Using Calculators to Teach Trigonometry

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Consider the plight of the ungifted mathematics students learning about trigonometric ratios for the first time. Typically, they are given some textbook diagrams indicating the measures of sides of a right triangle together with a definition of the tangent ratio. Soon after, similar diagrams and definitions are given for the sine and cosine ratios. Usually there is too little opportunity given for students to assimilate the concept of trigonometric ratios as constants with respect to particular angles. Perhaps the most demoralizing attribute of this kind of instruction is the very early introduction of the table of trigonometric ratios usually given to three or four decimal places. Students are given the typical flagpole and ladder problems to solve using the tables, but even when students do find solutions correctly, they are still very limited in their understanding of the table and trigonometric ratios.

The availability of inexpensive hand-held calculators makes feasible methods of instruction that can provide greater opportunity for understanding many mathematical concepts. This article describes one such method and our experiences in teaching by it. In the *Report of the Conference on Needed Research and Development on Hand-held Calculators in School Mathematics* (1976), Recommendation 6 states:

> Materials should be developed to exploit the calculator as a teaching tool at every point in the curriculum, to test a variety of ideas and possibilities pending emergence of calculator-integrated curriculums.

Agreeing with the recommendation, we constructed materials to use in an experiment to determine the effectiveness of hand-held calculators in learning trigonometric ratios. The materials:

1. Sets of four similar right triangles having acute angles that were multiples of 10 degrees from 10 to 80 degrees. (Although a right triangle with a 10-degree angle contains an angle of 80 degrees, a different set of four similar right triangles was constructed for each angle.)
2. Two large posters, one for each of the classrooms used, to provide a quick visual image of the opposite and adjacent sides of a right triangle and the hypotenuse.
3. A large poster for each of the eight groups taught, for them to develop their own short table of trigonometric ratios for acute angles that are multiples of 10 degrees.
4. Worksheets for recording data from measurements of sides and

angles of right triangles, for use in calculating trigonometric ratios and worksheets to help establish a clear concept of ratio and later to enable students to make applications and generalizations from their constructed tables.

The materials were used with four classes in a one-quarter mathematics course for Grade IX and X students recommended for students not planning to take Grade XI academic mathematics. We taught the students over a period of 18 school days during which each class met 13 times. Students in each of the four classes were randomly assigned to two groups, one group using the materials with the calculators and the other group using the materials without the calculators. In turn, the groups were randomly assigned for instruction so that each instructor had two calculator groups and two noncalculator groups. Instruction took place during the regular mathematics period in the regular mathematics room and in a nearby room. There were 131 students, and each of the eight groups contained 15 to 18 students.

All students satisfactorily measured angles and sides. The calculator groups calculated the trigonometric ratios quickly. However, the noncalculator groups, which had to work out ratios by pencil and paper long division, felt considerable frustration, and so did the teachers. This occurred despite the students' (in the noncalculator groups) working in groups of four and sharing the measurements and computation.

Whereas each student in the calculator group measured and calculated ratios for *all four* triangles in the set of four similar right triangles, students in the noncalculator groups measured *one* triangle, calculated the three ratios for one angle, and the four students in each group shared their results so that every student recorded and observed four computed ratios for the sine, cosine, and tangent of one angle. Every student then found the average of the four ratios after confirming that there were no gross errors in measurement or computation. The resulting average ratio was then placed prominently on a large poster on the wall.

The procedure was repeated for another angle that was a multiple of 10 degrees. With four groups in each class, this was sufficient to complete the class table of trigonometric ratios from 10 to 80 degrees in multiples of 10 degrees.

The posters were put up during each class so that students could see their tables developing and later, when the tables were completed, they could easily observe patterns the ratios exhibited.

In all groups, the ratios obtained were remarkably accurate. A comparison of the ratios on the posters with the correct trigonometric ratios rounded to two decimal places showed that, of the 192 ratios calculated by the 8 groups, 148 ratios were within .01 of the correct ratios, and 44 ratios were off by more than .01. Of these 44 larger errors, 27 were made with the tangent ratio: 19 for angles of 60, 70, and 80 degrees. The largest single error was a result of 5.37 instead of 5.67 for the tangent of 80 degrees. This result did not reduce the ability of the particular class to observe patterns from the table. Following are two of the tables, one prepared by a calculator group and one by a noncalculator group during the same period of instruction. Entries with an asterisk signify results in error by more than .01.

CALCULATOR GROUP B

Angle	sin	cos	tan
10°	.18	.99	.11
20°	.35	.95	.37
30°	.50	.87	.58
40°	.65	.77	.85
50°	.77	.65	1.18
60°	.87	.50	1.73
70°	.94	.35	2.68*
80°	.99	.18	5.49*

Table 1

NONCALCULATOR GROUP B

Angle	sin	cos	tan
10°	.19	.96*	.18
20°	.36	.93	.37
30°	.49	.86	.57
40°	.64	.76	.84
50°	.75*	.64	1.17*
60°	.86	.50	1.70*
70°	.92*	.34	2.69*
80°	.98	.17	5.70*

Table 2

Calculator Group B represented in Table 1 had the "best" results with four ratios off by more than .01. A calculator group also had the "worst" results with nine ratios off by more than .01. Altogether, for 96 ratios, the calculator groups were off by more than .01 18 times, while the noncalculator groups were off by .01 26 times.

The large well-spaced, uncluttered tables made the following generalizations easy to observe:

The sine and tangent ratios increased as the angle measure increased.

Contrastingly, the cosine ratio decreased as the angle increased.

The tangent ratio and the sine ratio were about the same for angles of 10 and 20 degrees.

At some point, the tangent ratio began to increase much faster than did the sine ratio.

The cosine ratio appeared to be the sine ratio "upside down."

When the angle was doubled, the corresponding ratios were not doubled.

The table was also used to motivate the students to think about the following questions:

Could one make reasonable estimates for angles that were multiples of five degrees?

What would the ratios be for angles of 0 and 90 degrees?

Students comprehended the first question clearly and suggested averaging two successive ratios. The second question was much more difficult and was not satisfactorily settled in most groups.

In a subsequent lesson, students obtained the estimated ratios for angles that were multiples of five degrees. This was done easily in both calculator and noncalculator groups. In another lesson, students calculated the quotients for the sine ratio divided by the cosine ratio. This was rapidly done in the calculator groups for all eight angles in the tables. Because of the time and tedium of long division in the noncalculator groups, the students shared the labor, each student performing one long division and contributing the result to the class set of quotients. They compared results with the tangent ratios in their tables. Unfortunately, in these students, little excitement was aroused by the close agreement of the quotients with the tangent ratios.

As stated earlier, these students were not planning to take Grade XI college preparatory mathematics, and many extremely weak students presented great challenges to the instructors. For many of them, it was difficult to grasp the concept of opposite and adjacent sides of an angle. However, the greatest difficulty and frustration for students and teachers occurred with long division. In one class, 13 students were given an exercise involving measurement of two sides of a triangle and conversion of the ratio of two sides from a fraction to a decimal. Only three students worked the division correctly. Five students reversed the dividend and divisor, while the other five made various errors such as incorrect subtractions and failure to place a zero in the quotient.

Despite such difficulties, all groups constructed the short table of trigonometric ratios satisfactorily. The tables were used after to solve problems and to continue to note patterns. When the conventional complete trigonometric table was distributed with entries to three-place decimals, there appeared to be a natural acceptance with an understanding that the three-place decimals represented ratios with which the students were now familiar.

As for performance differences between the calculator and noncalculator groups, on a quiz one week before the final test, the calculator groups did show some superiority. On a 16-point quiz, the mean of the calculator students was 11.8; the mean of the noncalculator students was 9.9. The difference was significant at the .05 level. However, on the posttest with 20 items, the calculator-instructed students had a mean of 9.90; the noncalculator-instructed students had a mean of 8.85. The difference was not significant at the .05 level. On a test of attitude focused on the instruction and topics taught during the experiment, there were no significant differences.

The materials and methods of instruction thus seemed to have no markedly different results in terms of achievement and attitude. However, both instructors worked much harder in the noncalculator groups. There were the long division difficulties, and more effort and planning was necessary to control the activities of the groups of four so that results could be shared efficiently. Errors in long division sometimes complicated the sharing of results. Too much time was spent on long division and too little on more interesting activities. The calculator groups were able to measure four times as many triangles as the noncalculator groups.

One may ask why the noncalculator groups used the materials and methods instead of the conventional textbook methods, which did not require as many computations. We decided that despite the computations, the students would attain better understanding of trigonometric ratios than with ordinary text materials not requiring actual measurements of similar right triangles. We also felt that the calculator groups had an advantage already and that the disadvantage in the noncalculator groups would be compounded if the students did not also have measurement activities enabling them to build their own tables.

Summary

This article presented a method of instruction on trigonometric ratios that seems appropriate where hand-hald calculators are available. The method worked about as well, in terms of achievement and attitude, where hand-held calculators

were not used, but the wear and tear on the teachers, if not the students, was greater. Clearly, with calculators it is feasible to obtain inductively the well-known trigonometric generalizations. For example, the calculator-equipped students were able to divide quickly the eight sine ratios in their tables by the corresponding cosine ratios so that they could compare them with the tangent ratios. Similarly, using measurements and calculators, it would not be difficult to compute the sum of the squares of the sine and cosine ratios and discover that the sum is about 1. A fine opportunity to discover the law of sines is made possible with measurements and calculators. While most students may not be surprised and delighted by the coincidence of almost identical quotients for such unlikely mates as sines of angles and measures of opposite sides, the calculator at least makes it feasible to make the discovery.

REFERENCE

Bell, Max and Marilyn Suydam. In Edward Esty and Joseph Payne (eds.) *Report of the Conference on Needed Research and Development of Hand-Held Calculators in School Mathematics*. Washington, D.C.: National Insititute of Education, 1976.

Calculators in the Classroom

The National Council of Teachers of Mathematics, recognizing the potential contribution of the calculator as a valuable instructional aid in the classroom, has adopted the following position statement:

> The National Council of Teachers of Mathematics encourages the use of calculators in the classroom as instructional aids and computational tools. Calculators give mathematics educators new opportunities to help their students learn mathematics and solve contemporary problems. The use of calculators, however, will not replace the necessity for learning computational skills.
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> As instructional aids, calculators can support the development and discovery of mathematical concepts. As computational tools, they reduce the time needed to solve problems, thereby allowing the consideration of a wider variety of applications. Furthermore, the use of calculators requires students to focus on the analysis of problems and the selection of appropriate operations. The effective use of calculators can improve student attitudes toward, and increase interest in, mathematics.
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> Other electronic devices, programmed to generate questions and activities and provide immediate feedback to students, are not to be confused with calculators or computers. These devices can be used to reinforce computational skills through drill.