Basic Concepts in Geometry

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Geometry has been studied since approximately 3000 B.C. During this period of 5000 years, mathematicians have proposed an endless number of geometric theorems. As new knowledge has been gained and new ideas presented, some of these theories have been rejected while others have been reinforced. The work of early mathematicians has provided a foundation of knowledge that has enabled later generations to continue research at a more advanced level.

In 300 B.C. Euclid's *Elements* were presented. Euclid's findings were based upon his own research as well as research of previous mathematicians. After defining basic terms such as a point and line, Euclid stated five axioms and five postulates. Euclid's *Elements* formed the foundation of Euclidean geometry. More recent mathematicians have revised the axioms and postulates of Euclid to remedy the defects. These defects include assumptions that were not stated, unconvincing proofs, and the omission of important postulates. Euclidean geometry is the study of ordinary two- and three-dimensional spaces studied by Euclid or the study of Euclidean spaces in any number of dimensions.[1] Euclidean geometry is also called parabolic geometry.

Analytic geometry was developed by Pierre de Fermat (1601-1665) and René Descartes (1596-1650). They decided to represent points in space by ordered sets of numbers. These numbers are called the coordinates. Analytic geometry can be divided into two subgroups, plane analytic geometry and solid analytic geometry. Plane analytic geometry gives the coordinates of a geometrical figure in two dimensions, and solid analytic geometry gives the coordinates in three or more dimensions. Analytic geometry is also known as coordinate geometry.

In the 1600s the first work on projective geometry was done, but was abandoned in the 1700s because of other mathematical problems that had to be studied. In the early 1800s Gaspard Monge (1746-1818) and J.V. Poncelet (1788-1867) resumed work on projective geometry. This geometry was an expansion of Euclidean geometry with the axioms of parallelism and order deleted. Theorems of Pappus and Desargues were used as classical theorems of projective geometry. One of the new ideas in projective geometry was the concept of infinity. Projective geometry includes the properties of a configuration which are preserved by all projections and the theorems involving projective properties. Projective geometry uses analytic geometry to represent figures and indicate projective transformations. Affine geometry is a subset of projective geometry. In affine geometry all transformations are linear. Two subsets of affine geometry are similarity geometry and equiareal geometry.

[1] *Mathematics Dictionary*, James/James (eds.) Toronto, 1959. p.175.

Absolute geometry uses all of Euclid's axioms except the axiom of parallelism. The propositions for absolute geometry are valid for Euclidean and non-Euclidean geometries. Absolute geometry can be pure or synthetic. Saccheri geometry is another name for absolute geometry.

Descriptive geometry is the study of the relationships of points, lines, planes and other surfaces in space. Architects and engineers use this type of geometry because it enables them to represent three-dimensional objects in two-dimensional drawings. The first studies of descriptive geometry were done in the late 1700s by Gaspard Monge.

Non-Euclidean geometry is analogous to Euclidean except that Euclid's postulate of parallelism is replaced. When the postulate of parallelism is changed, then the theorems depending on this postulate must also be revised. One type of non-Euclidean geometry was proposed almost simultaneously by N.L. Lobachevsky (1826) and János Bolyai (1832). This new geometry, hyperbolic, replaced Euclid's postulate of parallelism with a postulate stating that through a point P, which is not on the line l, there is more than one line parallel to l. Another type of non-Euclidean geometry was formulated by Georg Riemann (1826-1866). Riemann's elliptic geometry replaced Euclid's postulate of parallelism with a postulate stating that through a point P, which is not on the line l, there are no lines parallel to l.

Differential geometry is the application of calculus to study curves and surfaces. This geometry was developed by Monge and Gauss (1777-1855) in the late eighteenth and early nineteenth centuries. They were able to determine the shortest distance between two points on a curved surface. This was named a geodesic distance.

Topology is the branch of geometry that deals with patterns involving position and relative position. Topology is not concerned with the magnitude of angles, distance, and area. Karl Gauss was one of the main initiators of topology in 1833. Topology is so fundamental that its influence is apparent in almost all branches of math.

The study of geometry began hundreds of years ago. From the beginnings of Euclid several branches of geometry have been formed. As mathematicians further pursue this type of math, new questions will arise and doubts about the validity of previous research will form. New types of geometry will then be formed to account for new findings.

Bibliography

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