca.1600 A.D.) thought that just as ears are made for sound and eyes for color, the mind of man is meant "to consider quantity."

Galileo (ca.1600 A.D.) said that the book of the universe was written in mathematical language and its alphabet consisted of geometrical figures.

Bacon (ca.1600 A.D.) regarded mathematics merely as a servant to physics, and actually complained of the dominion which it was beginning to exercise in science.

Feynman (1965) writes, "mathematics can help physics, but they are two quite different activities - mathematics deals with the abstract world, and physics deals with the real world."

Eric Rogers (1960) suggests that we might describe mathematics as a "master architect designing the building in which science can grow at its best."

Just as there is a danger to science in over-glorifying mathematics and attempting to subordinate all of science to it, so is there a danger in calling mathematics the "handmaiden of science." Accelerations and retardations in the development of science can be traced, in large measure, to such attitudes.

This paper, in investigating the role of mathematics in the rise of science, chronologically examines various epochs of our past.

Pre-Greek and Greek Era

The mathematics of Egypt and Babylon preponderantly served a practical function. Pre-Greek mathematics was integral to astronomy, taxation, and the construction of moats and temples.

The most successful product of the Greek mind was the deductive quality of geometry. The first Greeks to grasp this possibility of abstraction in geometry were probably Thales (600-550 B.C.) and Pythagorus. Then around 300 B.C. one of the most famous masters of geometry, Euclid, set out to collect the theorems of his predecessors and to arrange them as a single self-contained work entitled *Elements*.

The next century produced two or more gifted mathematicians. Apollonius (ca.200 B.C.) discovered the so-called conic sections which later contributed directly to astronomy. In addition there was Archimedes (ca.250 B.C.) whose brilliance at mathematics was matched by his genius for mechanics.

In Greek astronomy, the first mathematically conceived system was that of Eudoxus (ca.370 B.C.). Spurred on by Plato's notion of reality, Eudoxus reduced the irregular movements of celestial bodies to uniform circular motion. His system consisted of 27 concentric spheres, one inside the other. The whole system was a purely geometrical hypothesis, calculated to represent the apparent paths of the planets. Later, Ptolemy (90-168 A.D.) wrote his famous astronomical treatise *De Algamest* in which he strove for a mathematical model of the universe, known today as the Ptolemaic system.

Greek physics did in no way bequeath a book comparable to the mathematical works of Euclid, Archimedes or Apollonius. Yet, Greek physics culminated in the system of Aristotle and held the stage of physics for almost 2000 years.

The Greeks created a general concept of mathematics and a general concept of physics. Yet, if one assesses the pragmatic outcomes of Greek mathematics and physics, and if one confronts the mathematics and physics of Greek antiquity with 20th century knowledge, then one may find that Greek physics as a whole *never* developed into a mathematical system. The system of Greek mathematics had severe limitations and shortcomings. For example, they were incapable of "founding" functions. Their symbolization did not advance beyond an elementary stage, the stage of abstraction from "direct actuality." (Full scale symbolization involves abstraction from abstraction.) These inadequacies made Greek mathematics unsuited for promoting the rise of theoretical physics as we know it today. In other words, these shortcomings prevented a type of scientific thinking from developing. Perhaps the Greeks did not introduce mathematics as a technique for mastering problems that arose in man's mind. Although Greek astronomy was mathematical and its mathematization made it successful, the Greeks never had the insight (necessary in pursuing mechanics, physics and other sciences) to articulate qualitative attributes by quantitative magnitudes. They never realized that writing physical laws as mathematical formulae and applying mathematical procedures to the formulae can sometimes lead to further explications and developments. The Greeks never arrived at such an insight in spite of the dramatic first steps taken by Archimedes. There was no determination to solve "scientific puzzles" through mathematical manipulation and empirical verification. This was so fateful that eventually even Greek mathematics tended to wither away.

Nevertheless, the Greeks did have areas of knowledge in which science and mathematics overlapped. Their astronomy was clearly mathematical in Eudoxus and Ptolemy. Pythagoreans envisioned some sort of mathematization of physics, although the extent and depth of their insights is not easy to appraise. Again, the laws of Archimedes on balancing the lever and on floating bodies clearly pertain to mathematical physics and were the first of their kind. And yet they did not have the effect of initiating a mathematical physics at that time. Even atomists like Epicurus and Democritus did not show any tendency to initiate a mechanics of the stature of the 17th or 18th century.

The Renaissance

The Greeks wrote hundreds of books on mathematics, treating it for the first time as worthy of study for its own sake. In the centuries of darkness that followed, much of this mathematical treasury was lost. But enough remained so that the scholars of the late Middle Ages once again launched a search for knowledge, giving the study of mathematics an impetus that kept on accelerating.

Men in the 16th and 17th centuries were already looking beyond arithmetic into the vistas of algebra. It was the French mathematician, Rene Descartes (1596-1650 A.D.) who first started writing algebraic equations. In his ambition to "remake the world" he developed a new branch of mathematics, analytic geometry, a technique for visualizing numbers as points on a graph, equations as geometric shapes, and shapes as equations. Trigonometry and logarithms also emerged from the Cartesian system of Descartes.

Then in 1665, England's Isaac Newton produced calculus, which for the first time permitted the mathematical analysis of all movement and change. Meanwhile, a German mathematician, Leibnitz (1646-1716), independently invented his version of calculus and in 1684 published his account of it. (Today the symbols derived by Leibnitz: $\frac{d}{dx}$, $\frac{d}{dy}$, $\frac{d}{dz}$, are more generally used than those derived by Newton: $\dot{x}$, $\dot{y}$, $\dot{z}$.)

In Rene Descartes' work, we meet a system of thought much more intensive, concentrated and intricately interlocked than the Greek system. In his system everything was to be accounted for mathematically - by configuration or by number. He regarded physics as reducible to mechanism, and even considered the human body as being analogous to a machine. The mechanization of his highly concentrated deductive system became the template for the structure of physical science.

Men of the 17th century were extremely conscious of the importance of mathematics to scientific development. Therefore, it is not surprising that the development of science after 1600 A.D. began with the establishment of this so-called "rational mechanics" which held the stage of science during the 17th, 18th and 19th centuries. Mechanics was a problem which only became manageable when, in a certain sense, it had been "geometrized." Motion became envisaged as occurring in the emptiness of Euclidean space. (The Aristotelian system had discouraged the idea of the composition of motion, and was uncongenial to any mathematical treatment of it.) Although Galileo was one of the first persons to treat motion quantitatively, he failed to achieve the perfect formulation of the modern law of inertia because he could not imagine a purely geometrical body sailing off into an utterly empty and directionless Euclidean space. The law of inertia had to wait for Descartes.

The 17th century produced the greatest single statement on the relation between mathematics and physics. It was Galileo's dictum that mathematics is a language of science. He went so far as to say that the mind was to be constantly directed only to those things, and to apply itself to only those problems which were amenable to measurement and calculation. Descartes, Torricelli, Kepler, Huygens, Newton, and others who succeeded Galileo, clarified this scientific value by their geometrizing problems concerning natural phenomena.

The mechanized study of motion may well have been the high point of 17th century science. The century witnessed one attempt after another not only to explain motion and other natural phenomena, but to interpret all changes of the physical universe in terms of a purely mechanistic universe. Kepler inaugurated the scientist's quest for a mechanistic universe.
To generations of astronomers, the basic celestial figure was a circle. Kepler broke away from this supposition by introducing ellipses. He did not find his ellipses in the tables of Tycho Brahe or the writings of Copernicus. He found them by searching untiringly in the work of Apollonius. Guided by astronomical observations, he was the first to grasp the true meaning of foci of conics. Conics and their theory were in no way Kepler's private mathematical invention. They had existed for nearly 2,000 years for anybody to find and use.

The determination to formulate all explanations in mechanistic terms had important effects upon the biological sciences. Harvey (1567-1650), in his enquiries into the circulation of the blood, had a purely mechanical approach. G.A. Borelli (1608-1679), in his book *The Motion of Animals*, wrote a chapter on the "Mechanical Propositions Useful for the More Exact Determination of the Motive Power of Muscles." This tendency to glorify mere mechanization led to the ubiquitous view that the animal body was nothing more than a piece of clockwork.

The effects of the new mechanistic outlook are vividly illustrated in the works of Robert Boyle (1627-1691). He is quoted to have said that he did not expect to "see any principles proposed more comprehensible and intelligible than the corpuscularian." This philosophical position is often called "mechanical philosophy," since it tends to give a mechanical explanation of the physical universe. One of

Boyle's works includes a discourse on the mechanical origin of heat and magnetism. According to him, chemistry itself could be reduced to micro-mechanics.

Later in the 17th century, a culminating event took place when Newton and Leibnitz introduced "derivatives" and laid the foundation for calculus and mechanics. The ultimate triumph came in publishing the *Principia*, in 1686. The *Principia* is important not so much because of its laws, definitions, concepts of time, space, and gravitational force, but because Newton constructed and deduced, by mathematical reasoning, what Kepler had only divined and postulated.

Outwardly, in the *Principia* there is hardly any mathematics invoked or presupposed which should not have been quickly accessible to Archimedes and Apollonius. No attempt had been made in the *Principia* to introduce Descartes' innovation of analytically using symbols and functions. Newton was quite skilled in the use of symbols and functions as he was very familiar with Descartes' work. In fact, Newton mastered the method of Descartes much better than Descartes himself. However, in the *Principia*, Newton's definitions of limit and derivative ("ultimate ratio") seem to be such that a personal disciple of Archimedes should have been able to compose them in principle. Why did this not occur? Greek thinking, in general, did not formulate such logical abstractions as: a relation of a relation, a property of properties, an aggregate of aggregates (for example, a rate of change of a rate of change - acceleration). Second derivatives were at the center of Newton's mechanics. It was this kind of limitation to Greek rationality that separated Archimedes from Newton; a hiatus which Archimedes could never succeed in crossing.

Inwardly, there is a difference between the Euclidean space that underlies the *Principia* and the Euclidean space that underlies Greek mathematics and physics. The Euclidean space of the *Principia* continues to emphasize Greek congruencies and similarities between figures. However, it does something new. Several significant physical entities found in the *Principia*, velocities, momenta and forces, are vectors. Vectorial composition and decomposition of these entities constitutes an innermost scheme of the entire theory. In the course of the 18th century, the vectorial statements of Newton and others were gradually transferred and reinterpreted into analytical statements. The 20th century widened the concept of a vector into the broader concept of a tensor.

To summarize the events in the 17th century, one could say that there was considerable scientific development where geometrical and mathematical methods could be easily and directly applied. Therefore, not only did the sciences make a remarkable development in the 17th century, but mathematics also progressed to a great extent. This is because the sciences, especially physics and dynamics, were pressing upon the frontiers of mathematics all the time. The sciences created a need for mathematics, and therefore mathematics flourished. The relationship between science and mathematics has never quite been the same since then. Today, both tend to go their separate ways and draw upon one another when the need arises. But it was not so in the 17th century. Science depended on mathematics and mathematics depended on science. Without the achievements of mathematics the scientific revolution, as we know it, would have been impossible.

18th-19th Centuries

Let us first look at the achievements of some of the great mathematicians and scientists in the 18th and

19th centuries. Then we shall analyze the role mathematics played in the rise of science at this point in history.

Because of Newton's success, mathematical theorists of the 18th and 19th centuries held fast to a philosophy of "mechanistic determination." The French mathematician, Pierre Simon de Laplace (1749-1827), perfected Newtonian analysis of the solar system in a great work entitled *Mechanique Celeste*. He also used calculus to explore and advance probability theory. The most celebrated partial differential equation was devised by Laplace.

$$\frac{\partial^2 v}{\partial^2 x} + \frac{\partial^2 v}{\partial^2 y} + \frac{\partial^2 v}{\partial^2 z} = 0$$

(This equation has been used to describe the stability of the solar system, the electric field around a charge, a steady distribution of heat, and many other phenomena.)

Euler (1707-1783), of Swiss fame, created a host of new uses for calculus as it applies to curves and surfaces. He has been called the father of modern topology.

Lagrange's (1716-1813) hallmarks were his famous works *Mechanique Analytique* and *Theorie des Fonctions Analytiques* - master textbooks in its subject. But by far his greatest achievement was the space of "generalized coordinates" of our mechanics of today.

The genius who dominated 19th century mathematics and physics was Carl Friedrick Gauss (1776-1855). He gave direction to the new movement toward generality in mathematics by imposing on it his own stern standard - a demand for absolute rigorous thinking. As a 17-year-old, he audaciously questioned certain rules of Euclid's geometry that generations of mathematicians had taken for granted, pointing out that many of them did not hold true on curved surfaces.

But it remained for Gauss's pupil, Riemann (1827-1866), to shatter the boundaries of traditional geometry by postulating not only curved spaces of three dimensions, but spaces made up of four and more dimensions. Fifty years later, the physicist, Albert Einstein, brought the process to a stunning climax by borrowing these abstractions and using them in his theory of relativity to describe the real universe.

Out of the 17th and into the 18th century, preference continued to be given to rational mechanics (mathematical analysis of everything possible). In this context several so-called 'principles of mechanics' were produced. Meanwhile, the theories of light, heat, electricity, and magnetism were not forgotten but they advanced at a slower pace until their turn for full attention came in the 19th century.

The mechanics of the 18th century and the first decades of the 19th century was virtually inseparable from the mathematics. Almost all the leading architects of the various parts of mechanics were eminent mathematicians: James Bernoulli (1667-1748), d'Alembert (1717-1783), Euler (1707-1783), Cauchy (1789-1857), Lagrange (1716-1813), Poisson (1791-1840), Laplace (1749-1827), Gauss (1776-1855), and Jacobi (1804-1851), to name a few. Correspondingly, most of their theorizing emanated from "pure thinking" with very marginal entanglements in direct experimentation.

The need for 17th century mathematics continued to grow. Scientific progress was now even more strongly dependent on mathematics than it was before. In this period, the amount of mathematics which was created for, and because of, mechanics (theoretical and applied) was enormous - especially in the area of analysis. The calculus of variation was instigated largely by mechanics of particles (finite systems), while other mathematical theories were instigated largely by mechanics of continua (hydro-dynamics, acoustics, general theory of elasticity). Virtually all of partial differential equations were created this way. Indeed, the mathematical theory of waves, which eventually became the hallmark of theoretical physics in all its parts, emerged from mechanics of continua. Fourier analysis was the result of the mechanics of continua and the theory of heat. The concept of potential energy originated in the Lagrangian theory of finite particles. Finally, it appears that the mechanics of continua had a share in the emergence of tensor theory.

Beginning with the 19th century, the relationship between mathematics and mechanics changed. Mathematics became more or less independent of mechanics and physics. It assumed a philosophical nature and began to develop for its own sake. Yet another kind of relationship between mathematics and theoretical physics developed. It was a rapport built more on parallelisms of pursuits rather than on identities of aims. Mathematical formulations were no longer created for a particular purpose.

From time to time in this century, theoretical physics was able to seize upon an unfamiliar ready-made piece of mathematics and use it instantly. It would have appeared as if the mathematics had been prefabricated especially for the theoretical physicist. For example, in the second half of the 19th century, statistical mechanics of the kinetic theory of matter was able to draw upon the mathematical theory of probability (initiated in the Renaissance age and developed by Laplace). Another example is the theory of relativity. It utilized the non-Euclidean geometry of the 19th century.

For all of physics, and gradually for other sciences as well, mechanics became a model of mathematization in the 19th century. Most of the development in electricity, magnetism, optics, and heat conduction was mathematically modeled on paradigms from mechanics of continua. Therefore, in many parts of physics the mathematics was uniformly the same, not only in technique but in the manner in which mathematical and physical conceptions were correlated with each other. However, there was one part of physics which did not conform to this general pattern - the theory of thermodynamics. It was mathematically linked to a novel kind of mechanics - statistical mechanics.

It would seem that the relationship between mathematics and science from the Greek times to the 19th century took a full circle. Generally speaking the Greeks regarded mathematics and science as two separate entities. Scientists in the Renaissance had a totally mechanistic outlook toward all knowledge. This resulted in a remarkable development of mathematics. In the 18th century, the scientific community's need for new mathematics continued to grow. The amount of mathematics that was created for and because of mechanics was enormous. Then, in the 19th century, the relationship between mathematics and science changed. Mathematics began to be independent of science. There developed a limited collaboration between physicists and mathematicians that remains unbroken today.

20th Century

It is intriguing that every so often it is possible to apply an almost forgotten mathematical development of yesterday to a scientific problem of today. The 20th century has some fine examples of this.

The power of mathematics has rarely been proven more effective than in relativity theory - a brilliant application of the geometry of curved surfaces to the treatment of space, time and motion. In his theory of general relativity, Einstein applied the 19th century ideas of Gauss and Reimann in suggesting the existence of a curved universe of four dimensions.

Another example of science drawing upon prefabricated mathematics is Boolean Algebra. Contemporary studies in network and information theory, mechanical and human, had to fall back on the work of George Boole (1815-1864). Boole developed symbolic logic to clarify difficult Aristotelian logic. Today, his sytem is widely used as a tool to augment sound reasoning and has practical uses in designing parts of telephone circuits and electronic computers.

In quantum physics, it happened that a scientific setting was fashioned out of a mathematics created 20 years previously. The original disparate quantum physics versions of Heisenberg and Schrodinger were merged into one by Schrodinger. The union of the two was mathematically brought about in the precincts of so-called Hilbert space. Since entering physics, this theory of operators has developed the concept of an operator from a tool in physics to a reality in nature, and it has raised the mathematization of physics to new levels. There is hardly a purely mathematical statement on operators in Hilbert space which some physicist would not interpret as an event, or as a property of an event in nature. In fact, it has become a general belief that mathematics and science have correspondence rules: if a purely unexpected mathematical formula arises, then a corresponding unknown occurrence in nature exists. Maxwell's prediction that light is an electromagnetic wave is a good example.

Meanwhile, pure mathematicians are climbing to new levels of abstraction. How their work will relate to future scientific knowledge no one really knows. It may be decades before science gets a chance to draw upon the mathematics of today.

Conclusion

We have made a cursory historical review of the role of mathematics in the rise of science. Some general observations are suggested.

(1) As seen from history, any area of inquiry capable of mathematization developed the earliest and fastest. This is why physics developed before chemistry, chemistry before biology, and biology before any of the social sciences. The characteristics of physical science are such that a vast range of phenomena can be handled by linear algebra or differential equations. On the other hand, the inexact sciences are less amenable to mathematical treatment and, therefore, have not developed so fast. Apparently, the mathematization of a science affects the role and nature of revolutions that may and do occur in it.

(2) Much of the newly created mathematics has, at the time of its creation, no overt bearing on applied science or even on theoretical science. There is today a warehouse of mathematical knowledge of which scientists have not yet taken advantage.

(3) It has become a part of the celebrated scientific methodology that, if a purely unexpected mathematical conclusion arises, then a corresponding unknown occurrence in nature should be detectable.

(4) Mathematical formulation of scientific statements bestows a peculiar kind of lucidity and precision upon them and establishes logical and cognitive relations among them. It also introduces challenging analogies and unifications. For instance, we have seen that most wave propagation phenomena, whether in acoustics, electricity or optics, are assumed to be governed by virtually the same set of differential equations.

Mathematics is not part of or subordinate to science. Mathematics is a unique realm of knowledge from which science borrows in order to develop a) a set of tools for inquiry into natural phenomena, and b) a language for the articulation of subsequent explanations.

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