MATH 20 ASSIGNMENT: A Student's Answer

by *Linda Quon*
Mathematics 20 Student
Stirling High School
Stirling, Alberta

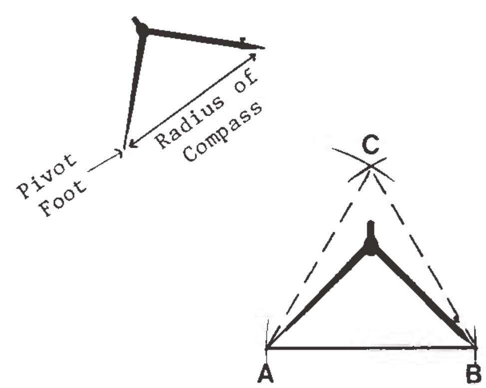
Part of our Math 20 assignment was to trisect a line segment using compass and straight edge only. As a typical teenager, I had plans on Friday evening, so as usual, homework was left until Sunday evening. Since my homework procedure was to do the easiest first, I had left the math assignment for the end.

I carefully thought over the hints given to us in class and tried to examine all the possibilities. Almost instantly I decided that *Postulate 23* was the answer. *Postulate 23* states that if a line is parallel to one side of a triangle and intersects the other sides at interior points, then the measures of one of those sides and the two segments into which it is divided are proportional to the measures of the three corresponding segments in the other side. The problem was how to apply it.

After short deliberation, I realized that *Postulate 23* did not apply. However, I did come up with the following solution as a result, which I applied to various other segments. As closely as I could measure, my method seemed to work and it was not until the next day that I found my answer to be only a close approximation and *not* a trisection.

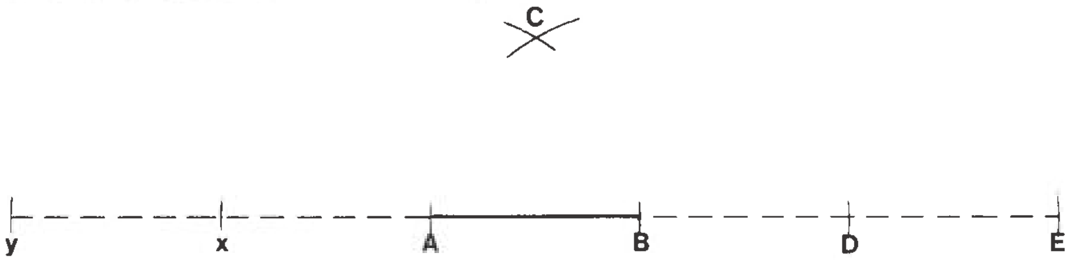
An Approximate Trisection of a Segment

1. Extend your compass so that its radius is equal to $m\overline{AB}$. Place the pivot foot on A and make an arc over AB as in the diagram. Now place the pivot foot on B and make an arc that will intersect the first arc. Call the point of intersection C. Notice, if you join A to C and B to C, you will have an equilateral triangle.
   NOTE: Once your compass has been set on d(A,B), *do not* adjust it throughout this entire process.

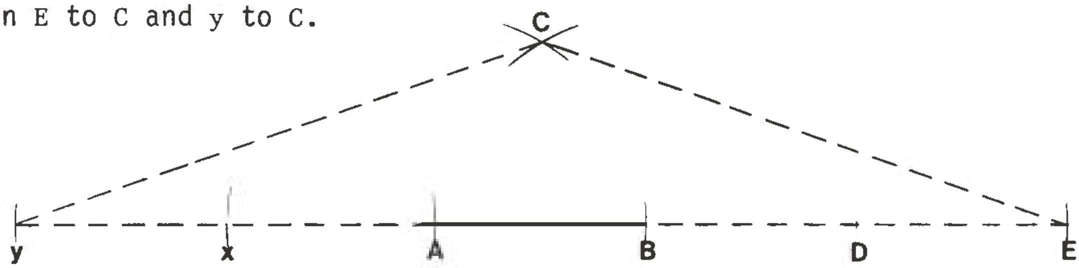


2. Extend $\overline{AB}$. With your compass pivot foot on B, intersect the line $\overleftrightarrow{AB}$ at D with an arc, move your pivot foot to D, and mark off another arc at E. Do the same on the other side.

   Now you have $\overline{yx} \cong \overline{xA} \cong \overline{AB} \cong \overline{BD} \cong DE$.

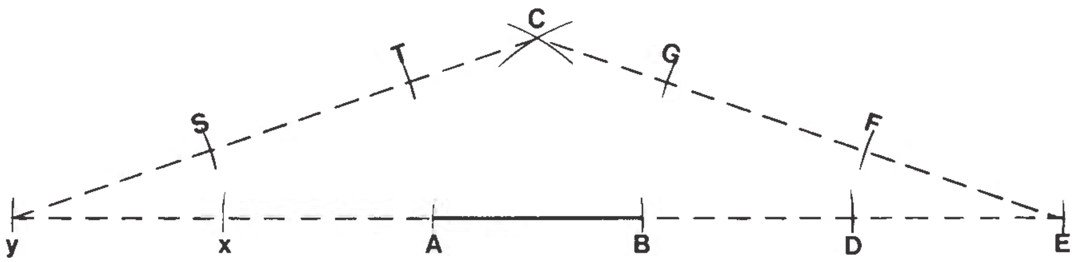


3. Join E to C and y to C.

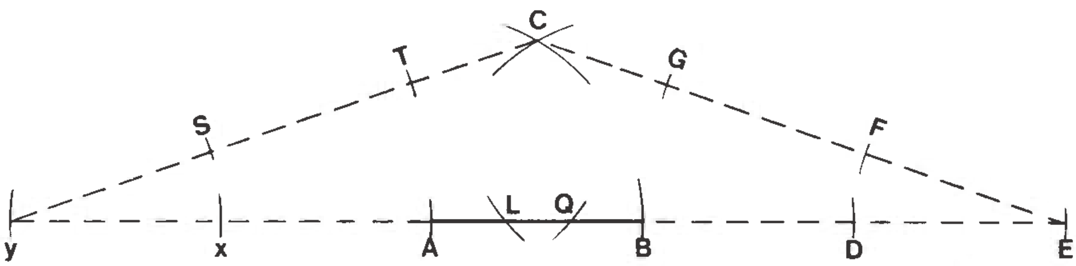


4. With pivot foot on E, make an arc that intersects $\overline{EC}$. Call it F. Now move your pivot foot to F and make an arc that intersects $\overline{FC}$. Call it G. Do the same on the other side.

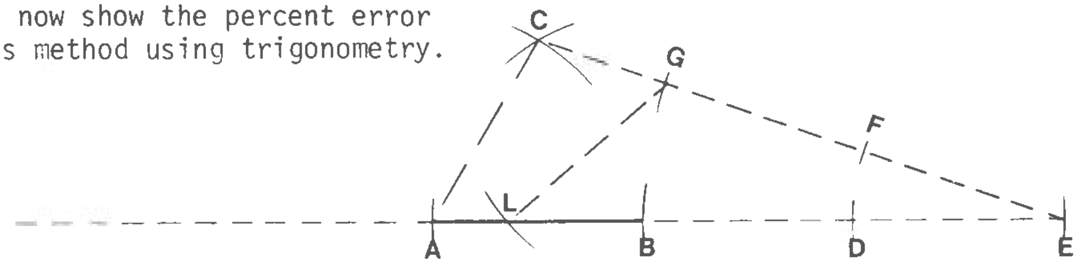
   Now you have $\overline{TS} \cong \overline{Sy} \cong \overline{yx} \cong \overline{xA} \cong \overline{AB} \cong \overline{BD} \cong \overline{DE} \cong \overline{EF} \cong \overline{FG}$.



5. With pivot foot on G, make an arc which intersects $\overline{AB}$. Call it L. Move your pivot foot to T, make another arc which intersects $\overline{AB}$. Call it Q. Your segment is now so close to being trisected that the naked human eye *cannot* detect that it isn't trisected.



Let us now show the percent error of this method using trigonometry.



Calculating Percent Error

Let $m\overline{AB} = 1$	
$\therefore m\overline{AB} = 1$	$m\overline{FG} = 1$
$m\overline{BD} = 1$	$m\overline{AC} = 1$
$m\overline{DE} = 1$	$m\overline{EG} = 2$
$m\overline{EF} = 1$	$m\overline{AE} = 3$

1. TO FIND $m\overline{CE}$:

$a^2 = b^2 + c^2 - 2bc \cos \cdot A^\circ$

$a^2 = (3)^2 + (1)^2 - (2 \cdot 3 \cdot 1 \cdot \tfrac{1}{2})$

$a^2 = 9 + 1 - (6 \cdot \tfrac{1}{2})$

$a^2 = 10 - 3$

$a^2 = 7$

$\boxed{m\overline{CE} = \sqrt{7}}$

2. TO FIND $m\measuredangle E$:

$$\frac{a}{\text{Sin } A^\circ} = \frac{b}{\text{Sin } B^\circ} = \frac{c}{\text{Sin } C^\circ} \text{ etc.}$$

$$\frac{\sqrt{7}}{\frac{\sqrt{3}}{2}} = \frac{1}{\text{Sin } E^\circ}$$

$$\frac{2\sqrt{7}}{\sqrt{3}} = \frac{1}{\text{Sin } E^\circ}$$

$$\text{Sin } E^\circ = \frac{\sqrt{3}}{2\sqrt{7}} = \frac{\sqrt{7}\cdot\sqrt{3}}{2\cdot 7} = \frac{\sqrt{21}}{14}$$

$\text{Sin } E^\circ = .32732684$

$E^\circ = 19.106605^\circ$

$\boxed{m\measuredangle E = 19.106605^\circ}$

3. TO FIND $m\measuredangle GLB$:

$$\frac{2}{\text{Sin GLB}^\circ} = \frac{1}{\frac{\sqrt{21}}{14}}$$

$$\text{Sin GLB}^\circ = \frac{2\sqrt{21}}{14} = \frac{\sqrt{21}}{7}$$

$\text{GLB}^\circ = 40.893395^\circ$

$\boxed{m\measuredangle \text{GLB}^\circ = 40.893395^\circ}$

4. TO FIND $m\measuredangle LGF$:

$m\measuredangle \text{GLB}^\circ + m\measuredangle E = 60^\circ$

$180^\circ - 60^\circ = 120^\circ$

$\boxed{m\measuredangle LGF = 120^\circ}$

5. TO FIND IF $m\overline{LE}$ IS REALLY $2\frac{2}{3}$:

$$\frac{m\overline{LE}}{\text{Sin } 120^\circ} = \frac{1}{\frac{\sqrt{21}}{14}}$$

$$m\overline{LE} = \frac{14}{\sqrt{21}} \text{ X Sin } 120^\circ$$

$$= \frac{14}{\sqrt{21}} \text{ X } \frac{\sqrt{3}}{2}$$

$$= \frac{7}{\sqrt{21}} \text{ X } \frac{\sqrt{3}}{1}$$

$$= \frac{7\sqrt{3}}{\sqrt{21}} = \frac{7\sqrt{3}\cdot\sqrt{21}}{21}$$

$$= \frac{\sqrt{3}\cdot\sqrt{21}}{3} = \frac{\sqrt{63}}{3} = \sqrt{7}$$

$\boxed{m\overline{LE} = 2.6457513}$

6. TO FIND PERCENT ERROR:

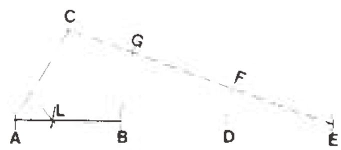
$$\%\ \text{error} = \frac{(2.\overline{666} - 2.6457513)}{2.6666666} \times 100$$

$$= \frac{.0209153}{2.6666666} \times 100 = .784324$$

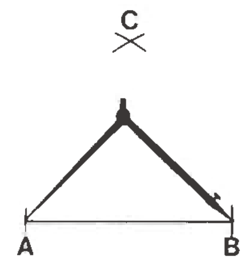
The percent error is .784324% - less than 1% error. L is about 8/1000 of a unit off the 1/3 mark!

A Shortcut

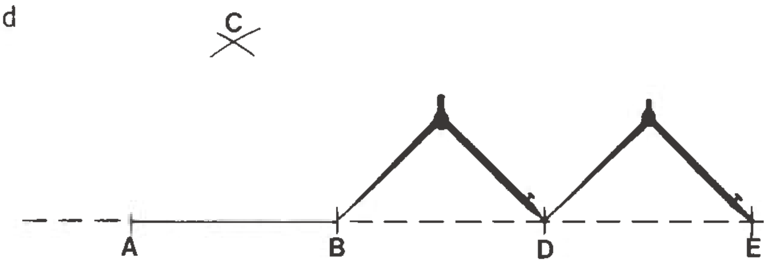
In this diagram, trigonometry has proven $\overline{LE} \cong \overline{CE}$. With that in mind, my method may be condensed to the following.



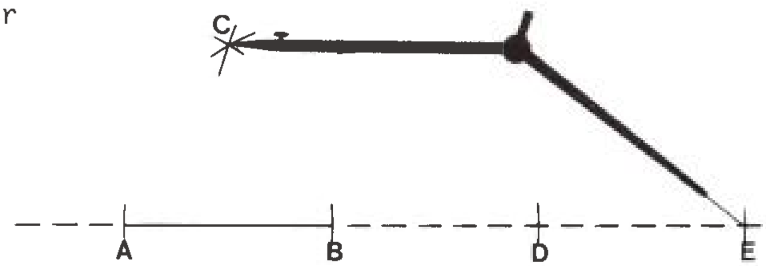
1. Measure d(A,B) with your compass. Mark off the arcs to make C.



2. Extend $\overleftrightarrow{AB}$. Measure off $\overline{BD}$ and $\overline{DE}$ so that $\overline{AB} \cong \overline{BD} \cong \overline{DE}$.



3. Extend your compass (with your pivot foot on E) to point C.



4. With the pivot foot still on E, move your compass so that it will make an arc that intersects $\overline{AB}$. That point of intersection should be point L as on the original diagram.