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THE ALBERTA TEACHERS' ASSOCIATION
MATHEMATICS COUNCIL

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Volume XVIII, Number 2

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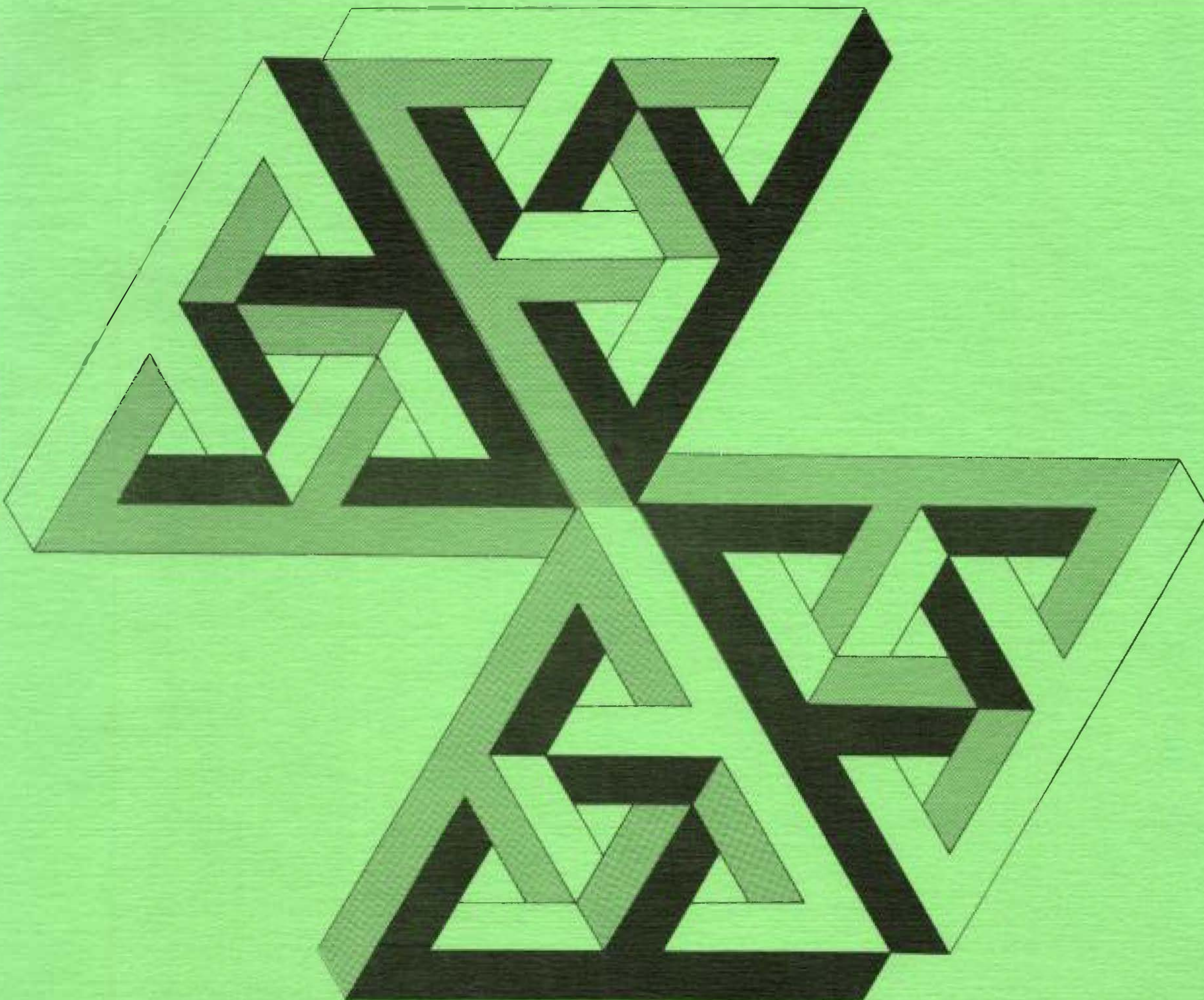

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THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

Volume XVIII, Number 2 November 1978

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EDPRESS

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The Editor's Page

Another successful conference is history. Held in Red Deer on October 13 and 14, the annual meeting featured an address by Dr. Jesse A. Rudnick, entitled "Beyond Skills and Concepts." Dr. Rudnick challenged teachers to go "beyond skills and concepts" when teaching math, emphasizing that teachers must not only know why they are teaching math, but be prepared to show students how to apply the subject.

Although we may call it "problem-solving" or "application," how often does math teaching result in the student transferring this knowledge to everyday life problems? This is the challenge for us, as we go "beyond skills and concepts," to teach students to apply math skills in real-life situations.

I have a tape of Dr. Rudnick's speech available for anyone interested. Requests must be made before January 1, 1979, however, as the original tape will be sent to another party at that time.

The conference's program planners saw the need to stress Dr. Rudnick's ideas, so the speakers in the Saturday sessions also focused on "Beyond Skills and Concepts."

During the luncheon program, Dr. Richard Guy of the University of Calgary attempted to teach us some new games to use as learning-aids. For a better understanding of the games, write to Dr. Guy for his forthcoming publication, which promises challenges for teachers and students alike.

Remember that in 1979 we host an NCTM "name-of-site" meeting in conjunction with our regular annual meeting. We are moving our location to Calgary for this one year, since Red Deer facilities are inadequate for such a large conference.

Ed Carriger
Editor

Notes from around Canada

The editor of Plus + + + *supplied us with the following material and encouraged us to include it in* Delta-K. *He hopes to tell us about important events, research, curriculum development and items of national interest. For this, he needs your help in sending information. Letters and material to the editor are welcome, and should be sent (in either French or English) to:*

E. J. Barbeau, Editor
University College, B201
University of Toronto
Toronto, Ontario, M5S 1A1

The Canadian Mathematics Education Study Group

Twice, in September, 1977 and in June, 1978, a group of mathematics educators, mostly from university mathematics departments and faculties of education, were invited to participate in conferences on the state of mathematical education research and teacher training in Canada. The proceedings of the first conference, *Educating Teachers of Mathematics: The Universities' Responsibility*, published by the Science Council, has appeared recently. This includes the texts of addresses by T.E. Kieren (University of Alberta), C. Gaulin (University of Laval), and A.J. Coleman (Queen's University) as well as the deliberations of the working groups.

At the second conference, besides invited addresses by G. Noelting (Quebec), G. Rising (Buffalo) and I. Weinzweig (Chicago), there were workshops at which discussions begun at the first conference on teaching, training, mathematization, and educational research were carried further.

In order to maintain the momentum of discussion and to keep the participants in touch, it was decided to form the Canadian Mathematics Education Study Group, a loose organization that will function through local, regional and national initiatives.

Further information can be obtained from the organizers: Professor A.J. Coleman (Mathematics Department, Queen's University, Kingston, Ontario K7L 3N6), Professor W. Higginson (Faculty of Education, Queen's University) or Professor D. Wheeler (Mathematics Department, Concordia University, 1455 de Maisonneuve Blvd. W., Montreal, P.Q. H3G 1M8). The address of the Science Council is 150 Kent Street, 7th floor, Ottawa, Ontario K1P 5P4.

PERMAMA: Inservice Teacher Training

In 1972, an innovative inservice teacher training programme, "PERfectionnement des MAîtres en MAthematique," (PERMAMA), was established by Téléuniversité du Québec, a branch of the University of Quebec. Intended for high school mathematics teachers, it is a credit programme leading to a certificate. Participants register for courses in one of 97 local or regional centres. They work through prepared modules and meet regularly for discussions directed by a "moniteur-animateur" at the centre. About 60 modules are currently available.

A fuller account is found in the Science Council of Canada report, *Educating Teachers of Mathematics: The Universities' Responsibility* or may be obtained from Professor Claude Gaulin, Faculté des Sciences de l'Education, Université Laval, Québec, P.Q. G1K 7P4.

Survey of Educational Research in Canada

Dr. D.R. Drost, Department of Curriculum and Instruction, Memorial University, St. John's Nfld. A1B 3X8, is compiling a catalogue of Canadian research related to mathematics education. Please send a list of studies currently in progress or completed in the last five years, including name of researchers, title of study, grade level, and date. These might be graduate theses, personal published reports, both internal and external, or assessment studies. This is the first step of the research subgroup of the Canadian Mathematics Education Study Group toward making knowledge of the work of Canadian researchers more widespread.

Crux Mathematicorum

Crux Mathematicorum is a problems journal edited by Léo Sauvé and published ten times annually by Algonquin College under the sponsorship of the Carleton-Ottawa Mathematics Association (affiliated with O.A.M.E.). Besides some delightful problems contributed by readers, most of which are suitable for high school teachers and lively students, there are brief articles, many on elementary topics in geometry and number theory. The managing editor is F.G.B. Maskell, Mathematics Department, Algonquin College, 200 Lees Avenue, Ottawa, Ontario K1S 0C5, to whom requests for subscriptions, sample copies, or further information may be sent. Bound-back volumes are available (Vol. I-II, $10; Vol. III, $10).

Canadian Mathematical Society Education Committee

The Canadian Mathematical Society, the national organization of university mathematicians and sponsor of the Canadian Mathematical Olympiad, has recently re-established its standing education committee. It is chaired by Professor George Bluman of the Mathematics Department of the University of British Columbia. The committee is interested in determining the current state of mathematical education in schools across Canada and exploring avenues of cooperation between teachers, mathematical educators and research mathematicians.

Did You Know...

...an interesting mathematics-in-the-social-sciences topic can be found on pages 304-332 of the 1968 volume of the *Villanova Law Review?* There a lawyer, J. F. Banzhaf III, argued by means of permutations, combinations, and the computer that in the 1960 presidential election, a voter in New York State had 3.312 times the power of a voter in the District of Columbia. Banzhaf's measure of "power" just made its first appearance in a commerical textbook (M. L. Bittinger and J. C. Crown, *Finite Mathematics,* Addison-Wesley, 1977).

New York State Mathematics Teachers' Journal, Spring/Summer 1978

Using Calculators to Teach Trigonometry

Walter Szetela
University of British Columbia

Robert Campbell, secondary mathematics coordinator, McRoberts Junior Secondary School, Richmond, B.C.

Reprinted from Vector, *June 1978.*

Consider the plight of the ungifted mathematics students learning about trigonometric ratios for the first time. Typically, they are given some textbook diagrams indicating the measures of sides of a right triangle together with a definition of the tangent ratio. Soon after, similar diagrams and definitions are given for the sine and cosine ratios. Usually there is too little opportunity given for students to assimilate the concept of trigonometric ratios as constants with respect to particular angles. Perhaps the most demoralizing attribute of this kind of instruction is the very early introduction of the table of trigonometric ratios usually given to three or four decimal places. Students are given the typical flagpole and ladder problems to solve using the tables, but even when students do find solutions correctly, they are still very limited in their understanding of the table and trigonometric ratios.

The availability of inexpensive hand-held calculators makes feasible methods of instruction that can provide greater opportunity for understanding many mathematical concepts. This article describes one such method and our experiences in teaching by it. In the *Report of the Conference on Needed Research and Development on Hand-held Calculators in School Mathematics* (1976), Recommendation 6 states:

> Materials should be developed to exploit the calculator as a teaching tool at every point in the curriculum, to test a variety of ideas and possibilities pending emergence of calculator-integrated curriculums.

Agreeing with the recommendation, we constructed materials to use in an experiment to determine the effectiveness of hand-held calculators in learning trigonometric ratios. The materials:

1. Sets of four similar right triangles having acute angles that were multiples of 10 degrees from 10 to 80 degrees. (Although a right triangle with a 10-degree angle contains an angle of 80 degrees, a different set of four similar right triangles was constructed for each angle.)
2. Two large posters, one for each of the classrooms used, to provide a quick visual image of the opposite and adjacent sides of a right triangle and the hypotenuse.
3. A large poster for each of the eight groups taught, for them to develop their own short table of trigonometric ratios for acute angles that are multiples of 10 degrees.
4. Worksheets for recording data from measurements of sides and

angles of right triangles, for use in calculating trigonometric ratios and worksheets to help establish a clear concept of ratio and later to enable students to make applications and generalizations from their constructed tables.

The materials were used with four classes in a one-quarter mathematics course for Grade IX and X students recommended for students not planning to take Grade XI academic mathematics. We taught the students over a period of 18 school days during which each class met 13 times. Students in each of the four classes were randomly assigned to two groups, one group using the materials with the calculators and the other group using the materials without the calculators. In turn, the groups were randomly assigned for instruction so that each instructor had two calculator groups and two noncalculator groups. Instruction took place during the regular mathematics period in the regular mathematics room and in a nearby room. There were 131 students, and each of the eight groups contained 15 to 18 students.

All students satisfactorily measured angles and sides. The calculator groups calculated the trigonometric ratios quickly. However, the noncalculator groups, which had to work out ratios by pencil and paper long division, felt considerable frustration, and so did the teachers. This occurred despite the students' (in the noncalculator groups) working in groups of four and sharing the measurements and computation.

Whereas each student in the calculator group measured and calculated ratios for *all four* triangles in the set of four similar right triangles, students in the noncalculator groups measured *one* triangle, calculated the three ratios for one angle, and the four students in each group shared their results so that every student recorded and observed four computed ratios for the sine, cosine, and tangent of one angle. Every student then found the average of the four ratios after confirming that there were no gross errors in measurement or computation. The resulting average ratio was then placed prominently on a large poster on the wall.

The procedure was repeated for another angle that was a multiple of 10 degrees. With four groups in each class, this was sufficient to complete the class table of trigonometric ratios from 10 to 80 degrees in multiples of 10 degrees.

The posters were put up during each class so that students could see their tables developing and later, when the tables were completed, they could easily observe patterns the ratios exhibited.

In all groups, the ratios obtained were remarkably accurate. A comparison of the ratios on the posters with the correct trigonometric ratios rounded to two decimal places showed that, of the 192 ratios calculated by the 8 groups, 148 ratios were within .01 of the correct ratios, and 44 ratios were off by more than .01. Of these 44 larger errors, 27 were made with the tangent ratio: 19 for angles of 60, 70, and 80 degrees. The largest single error was a result of 5.37 instead of 5.67 for the tangent of 80 degrees. This result did not reduce the ability of the particular class to observe patterns from the table. Following are two of the tables, one prepared by a calculator group and one by a noncalculator group during the same period of instruction. Entries with an asterisk signify results in error by more than .01.

CALCULATOR GROUP B			
Angle	sin	cos	tan
10°	.18	.99	.11
20°	.35	.95	.37
30°	.50	.87	.58
40°	.65	.77	.85
50°	.77	.65	1.18
60°	.87	.50	1.73
70°	.94	.35	2.68*
80°	.99	.18	5.49*

Table 1

NONCALCULATOR GROUP B			
Angle	sin	cos	tan
10°	.19	.96*	.18
20°	.36	.93	.37
30°	.49	.86	.57
40°	.64	.76	.84
50°	.75*	.64	1.17*
60°	.86	.50	1.70*
70°	.92*	.34	2.69*
80°	.98	.17	5.70*

Table 2

Calculator Group B represented in Table 1 had the "best" results with four ratios off by more than .01. A calculator group also had the "worst" results with nine ratios off by more than .01. Altogether, for 96 ratios, the calculator groups were off by more than .01 18 times, while the noncalculator groups were off by .01 26 times.

The large well-spaced, uncluttered tables made the following generalizations easy to observe:

The sine and tangent ratios increased as the angle measure increased.

Contrastingly, the cosine ratio decreased as the angle increased.

The tangent ratio and the sine ratio were about the same for angles of 10 and 20 degrees.

At some point, the tangent ratio began to increase much faster than did the sine ratio.

The cosine ratio appeared to be the sine ratio "upside down."

When the angle was doubled, the corresponding ratios were not doubled.

The table was also used to motivate the students to think about the following questions:

Could one make reasonable estimates for angles that were multiples of five degrees?

What would the ratios be for angles of 0 and 90 degrees?

Students comprehended the first question clearly and suggested averaging two successive ratios. The second question was much more difficult and was not satisfactorily settled in most groups.

In a subsequent lesson, students obtained the estimated ratios for angles that were multiples of five degrees. This was done easily in both calculator and noncalculator groups. In another lesson, students calculated the quotients for the sine ratio divided by the cosine ratio. This was rapidly done in the calculator groups for all eight angles in the tables. Because of the time and tedium of long division in the noncalculator groups, the students shared the labor, each student performing one long division and contributing the result to the class set of quotients. They compared results with the tangent ratios in their tables. Unfortunately, in these students, little excitement was aroused by the close agreement of the quotients with the tangent ratios.

As stated earlier, these students were not planning to take Grade XI college preparatory mathematics, and many extremely weak students presented great challenges to the instructors. For many of them, it was difficult to grasp the concept of opposite and adjacent sides of an angle. However, the greatest difficulty and frustration for students and teachers occurred with long division. In one class, 13 students were given an exercise involving measurement of two sides of a triangle and conversion of the ratio of two sides from a fraction to a decimal. Only three students worked the division correctly. Five students reversed the dividend and divisor, while the other five made various errors such as incorrect subtractions and failure to place a zero in the quotient.

Despite such difficulties, all groups constructed the short table of trigonometric ratios satisfactorily. The tables were used after to solve problems and to continue to note patterns. When the conventional complete trigonometric table was distributed with entries to three-place decimals, there appeared to be a natural acceptance with an understanding that the three-place decimals represented ratios with which the students were now familiar.

As for performance differences between the calculator and noncalculator groups, on a quiz one week before the final test, the calculator groups did show some superiority. On a 16-point quiz, the mean of the calculator students was 11.8; the mean of the noncalculator students was 9.9. The difference was significant at the .05 level. However, on the posttest with 20 items, the calculator-instructed students had a mean of 9.90; the noncalculator-instructed students had a mean of 8.85. The difference was not significant at the .05 level. On a test of attitude focused on the instruction and topics taught during the experiment, there were no significant differences.

The materials and methods of instruction thus seemed to have no markedly different results in terms of achievement and attitude. However, both instructors worked much harder in the noncalculator groups. There were the long division difficulties, and more effort and planning was necessary to control the activities of the groups of four so that results could be shared efficiently. Errors in long division sometimes complicated the sharing of results. Too much time was spent on long division and too little on more interesting activities. The calculator groups were able to measure four times as many triangles as the noncalculator groups.

One may ask why the noncalculator groups used the materials and methods instead of the conventional textbook methods, which did not require as many computations. We decided that despite the computations, the students would attain better understanding of trigonometric ratios than with ordinary text materials not requiring actual measurements of similar right triangles. We also felt that the calculator groups had an advantage already and that the disadvantage in the noncalculator groups would be compounded if the students did not also have measurement activities enabling them to build their own tables.

Summary

This article presented a method of instruction on trigonometric ratios that seems appropriate where hand-hald calculators are available. The method worked about as well, in terms of achievement and attitude, where hand-held calculators were not used, but the wear and tear on the teachers, if not the students, was greater. Clearly, with calculators it is feasible to obtain inductively the well-known trigonometric generalizations. For example, the calculator-equipped students were able to divide quickly the eight sine ratios in their tables by the corresponding cosine ratios so that they could compare them with the tangent ratios. Similarly, using measurements and calculators, it would not be difficult to compute the sum of the squares of the sine and cosine ratios and discover that the sum is about 1. A fine opportunity to discover the law of sines is made possible with measurements and calculators. While most students may not be surprised and delighted by the coincidence of almost identical quotients for such unlikely mates as sines of angles and measures of opposite sides, the calculator at least makes it feasible to make the discovery.

REFERENCE

Bell, Max and Marilyn Suydam. In Edward Esty and Joseph Payne (eds.) *Report of the Conference on Needed Research and Development of Hand-Held Calculators in School Mathematics*. Washington, D.C.: National Insititute of Education, 1976.

Calculators in the Classroom

The National Council of Teachers of Mathematics, recognizing the potential contribution of the calculator as a valuable instructional aid in the classroom, has adopted the following position statement:

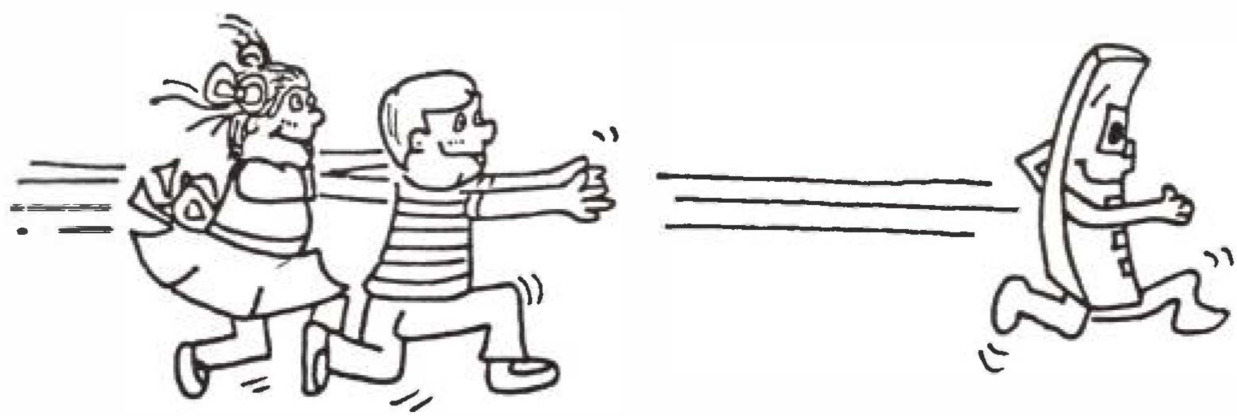
> The National Council of Teachers of Mathematics encourages the use of calculators in the classroom as instructional aids and computational tools. Calculators give mathematics educators new opportunities to help their students learn mathematics and solve contemporary problems. The use of calculators, however, will not replace the necessity for learning computational skills.
>
> As instructional aids, calculators can support the development and discovery of mathematical concepts. As computational tools, they reduce the time needed to solve problems, thereby allowing the consideration of a wider variety of applications. Furthermore, the use of calculators requires students to focus on the analysis of problems and the selection of appropriate operations. The effective use of calculators can improve student attitudes toward, and increase interest in, mathematics.
>
> Other electronic devices, programmed to generate questions and activities and provide immediate feedback to students, are not to be confused with calculators or computers. These devices can be used to reinforce computational skills through drill.

Kids 'n Calculators

by *Dick Holmes*
Mathematics Consultant
Calgary Board of Education

Illustrations by *Mrs. Bev Hubert*
Calgary Board of Education

Reprinted from "Kids 'n Calculators," Vector, *June 1978*



Introduction of the Calculator

Schools and teachers will have to answer these among other questions before introducing the calculator in the classroom.

1. What is the purpose of the calculator in the classroom?
2. Who will use the calculators?
3. To what extent will the calculators be used?
4. What new things can be done or what things can be done more effectively with the addition of calculators?
5. What pitfalls will possibly occur because of the use of the machine?
6. What security problems require attention?
7. What considerations need be given in selecting a calculator?
8. What public relations work needs to be done with parents <u>before</u> introducing the calculators?

Classroom Uses

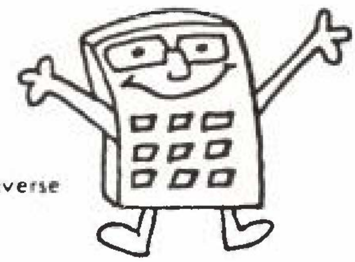
Some of the ways in which electronic calculators may be beneficial in the classroom include:

1. Immediate reinforcement for checking.
2. Accuracy of computation.
3. Analyze computational steps.
4. Develop and reinforce estimation skills.
5. Learning resource for self-discovery and exploration.
6. Explore patterns of numbers.

7. Motivation and success.
8. Introduce order of operations and use of parentheses.
9. Approximate and find square roots.
10. Study statistics, exponents, consumer topics.
11. Compute perimeter, area and volume.
12. Round decimals and find percent.
13. Calculations in problem solving.

Selection of a Suitable Calculator

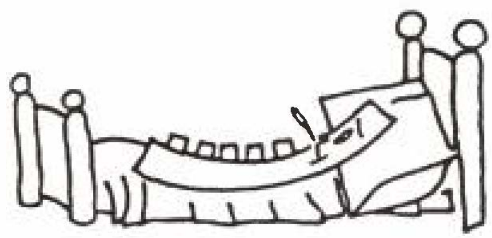
In selecting a calculator for classroom use, some considerations should include:



1. What functions are needed?
2. Has the calculator a floating decimal point?
3. Does the machine have a constant key or a key which performs this function?
4. What logic does the machine use (algebraic, arithmetic, reverse polish)?
5. Is an adaptor available for the machine?
6. Is the display clear and large?

Care of the Calculator

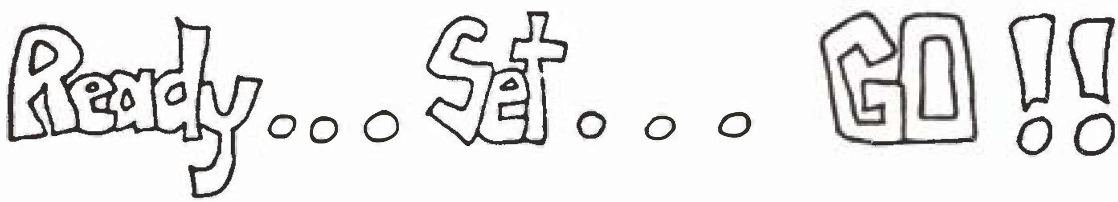
1. Check to ensure the switch is off when the calculator is not in use.
2. Use a finger to push the keys - not a pen or pencil.



Before Using the Calculator

When exploring the calculator before use, check to see:

1. That all display lights are working (a row of 8's uses all the lights.)
2. That the calculator is functioning correctly (do a few large multiplication questions for which the answer is known).
3. What the calculator does when division by zero occurs.
4. How the machine indicates an overflow.
5. If the calculator is programmed to round or truncate decimals.



Multiplication

You may use your calculator to find the product of numbers in a number of different ways.

Find the answer to the following questions by putting the numbers directly into the display and using the multiplication key.

23 x 46 =

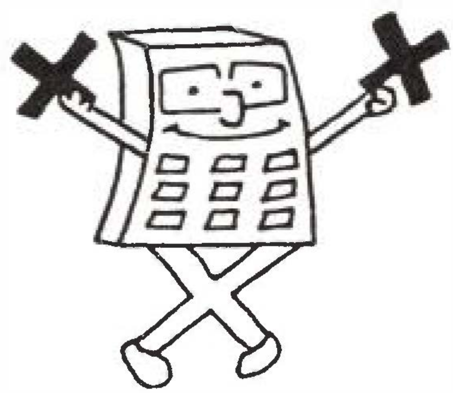
16 x 54 =

256 x 4569 =

Another method of multiplying with the calculator is finding partial products and then finding the sum of the parts. Can you see where all the numbers come from in these examples? (Note the place holding positions of each multiplicand.)

23			12 43	
x 46			16 56	
18	(6 x 3)		24 08	(56 x 43)
12	(6 x 2)		6 72	(56 x 12)
12	(4 x 3)		6 88	(16 x 43)
8	(4 x 2)		1 92	(16 x 12)
1058			2 05 84 08	

This method of multiplying will be very useful when the product of two numbers is too large for the calculator display. It will be possible to find the answer using a calculator.



Find the product of each of the following:

a. 12 345 x 67 809

b. 246 813 x 135 792

c. 473 625 x 694 537

Estimation

Estimation is an important skill in mathematics. Use rounding skills to help you select the largest number in each row. Use your calculator to check your answers.

38 x 52	12 x 88	29 x 61
502 - 347	651 - 459	298 - 147
352 ÷ 23	82 ÷ 6	1468 ÷ 72

Large Numbers

Find the value of these expressions:

$6^3 =$

$5^5 =$ $\qquad (2 + 1)^5 - (12 - 8)^3$

$7^1 =$

$10^1 =$ $\qquad \dfrac{0.000006 \times 0.00033 \times 17\,000\,000}{0.0034 \times 1\,100\,000\,000 \times 0.00004}$ (Scientific Notation)

Which of the following are Pythagorean Triples?

10, 24, 25	16, 24, 30
6, 8, 10	423, 1064, 1145

Unit Price

Find the amount saved by buying the larger size.

2 kg for 68¢; 5 kg for $1.39

3 m for $12.48; 10 m for $35.99

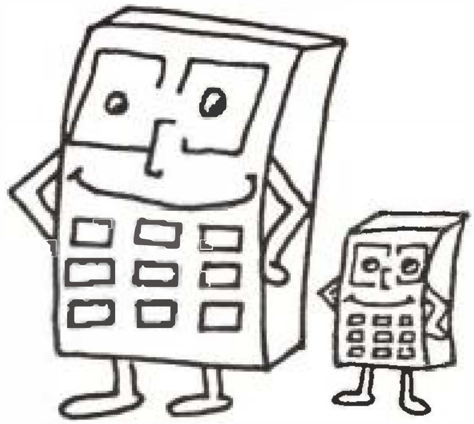
12 cc for 79¢; 39 cc for $2.50

Which is the better buy?

156 g for 45¢; 392 g for 87¢

10 kg for $2.99; 25 kg for $7.45

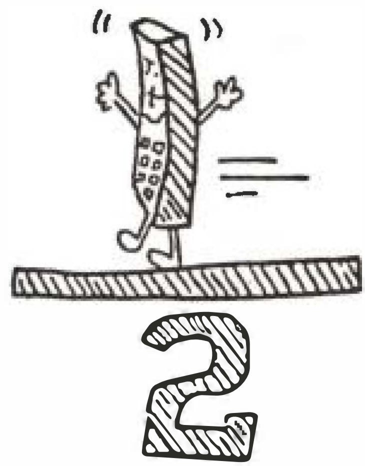
180 ml for 12¢; 500 ml for 38¢



Fractions to Decimals

Use your calculator to find the decimal expression for these fractions:

$\frac{1}{3} =$	$\frac{1}{5} =$	$\frac{1}{9} =$
$\frac{2}{3} =$	$\frac{2}{5} =$	$\frac{2}{9} =$
$\frac{3}{7} =$	$\frac{3}{5} =$	$\frac{3}{9} =$
$\frac{1}{2} =$	$\frac{4}{5} =$	$\frac{4}{9} =$



Order of Operations

Use your calculator to perform the indicated operations. Enter the numbers and operations in the order in which they appear.

$4 + 5 + 3 \div 3 =$

$4 + 4 \times 6 \div 4 =$

$12 - 4 \div 2 + 3 =$

$7 - 3 \times 6 \div 3 =$

Use your calculator to find the value of each expression. Keep in mind the correct order of performing operations. Rewrite each question using parentheses to indicate the correct order of performance.

$4 + 5 + 3 \div 3 =$

$4 + 4 \times 6 \div 4 =$

$12 - 4 \div 2 + 3 =$

$7 - 3 \times 6 \div 3 =$

Put signs (+, -, x, ÷) in the blanks indicating the operation to be performed to make true statements. If parentheses are required, show them. Use your calculator to show your work.

4	____	5	____	3	____	3 = 29
4	____	4	____	6	____	4 = 2
12	____	4	____	2	____	3 = 18
7	____	3	____	6	____	3 = 2

Percent

The manager of Ace Hardware recently announced price increases for a number of articles. The original and new prices are given. Find the amount of the increase in dollars and percent.

Original	New	Amount of Increase	Percent Increase
$ 40	$ 45		
$348.26	$696.56		

A local store advertised a month-end clearance sale by listing the percentage reduction of various articles. Find the amount of the reduction and the sale price.

Original	Percent Reduction	Amount of Reduction	Sale Price
$ 65	20%		
$ 99	33 1/3%		
$1256.89	10%		
$ 175.20	25%		

Change the following percents to decimal fractions.

34% =

67.3% =

0.06% =

132% =

Functions

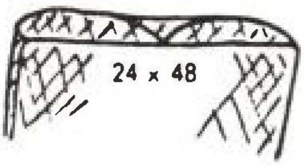
Use a calculator to complete the following tables:

(x, y)	2x + 3y + 4
(2, 3)	
(0, 0)	
(4, 1)	
*(-1,3)	

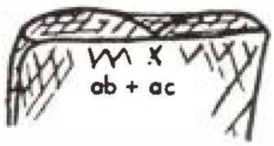
(x, y)	x + x x y
(3, 2)	
(5, 3)	
(1, 1)	

Equivalent Expressions

Mark the expressions that are equivalent to the goal expression. Use your calculator to check your answers.



48 x 24	27 x 51
2 x 2 x 2 x 3 x 2 x 2 x 2 x 2 x 3	2(10) + 4(1) x 4(10) + 8(1)
24(40 + 8)	(50 x 24) - (2 x 24)
20 x 52	50 x 22



a(b + c)	a x b x c
ac + ab	ba + ca
(c + b)a	a x b x 1 + a x c x 1

Rules for Divisibility

All even numbers are divisible by 2 with no remainder. It is possible to state a rule for divisibility by 2 which has no exceptions. The rule can be stated as follows:

Rule: A number is divisible by 2 if it is an even number.

Another rule that most people are familiar with is the rule for numbers that are divisible by 5. Try these on your calculator. Answer yes or no as to whether or not the numbers are divisible by 5.

200 ____ 56 ____ 14 685 ____ 280 ____

Can you state the rule?

Rule: A number is divisible by 5 if it ________________________________.

Now try these!

Can you find a rule for numbers that are divisible by 9? Use your calculator to help you answer yes or no as to whether or not these numbers are divisible by 9.

123 ____ 126 ____ 18 ____ 270 ____ 234 ____

Find more numbers which are divisible by 9.

____ ____ ____ ____ ____

Examine carefully those numbers which are divisible by 9. Can you state the rule?

Rule: ____.

It is possible to state rules for divisibility by 3, 4, 6, 8, 7 (this one is difficult). How many rules can you find?

Square Root

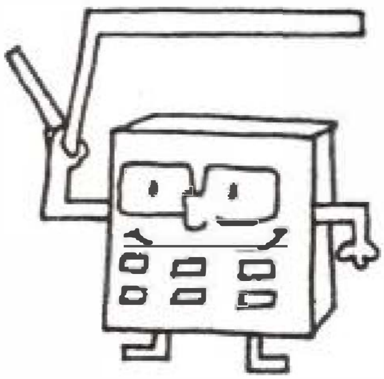
Some calculators have a square root key. However, it is very easy to find the square root of a number using your calculator without using a square root key.

Let us find the square root of 320.

1. Guess a number which you think will be close to the square root of 320. (e.g. 16)

2. Divide 320 by the number you selected.
(e.g. $\frac{320}{16} = 20$

If your guess is the same as the square root of the number, the quotient should be the same as your guess.
If the number and the quotient are not the same:



3. Add the quotient and your guess and divide by 2. (e.g. $\frac{20 + 16}{2} = 18$)

4. Divide 320 by the new numbers.
(e.g. $\frac{320}{18} = 17.777$)

If the divisor and quotient are the same number, you have found the square root.
If they are not the same:

5. Add the quotient and the divisor and divide by 2. (e.g. $\frac{18 + 17.777}{2} = 17.888$

6. Divide 320 by the result.

7. Continue this pattern until the desired accuracy has been obtained.

This method of finding the square root of a number was developed by Isaac Newton, hence, the name "Newton's Method" of finding the square root. Use "Newton's Method" to find the square root of the following:

a. $\sqrt{850}$

b. $\sqrt{29.63}$

c. $\sqrt{.3456}$

d. $\sqrt{90}$

Cube Root

It is possible to find the cube root of a number using a method very similar to the one used for finding the square root of a number.

Let us find the cube root of 320.

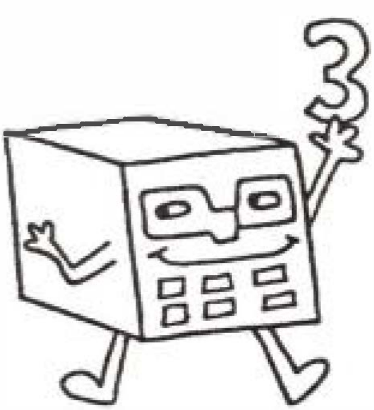
1. Guess a number which you think will be close to the cube root of 320. (e.g. 5)

2. The number ÷ guess ÷ guess + guess ÷ 3.
(e.g. 320 ÷ 5 ÷ 5 + 5 + 5 ÷ 3 = 7.6)

If your guess is the same as the cube root of the number, then the answer from step 2 cubed will equal the number.
(e.g. $7.6^3 \neq 320$)

If your guess was not the cube root, then:

3. Repeat step 2 using the result from step as your new guess.
(e.g. 320 ÷ 7.6 ÷ 7.6 + 7.6 + 7.6 ÷ 3 = 6.91)



4. Continue this pattern until the desired accuracy has been obtained.

Find the cube root of each of the following:

a. 2197 c. 515

b. 65 456 d. 3003

Number Patterns

Many interesting number patterns can be examined quickly with the use of your calculator. Try these. Use your calculator to find the first three or four answers; then complete the pattern without doing any actual calculation. When you have finished, use your calculator to check your answers.

1 x 1 =	66 x 66 =
11 x 11 =	666 x 666 =
111 x 111 =	6666 x 6666 =
1111 x 1111 =	66 666 x 66 666 =
11 111 x 11 111 =	666 666 x 666 666 =
111 111 x 111 111 =	6 666 666 x 6 666 666 =
1 111 111 x 1 111 111 =	
11 111 111 x 11 111 111 =	
111 111 111 x 111 111 111 =	

37 x 3 =	37 037 x 3 =
37 x 6 =	37 037 x 6 =
37 x 9 =	37 037 x 9 =
37 x 12 =	37 037 x 12 =
37 x 15 =	37 037 x 15 =
37 x 18 =	37 037 x 18 =
37 x 21 =	37 037 x 21 =
37 x 24 =	37 037 x 24 =

Find $n^3 - n$ when n = 2, 3, 4, 5, 6, 7.

Can you state a rule for the highest common factor of the answers?

Rule: $n^3 - n$ is always a multiple of ____________________

Find $x^2 + x + 41$ when x = 0, 1, 2, 3 . . .

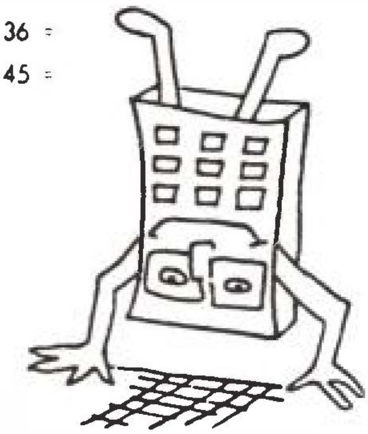
Is the result always a prime number? __________

Nimble Nine

The number nine is a fascinating number. Can you find the pattern for the following after doing only two or three on your calculator?

1 2 3 4 5 6 7 8 9 x 9 =
x 18 =
x 27 =
x 36 =
x 45 =
x 54 =
x 63 =
x 72 =
x 81 =

2 4 6 9 1 3 5 8 x 9 =
x 18 =
x 27 =
x 36 =
x 45 =



9 9 9 9 9 x 1 =
x 2 =
x 3 =
x 4 =
x 5 =
x 6 =
x 7 =

9 x 9 =
99 x 99 =
999 x 999 =
9999 x 9999 =
9 999 x 9 999 =
99 999 x 99 999 =
999 999 x 999 999 =

1 ÷ 9 =
2 ÷ 9 =
3 ÷ 9 =
4 ÷ 9 =
5 ÷ 9 =
6 ÷ 9 =
7 ÷ 9 =
8 ÷ 9 =
9 ÷ 9 =

9 x 9 + 7 =
9 x 98 + 6 =
9 x 987 + 5 =
9 x 9876 + 4 =
9 x 98 765 + 3 =
9 x 987 654 + 2 =
9 x 9 876 543 + 1 =

Motivation and Fun

Most numbers on the calculator, when looked at upside down, resemble letters of the alphabet. The letters represented are:

1	3	4	5	7	8	9	0	2	
I	E	h	S	L	B	G	O	Z	with a little imagination

Students can write problems for other students to solve; math skills can be reviewed, codes can be decoded, etc.

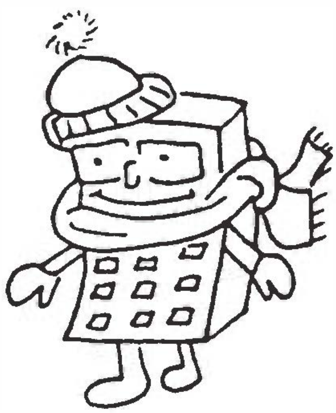
From Games, Tricks and Puzzles for a Hand Calculator

What did the cannibals say when they saw their dinner guest getting angry?

$$\frac{228440}{4} - 1.66 = \underline{\qquad\qquad\qquad}$$

What did Snoopy add to his doghouse as a result of his dog-fights with the Red Baron?

$(3 \times 303 + 50)7 \times 8 =$ ____________________



What kind of a double doesn't a golfer want at the end of a round of golf?

$1956 \times 4 + 153 \times 4 =$ ____________________

From The Calculator Book

A dingaling

$$\frac{56^2 + 8^3 + 243968}{2^5} = \text{\_\_\_\_\_\_\_\_\_\_\_\_}$$

Good for travel in a winter wonderland

$$\frac{25(18765 + 11115)}{4} + 65^3$$

Enter your age, then double it. Now add 5 and multiply by 50. Add the amount of change in your pocket, up to one dollar. Subtract the number of days in a year; add 115; divide by 100. Result - your age, then a decimal, then the amount of change.

Choose a number; add 10; multiply by 2; divide by 4; subtract 5; multiply by 2.

Games with Calculators

A few games that can be used in the classroom and that will reinforce computational and estimation skills are cited here.

Nim

The original game of Nim was played with a pile of sticks. Players took turns taking one, two or three sticks from the pile until only one stick remained. The player having to take the last stick lost the game.

A similar game can be played by two people using a calculator. The calculator should read 0 to begin the game. The first player pushes the 1, 2 or 3 key, followed by the + key and hands the machine to the second player. This player pushes the 1, 2 or 3 key, followed by the + key and hands the calculator back to the first player. This continues, players alternating turns and adding 1, 2 or 3 until 21 has been reached. The player making this total loses the game. A simple variation to this but changing the winning strategy would be to have the player making 21 win the game.

To make the game more difficult or for variety in strategy, use other keys and a different goal number, e.g. Use 1, 2, 3, 4, 5, 6 with goal number 50
Use 1, 4, 7 with goal number 50

One Only

Each player uses his own calculator. Select a number and using only the selected number key and any of the function keys (+, -, x, ÷ or =) get the calculator to read some pre-selected goal number.

Example: number 4; goal 13 (4 x 4 + 4 ÷ 4 + 4 + 4)

There will usually be a number of ways to get the goal number. A possible variation would be to achieve the goal in as few moves as possible.

Zero

A game to develop estimating skills and number awareness can be played by one person using a calculator. Select any six-digit number and put it in the display. Using any operation (+, -, ÷, =) and any two-digit numbers get the calculator to display 0 in four moves. A move consists of one operation with one two-digit number. Multiplication and division by 0 are not allowed.

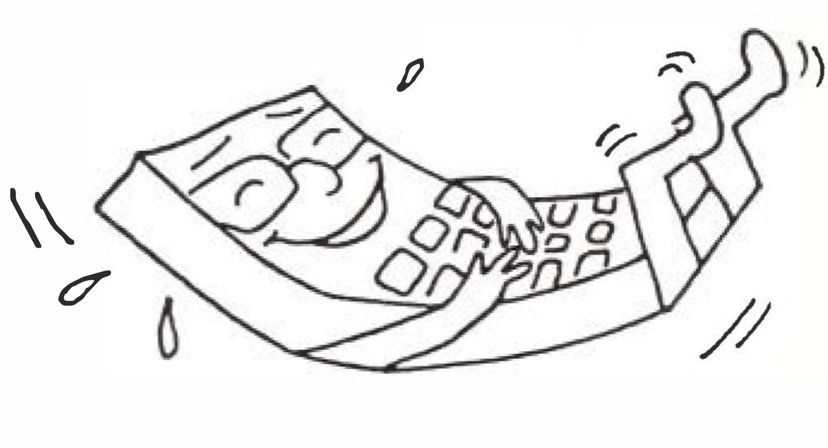
To make the game more difficult do not allow any two digits in the six-digit number to be the same.

One Hundred

The object of the game is to use each of the number keys once with any combination of the function keys (+, -, x, ÷, =) to get the calculator to read 100.

Many variations of this game can be played.

Example: use the number keys in any order
use the number keys in order from smallest to largest
use the number keys in order from largest to smallest



LESSON PLANS

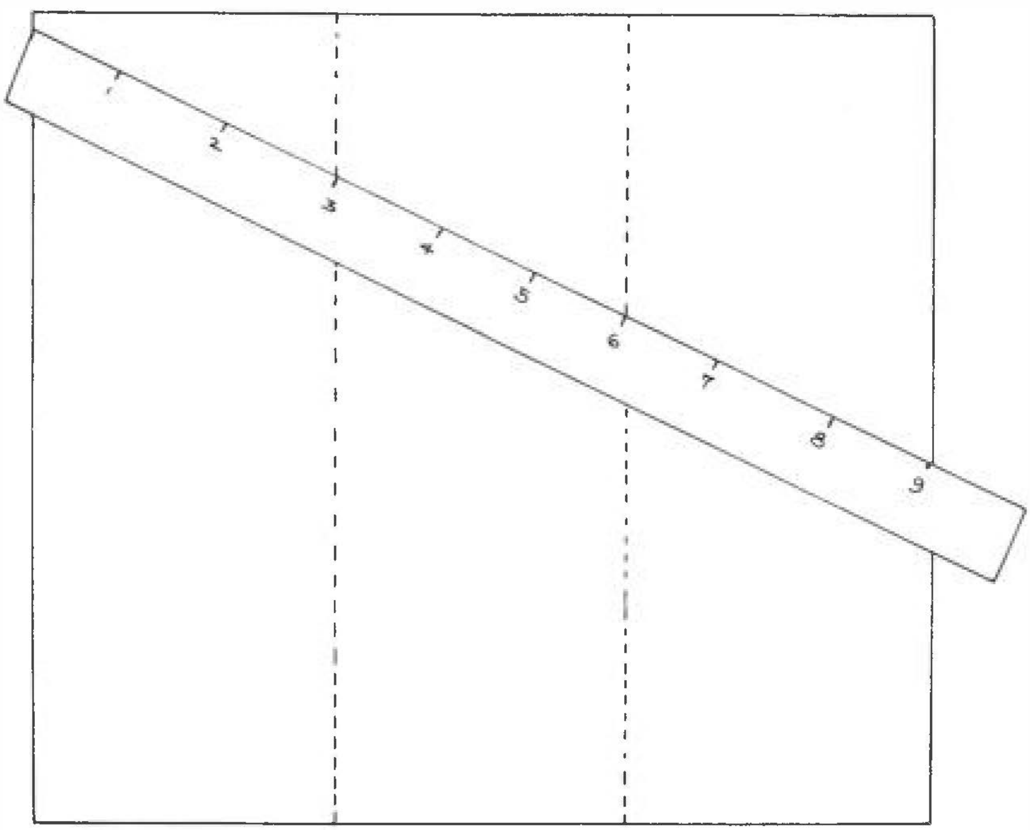
Do It the Easy Way

contributed by david morgan
carson graham senior secondary school, north vancouver

Reprinted from Vector, *Volume 19, Number 2, December 1977*

We wish to divide an 8½" wide piece of paper into three equal parts. (Sorry for not going metric.) Using arithmetic this is a mess. Calculation gives 3 5/6" divisions, which does not appear on my ruler, so forget it.

There is an easier way. Place a ruler obliquely across the page so that 0" is at one edge and 9" is at the other edge. (See diagram.) Mark the paper at 3" and 6." These marks divide the page neatly into thirds. Another set of marks can be made elsewhere on the page and lines can be ruled.



I have used this procedure several times in preparing dittos and doing carpentry. It can be used to divide something into thirds, quarters, sevenths or what have you. I use a metric rule, but I still use this trick.

It is interesting that I was taught how to trisect lines in school, but it took an old welder to show me a practical application.

Christmas Math Songs

contributed by henry enns (port coquitlam) and jack schellenberg (winston churchill) vancouver

Reprinted from **Vector**, *Volume 19, Number 2, December 1977*

TUNE – Santa Claus Is Coming To Town
Oh, you'd better take care completing the square;
you'd better not try dividing by y!
Math exams are coming to town.
We're making a list, don't shake in your boots;
Just watch out for extraneous roots –
Math exams are coming to town.
You know you'll have quadratics
And exponentials too
You rationalize denomi
Nators like the root of two.
So, you'd better be bright and calculate right –
You'd better check roots for the one that suits;
Math exams are coming to town.

TUNE – Jingle Bells
A day or two ago, I tried to solve for c;
When all the time, you know, I should have solved for b.
But after many tries, and sessions after school,
I uttered loud and joyful cries – when I found out this rule:
Oh, a and b, b and c – write them on the page:
Sometimes put down x and y – they seem to be the rage.
Don't give up – play it cool – make a guess or two.
And keep the paper neat and clean, and there's a pass for you.

TUNE – I'm Dreaming of a White Christmas
I'm dreaming of a quadratic, just like the one in our textbook,
Where solutions caper across the paper,
And make me think that I am smart.
I'm dreaming of a quadratic – And to myself each night I write:
'May quadratics give you no fright – And May all the answers be right.'

TUNE – Rudolph the Red-Nosed Reindeer
Zero, that funny cipher has a shape that looks like 'O.'
And if you want to use it, there are things you need to know.
Never divide by zero; if you do, you will be sad,
Getting a crazy answer, making your report look bad.
But treat zero as your friend – use him carefully –
'Safe to multiply or to add' – That's the rule for zero, lad!
Zero, that screwball number wants to be a comrade true,
But never divide by zero, **or you'll be getting zero, too!**

TUNE – O Tannenbaum
O G'ometry, geometry, I am fearful about thee!
Geometry, my bugaboo, a subject I will ne'er let through.
You keep my brain in dizzy whirls
You're tough for boys and tough for girls;
Oh G'ometry, geometry, what Satan's imp invented thee?

Fun with Holiday Facts and Figures

contributed by dave ellis, on exchange in edinburgh, scotland

Reprinted from Vector, *Volume 19, Number 2, December 1977*

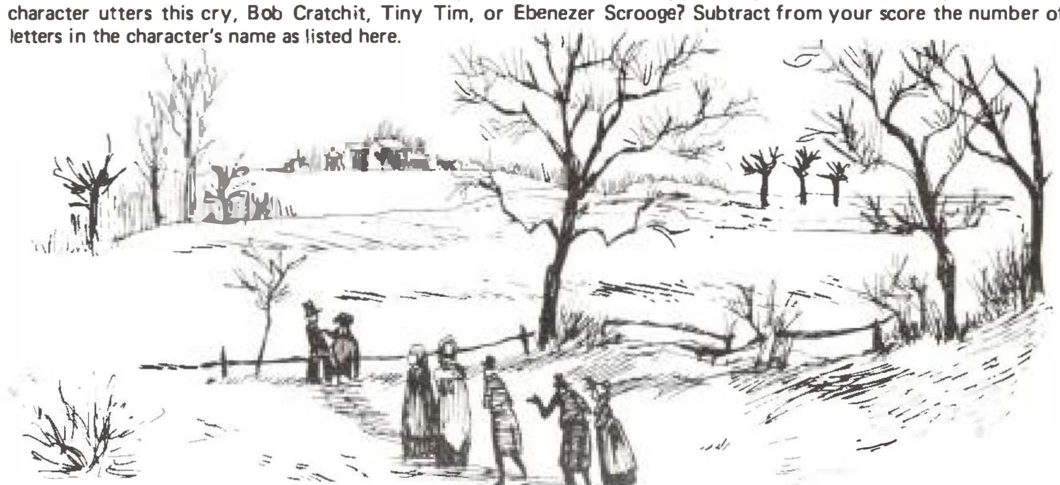
1. Start with the number of the day in December upon which Christmas falls.
2. To this figure add the number of wise men who came bearing gifts to the infant Jesus.
3. Divide your score by the total number of carolers in the following group: *A trio of tenors, an alto and a pair of sopranos and a solitary basso.*
4. Multiply your score next by the number of e's in this sentence.
5. **Careful now.** Subtract from your score the grand total of the four numbers spoken in the following rhyme:

   Said Mrs. Claus to Mr. Claus while they ate on a November night,
   'It's time to start your diet, dear, for you'll not fit the chimneys right.'

6. Add next the number of ornaments in two boxes of ornaments if each box contains a dozen and a half.
7. If O. Henry wrote the Christmas story, 'The Gift of the Magi,' divide your score by 4; if he did not, divide your score by 3.
8. Next: Add to your score the number of people seated at this holiday table:

   Mom and Dad were at opposite ends of the table and my brother and I and his girlfriend sat across from her cousin and his wife and son.

9. In Charles Dickens' 'A Christmas Carol,' the story ends with the much quoted cry, 'God bless us, every one!' Which character utters this cry, Bob Cratchit, Tiny Tim, or Ebenezer Scrooge? Subtract from your score the number of letters in the character's name as listed here.



10. Divide your score by the number of misspelled words in the following group:

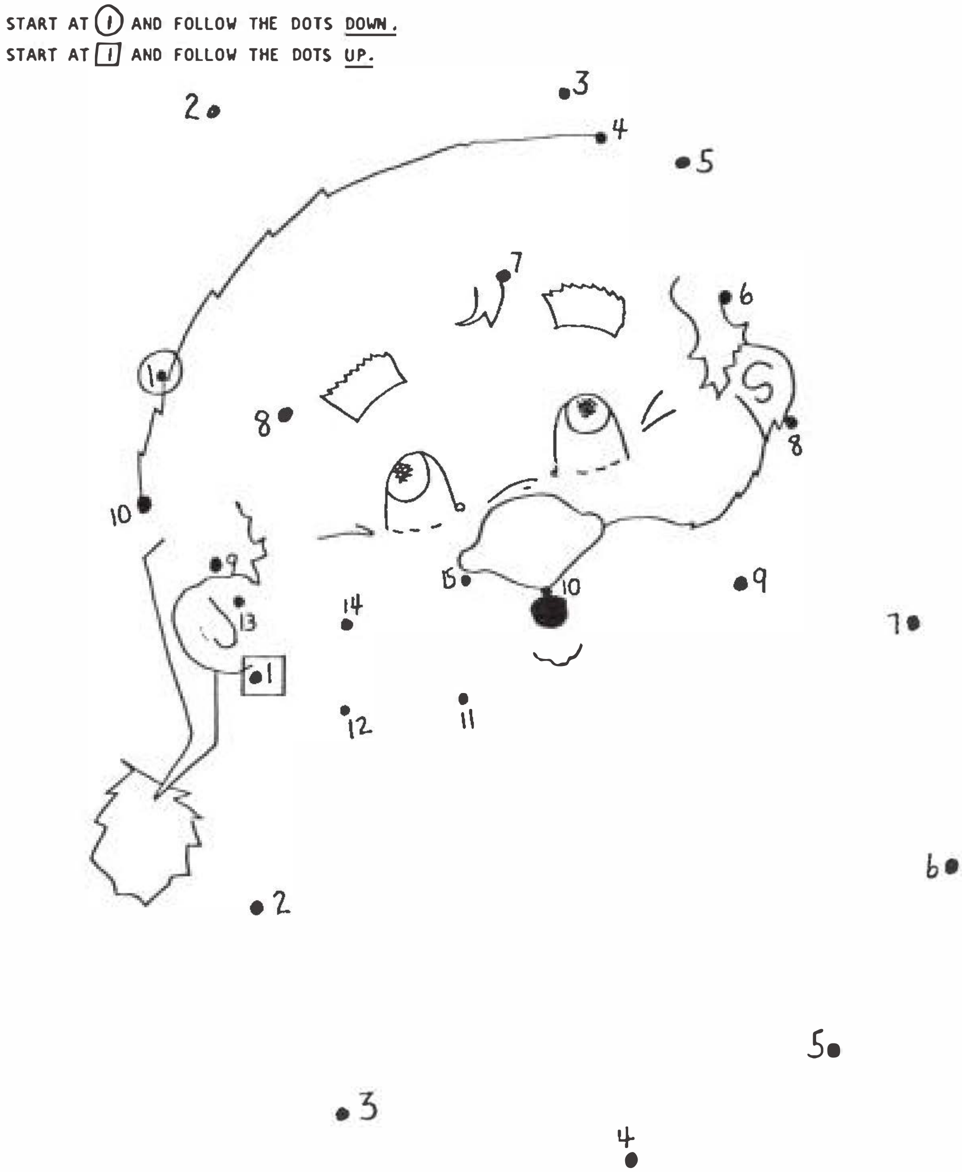
    EVERGREEN   RESOLUTION   MISTLETOW   HOLLY   RAINDEER

11. Irving Berlin wrote the song 'White Christmas.' If this statement is not false, subtract 6 from your score; if this statement is not true, subtract 5 from your score.
12. Our answer is the number of the day in January that is New Year's Day. IS YOURS?

Your Christmas Package

Reprinted from QAMT Journal, *Volume 1, Number 2, Christmas 1977*

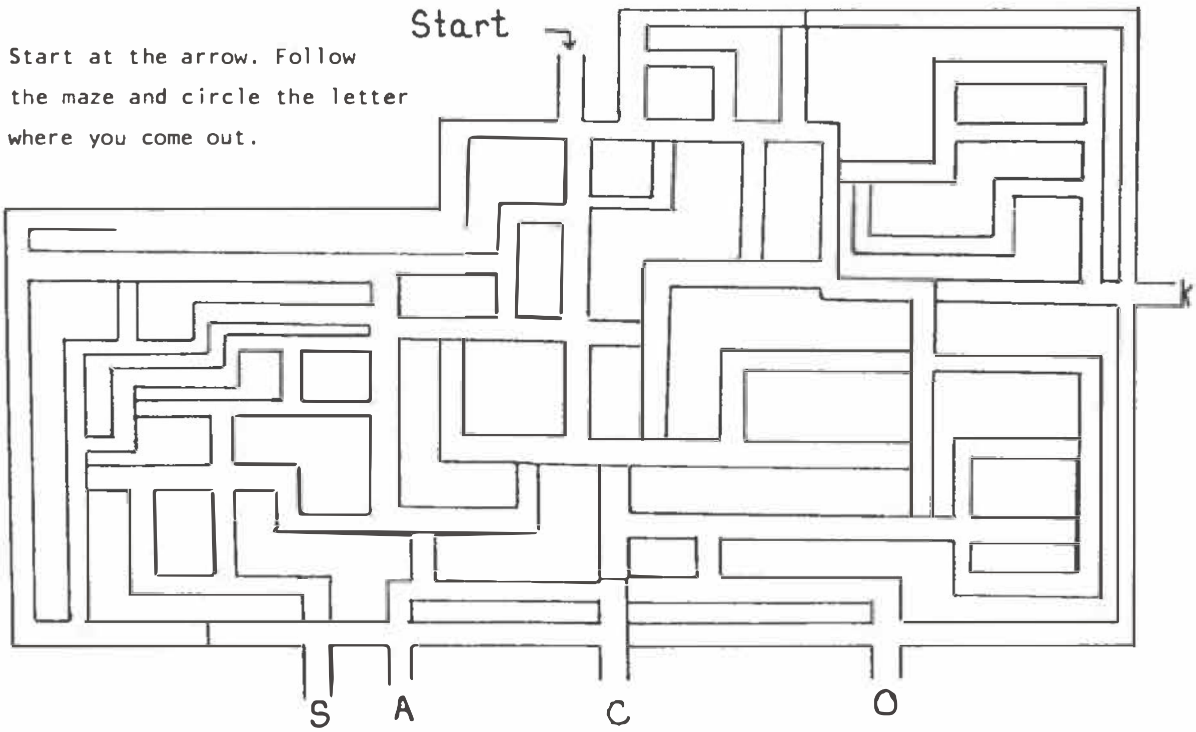
The following pages contain an assortment of puzzles you can use in your classroom(s) during Christmas week. The Santa Claus puzzle is intended for Kindergarten and was created by Susan Jeannotte. *The first "secret message" puzzle was put together by* Gayle Legault *and* Susan Jeannotte, *and can be used by grades 1-3.* Gundie Robertson *created the next "secret message," and suggests it be used by grades 4-6. The final puzzle is the handiwork of* Jack Benoit, *and is intended for junior high school math students. So use your school's copying equipment and make a stencil of the appropriate puzzle for your class(es).*



SOLVE EACH PUZZLE AND YOU WILL GET A LETTER.

PUT THE LETTERS TOGETHER AND FIND THE SECRET MESSAGE.

Start at the arrow. Follow the maze and circle the letter where you come out.

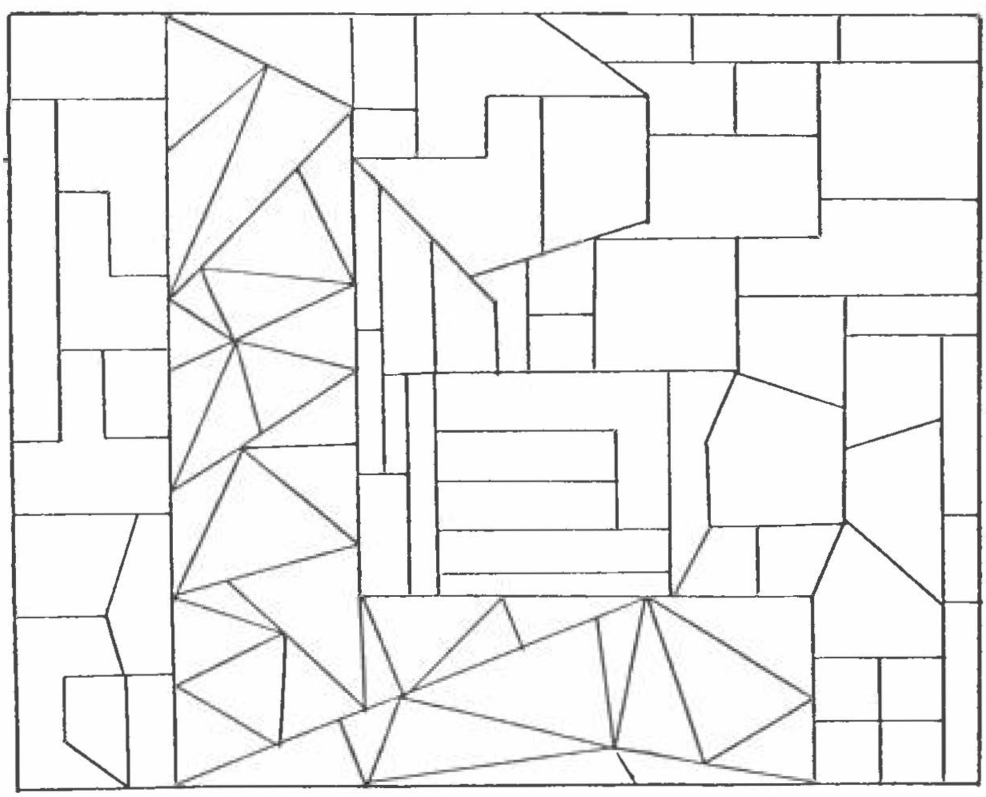


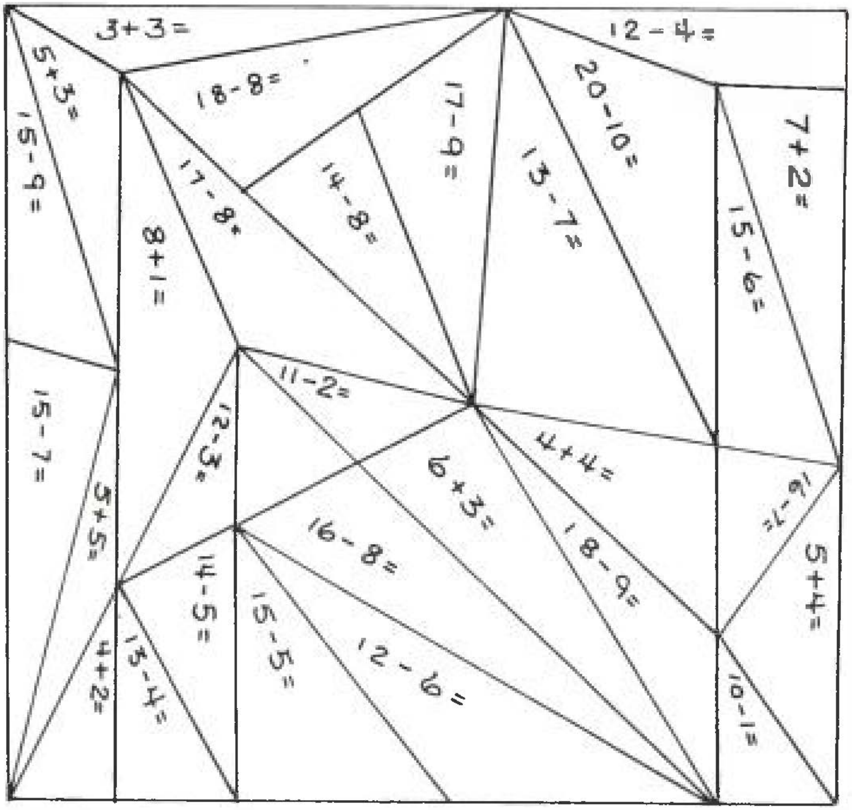
Color all the squares RED.

Color all the rectangles YELLOW.

Color all the triangles GREEN.

Color all the odd shaped figures BLUE.





Solve all equations.

Color answers of ...

8 yellow

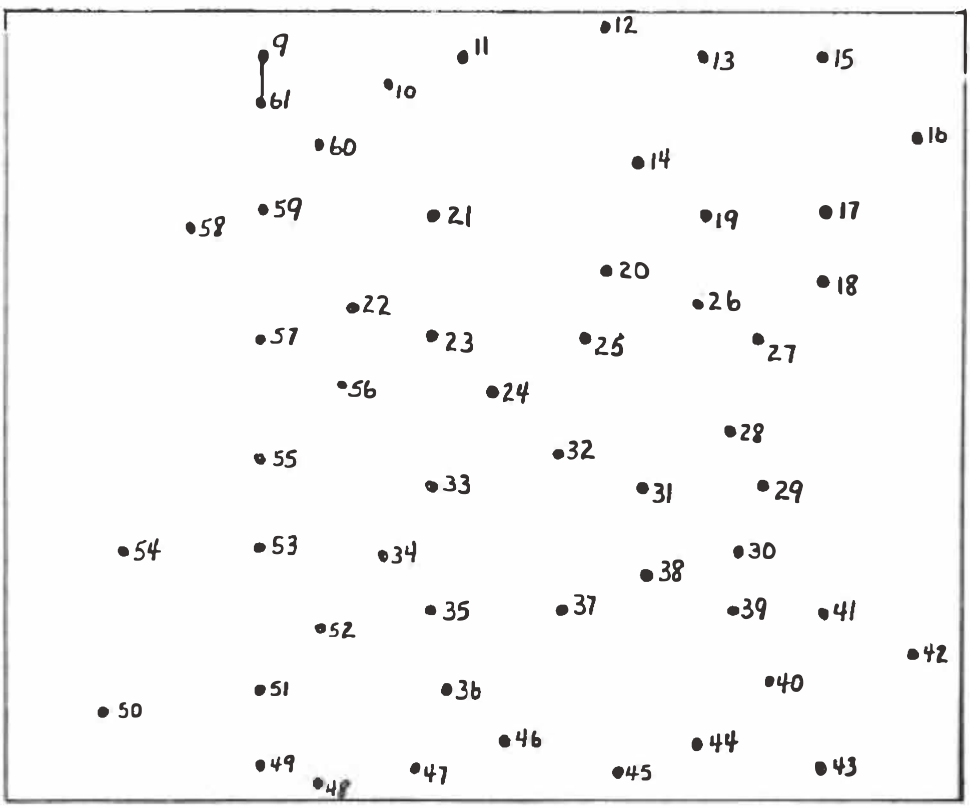
10 green

6 blue

9 red

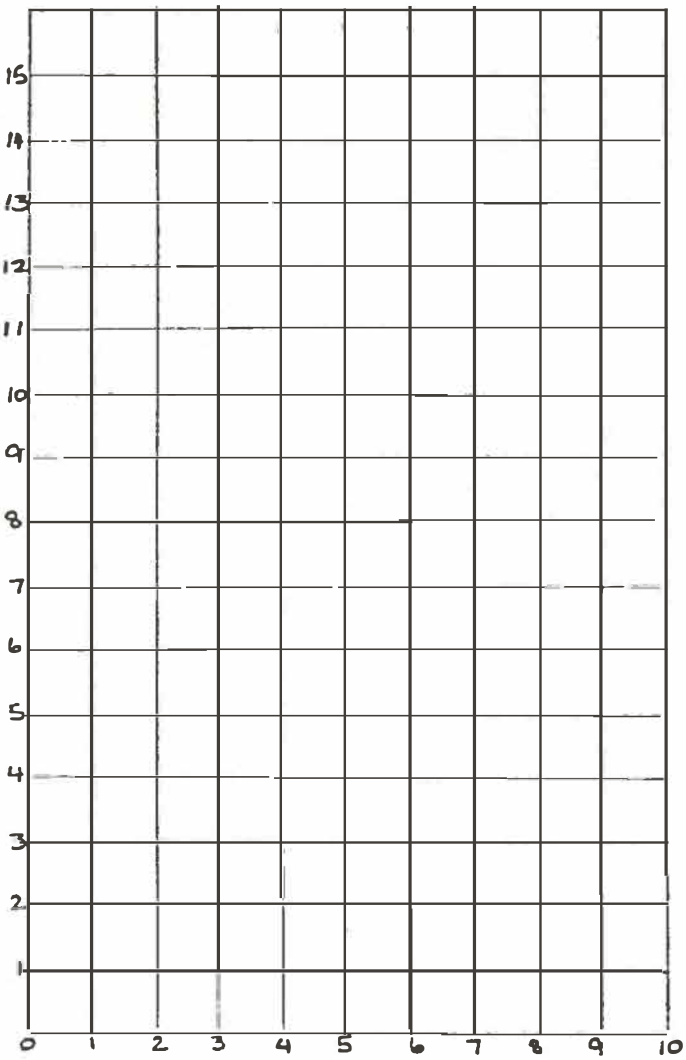
What letter did you find?

Start at 9.
Follow the odd-numbered dots in order.



DIRECTIONS

SOLVE EACH PUZZLE AND YOU WILL GET A LETTER. WHEN YOU HAVE FINISHED, PUT THE LETTERS TOGETHER. THEY WILL BE SCRAMBLED. UNSCRAMBLE THEM TO READ THE MESSAGE.

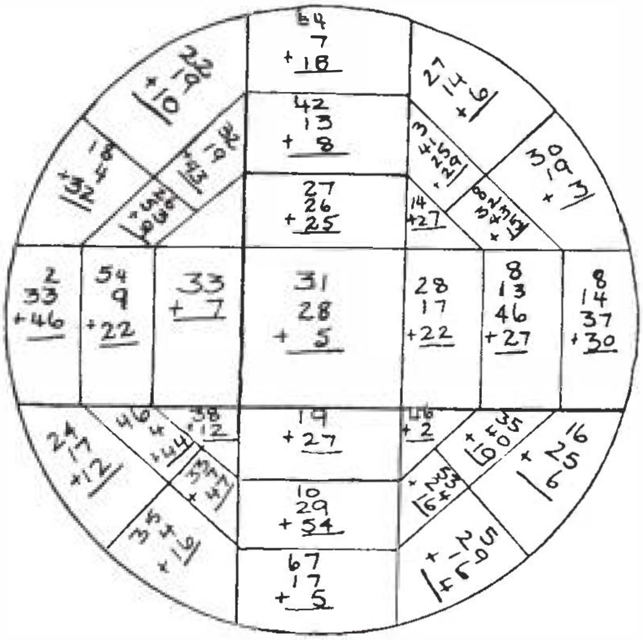


(2,2) (6,2) (2,2) (2,6) (5,6) (2,6) (2,10) (6,10)

Directions: Join the points in order. You will find a letter.

3×7 = ______	400 = G
183 ÷ 3 = ______	146 = R
173 − 87 = ______	763 = A
72 + 185 = ______	425 = N
Ans. = ______	207 = S

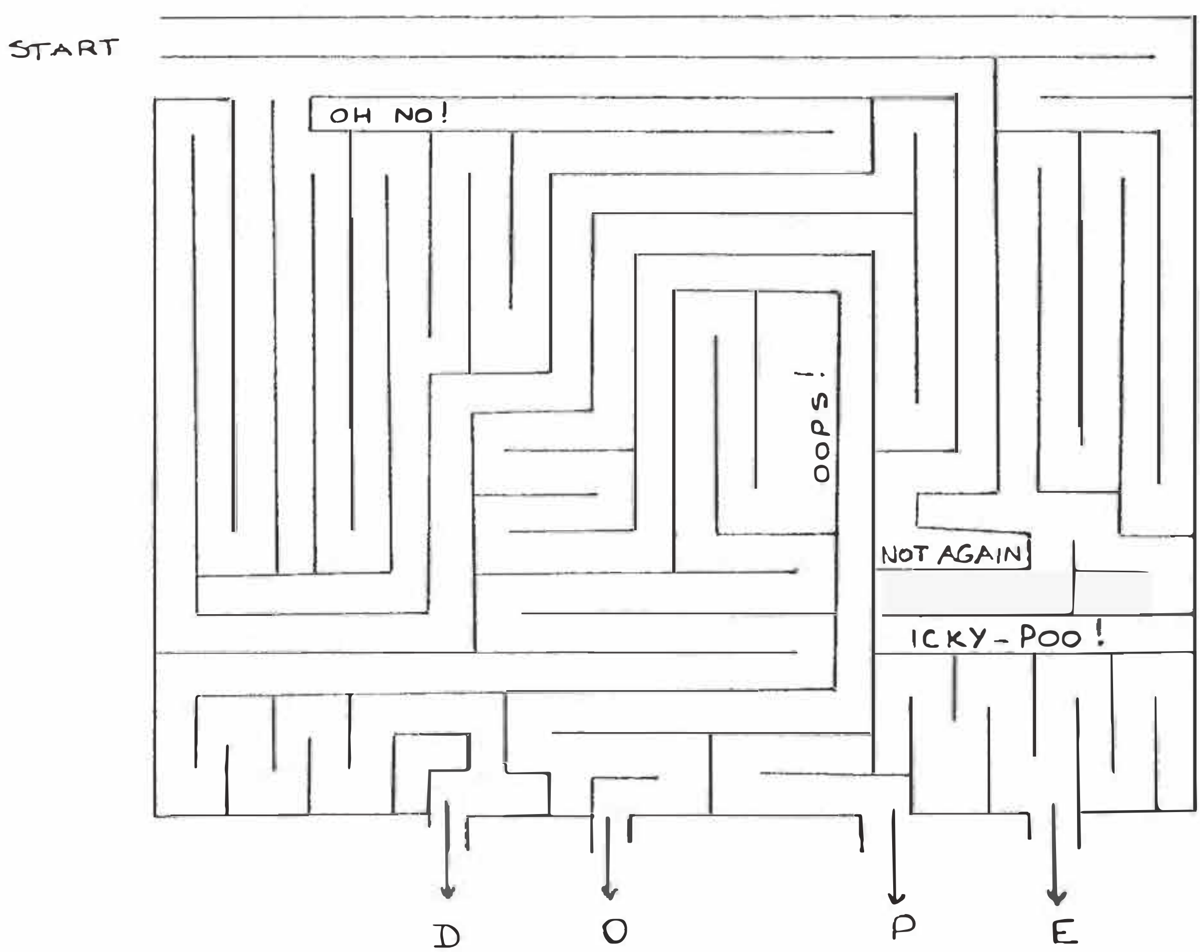
Do the above examples. Find the sum of your answers. It will equal a letter.



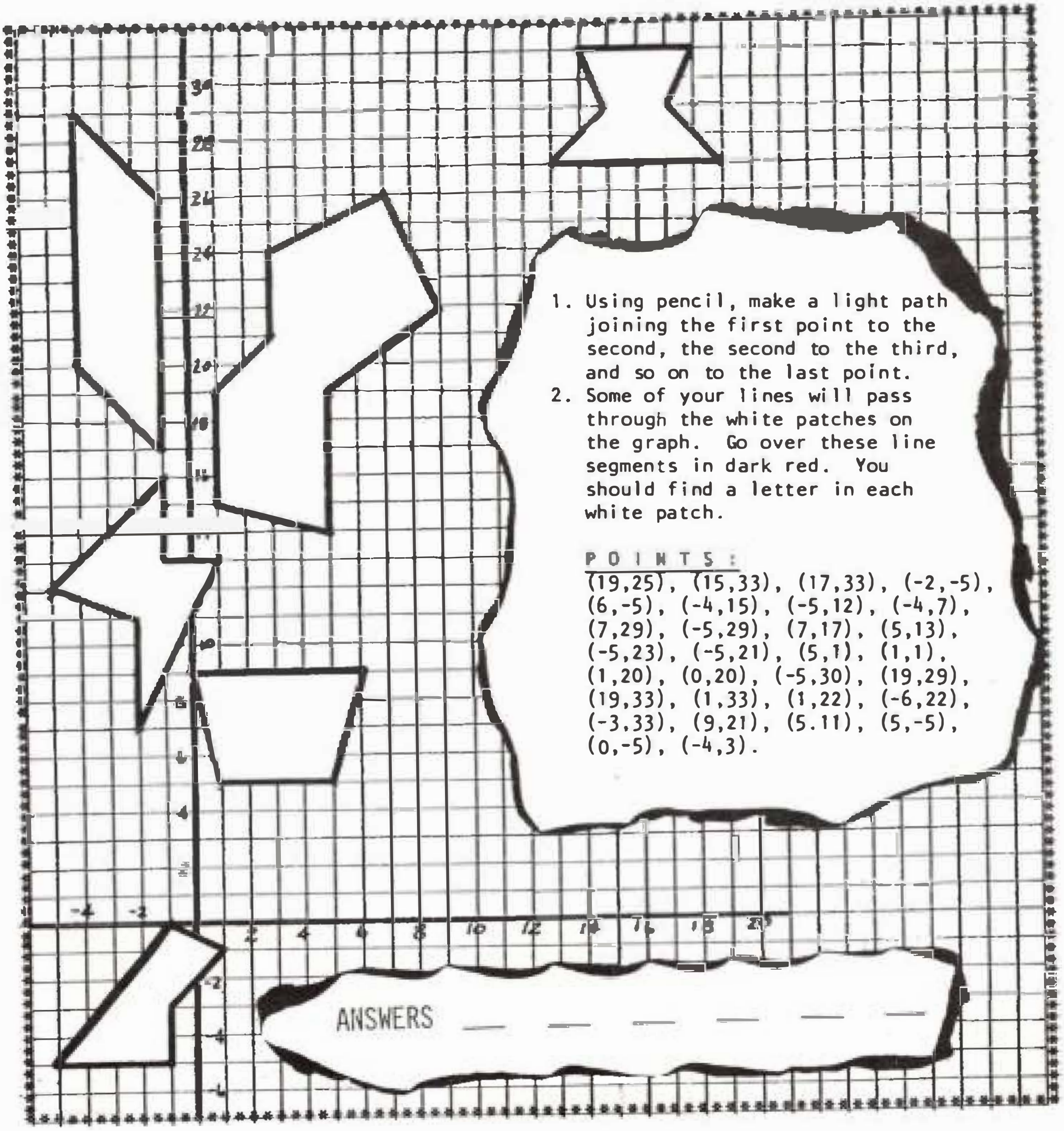
Color section BLUE when tens place is 4 or 5.

Color section RED when tens place is 6 or 7.

Color section GREEN when tens place is 8 or 9.



THESE PAGES CONTAIN 4 PUZZLES WHICH MUST BE SOLVED TO GET CERTAIN LETTERS. GET THE 14 LETTERS, UNSCRAMBLE THEM, AND DISCOVER THE SECRET MESSAGE.

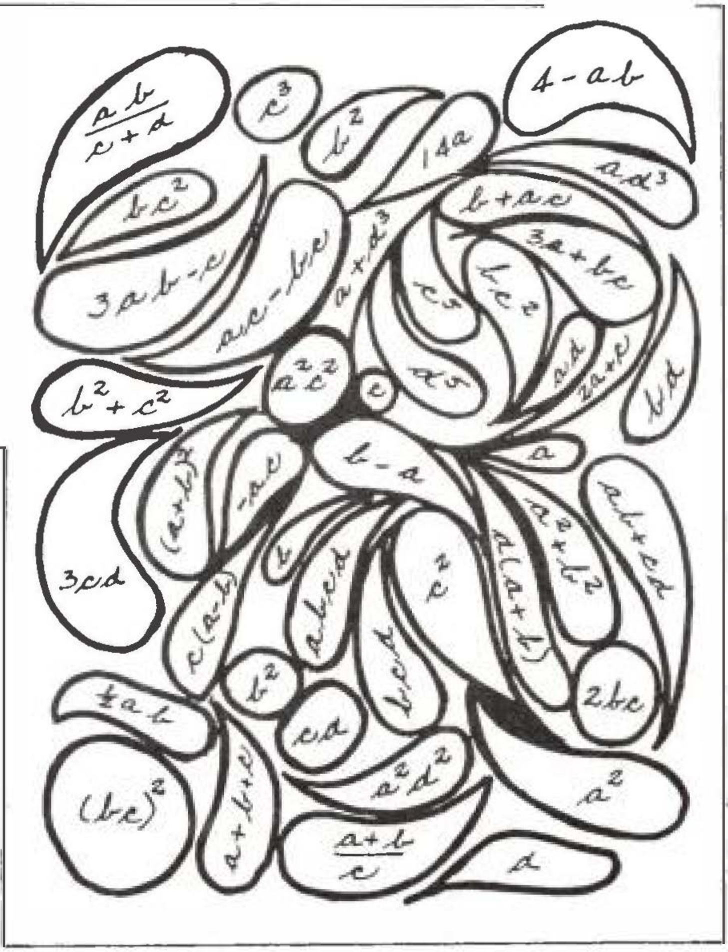


1. Using pencil, make a light path joining the first point to the second, the second to the third, and so on to the last point.
2. Some of your lines will pass through the white patches on the graph. Go over these line segments in dark red. You should find a letter in each white patch.

POINTS:
(19,25), (15,33), (17,33), (-2,-5), (6,-5), (-4,15), (-5,12), (-4,7), (7,29), (-5,29), (7,17), (5,13), (-5,23), (-5,21), (5,1), (1,1), (1,20), (0,20), (-5,30), (19,29), (19,33), (1,33), (1,22), (-6,22), (-3,33), (9,21), (5.11), (5,-5), (0,-5), (-4,3).

IN THE ACCOMPANYING DIAGRAM, USE $a = 1$, $b = 2$, $c = -1$, $d = 0$, AND BLACKEN EACH AREA WHOSE VALUE IS 1. THE RESULT SHOULD FORM A LETTER. THERE ARE THREE OF THIS LETTER IN THE SECRET MESSAGE.

ANSWER:_____



GIVEN: A = {a, b, c, d, e}
B = {a, c}
C = {b, d}
D = {a, d}
E = {c, e}

EVALUATE:

$B \cap [(A \cap E) \cup (D \cap C)]$

ANSWER:____.

ANSWER EACH OF THE FOLLOWING QUESTIONS CORRECTLY, AND ADD ALL OF YOUR ANSWERS. USE THE RESULTING NUMBER TO DETERMINE THE LETTERS FOR THE SECRET MESSAGE.

1. The 15th term of 1,4,9,16,25,... ____
2. How many prime numbers between 10 and 50? ____
3. The value of $(-3)^5$ is ____
4. Number of sides in a pentagon ____
5. Work out 111111^2. The sum of the digits of your answer ____
6. The only 2-digit number that is both a perfect square and a perfect cube ____
7. $a^2 \quad b^2 \quad c^2$ suggests someone's Theorem. How many letters in his name? ____

If your answer is 217 use s a p
.... 324 use c u t
.... 108 use s i t
.... 773 use b u g
.... 58 use p e a

ANSWER: ___ ___ ___

SECRET MESSAGE - TWO WORDS:

__ __ __ __ __

__ __ __ __ __ __ __ __ __

Problem-Solving Contest in *The Mathematics Student*

Beginning with the October, 1978 issue, *The Mathematics Student* will feature a "Competition Corner" (formerly "Problem Section"), which will include a continuous problem-solving contest. Each of the eight issues published during the school year will contain five problems of varying difficulty; each successful solution will carry a credit of 2-5 points. At the end of the school year, winners will be declared on the basis of cumulative scores and prizes will be awarded at each grade level. The overall winner of the contest will receive "The Polya Prize for Problem-Solving" as a special recognition.

Please encourage your student to take part in this highly educational endeavor. For further information, write to David Logothetti (Department of Mathematics, The University of Santa Clara, Santa Clara, CA 95053) or to George Berzsenyi (Department of Mathematics, Lamar University, Beaumont, TX. 77710). Subscription information and sample copies of *The Mathematics Student* are available from the NCTM Headquarters.

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