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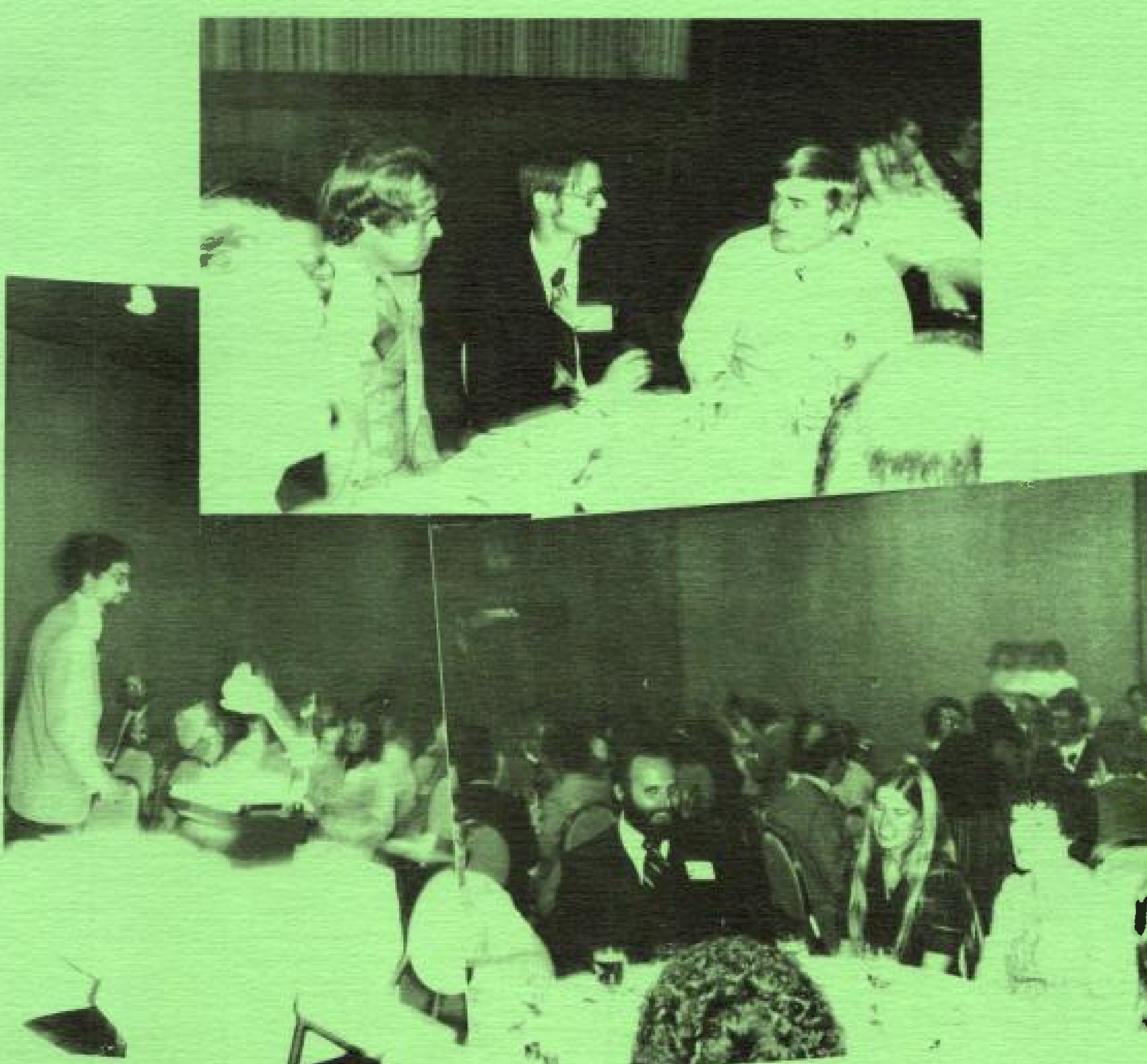
delta-k

THE ALBERTA TEACHERS' ASSOCIATION
MATHEMATICS COUNCIL

Volume XVIII, Number 3

March 1979



Scenes from the 18th Annual Conference
October 13 and 14, 1978

NATIONAL COUNCIL OF TEACHERS
OF MATHEMATICS

57th Annual Meeting

18-21 April 1979 **Boston, Massachusetts**

Program Highlights

250 Sessions
125 Workshops and worksessions
80 Short subjects
Serendipity of ideas
Marathon make-and-take workshop
Hands-on computer workshops
Information-sharing sessions—YOU and YOUR NCTM committees
Round-table research discussions
Commercial exhibits
Sale of NCTM publications
Reception

Featured Speakers

Richard Cortright
Martha Denckla
Jasper Harvey
Shirley Hill
Paul Szep

Topics to Be Emphasized

Mathematics for the exceptional child
Problem solving

Special Events

Presidential address
Opening and closing general session
Banquet

Sightseeing Opportunities

Museum of Science and the Hayden Planetarium
Plymouth Pilgrimage
The Witch City
Newport-by-the-Sea
Gloucester Harbor
Sturbridge Village
The Museum of Fine Arts
Greater Boston
Hood Sail Makers

Hotel rooms have been reserved in several Boston hotels and motels. Housing forms will be available in the complete program booklet. All convention activities will take place at the Sheraton-Boston Hotel and the John B. Hynes Veterans Auditorium. The program will begin on Wednesday, 18 April, at 8:00 P.M. and end on Saturday, 21 April, at 1:00 P.M.

The **complete program booklet** for the 57th Annual Meeting will be mailed in January to all NCTM members in the United States, U.S. Territories, and Canada. Additional copies may be requested from the NCTM Headquarters Office, 1906 Association Dr., Reston, VA 22091.

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THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

Volume XVIII, Number 3 — March 1979

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Mathematics Council Executive 1978-79

123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789

President
Robert Holt Res 462-1847
4255 - 74 Street Bus 434-0464
Edmonton T6K 0W1

Past President
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Dept. of Elem. Educ. Bus 432-4188
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Edmonton T6G 2G5

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Calgary T2J 3K8

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Lacombe T0C 1S0

delta-k Editor and
Publicity Chairman
Ed Carriger Res 843-6138
R.R. #1, Site 2, Box 3 Bus 843-6528
Bluffton T0C 0M0

Faculty of Education Rep.
and *Monograph* Editor
John Percevault Res 328-1259
Faculty of Education Bus 329-2111
University of Lethbridge ext. 257
Lethbridge T1K 3M4

Department of Education
Representative
Bruce Stonell Res 346-7814
Regional Office Bus 343-5262
of Education
4th Floor, 4814 Ross St.
Red Deer T4N 1X4

Mathematics Representative
Dr. Geoffrey J. Butler Res 435-1433
Department of Mathematics Bus 432-3988
University of Alberta
Edmonton T6G 2G1

ATA Representative
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Barnett House Bus 453-2411
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Edmonton T5N 2R1

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Okotoks T0L 1T0

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Director and NCTM Rep. Bus 429-5621
Edmonton Pub. School Bd. ext. 264
10010 - 107A Avenue
Edmonton T5H 0Z8

The Editor's Page

This issue of *delta-k* was unfortunately delayed due to a car accident I was involved in. Also, after a six-week rest at the U of A hospital, I felt that a holiday was in order, which should be ended by the time you read this.

During the Easter week holiday in April, I will be taking a trip to Boston, Mass., as NCTM representative for MCATA. The report on that trip will come with next September's *delta-k* and/or next October's business meeting.

Those of you who applied for membership in NCTM at the time of our fall conference may wonder why your memberships have not yet been acknowledged. Unfortunately the process has been slightly delayed because of an error in quoted rates on the part of MCATA executive. The error has now been corrected, and memberships are being handled at the time of this writing.

Remember - an NCTM "name of site" will be held in October, in Calgary, where we will host a meeting that attracts participants from western Canada and the northwestern United States and a few from more distant parts of the continent.

I am very pleased to report that this and the next edition of *delta-k* will be filled with articles that are original submissions. This is very encouraging for me after having to frequently use reprinted articles because of a lack of original material. Also, most or all of the activities are more universal in nature.

I recently received a copy of *Ergo*, a paper published by Athabasca University, 14515 - 122 Avenue, Edmonton T5L 2W4. The copy was sent to me as publicity chairman of MCATA. After perusing the paper, I would say that many of you who are interested in broadening your education into areas of interest for personal satisfaction would find this publication useful. Since there is no information in the paper concerning subscriptions, I must assume that a letter to *Ergo* would get you on the mailing list, and thus, information on how to further your education through "TV" and "Radio" courses. I recommend the paper to all who desire home study and have the time and initiative to pursue it.

Ed Carriger
Editor

Basic Concepts in Geometry

by *Joanne Newman* and *Ved P. Madan*
Red Deer College
Red Deer, Alberta

Geometry has been studied since approximately 3000 B.C. During this period of 5000 years, mathematicians have proposed an endless number of geometric theorems. As new knowledge has been gained and new ideas presented, some of these theories have been rejected while others have been reinforced. The work of early mathematicians has provided a foundation of knowledge that has enabled later generations to continue research at a more advanced level.

In 300 B.C. Euclid's *Elements* were presented. Euclid's findings were based upon his own research as well as research of previous mathematicians. After defining basic terms such as a point and line, Euclid stated five axioms and five postulates. Euclid's *Elements* formed the foundation of Euclidean geometry. More recent mathematicians have revised the axioms and postulates of Euclid to remedy the defects. These defects include assumptions that were not stated, unconvincing proofs, and the omission of important postulates. Euclidean geometry is the study of ordinary two- and three-dimensional spaces studied by Euclid or the study of Euclidean spaces in any number of dimensions.[1] Euclidean geometry is also called parabolic geometry.

Analytic geometry was developed by Pierre de Fermat (1601-1665) and René Descartes (1596-1650). They decided to represent points in space by ordered sets of numbers. These numbers are called the coordinates. Analytic geometry can be divided into two subgroups, plane analytic geometry and solid analytic geometry. Plane analytic geometry gives the coordinates of a geometrical figure in two dimensions, and solid analytic geometry gives the coordinates in three or more dimensions. Analytic geometry is also known as coordinate geometry.

In the 1600s the first work on projective geometry was done, but was abandoned in the 1700s because of other mathematical problems that had to be studied. In the early 1800s Gaspard Monge (1746-1818) and J.V. Poncelet (1788-1867) resumed work on projective geometry. This geometry was an expansion of Euclidean geometry with the axioms of parallelism and order deleted. Theorems of Pappus and Desargues were used as classical theorems of projective geometry. One of the new ideas in projective geometry was the concept of infinity. Projective geometry includes the properties of a configuration which are preserved by all projections and the theorems involving projective properties. Projective geometry uses analytic geometry to represent figures and indicate projective transformations. Affine geometry is a subset of projective geometry. In affine geometry all transformations are linear. Two subsets of affine geometry are similarity geometry and equiareal geometry.

[1] *Mathematics Dictionary*, James/James (eds.) Toronto, 1959. p.175.

Absolute geometry uses all of Euclid's axioms except the axiom of parallelism. The propositions for absolute geometry are valid for Euclidean and non-Euclidean geometries. Absolute geometry can be pure or synthetic. Saccheri geometry is another name for absolute geometry.

Descriptive geometry is the study of the relationships of points, lines, planes and other surfaces in space. Architects and engineers use this type of geometry because it enables them to represent three-dimensional objects in two-dimensional drawings. The first studies of descriptive geometry were done in the late 1700s by Gaspard Monge.

Non-Euclidean geometry is analogous to Euclidean except that Euclid's postulate of parallelism is replaced. When the postulate of parallelism is changed, then the theorems depending on this postulate must also be revised. One type of non-Euclidean geometry was proposed almost simultaneously by N.L. Lobachevsky (1826) and János Bolyai (1832). This new geometry, hyperbolic, replaced Euclid's postulate of parallelism with a postulate stating that through a point P, which is not on the line l, there is more than one line parallel to l. Another type of non-Euclidean geometry was formulated by Georg Riemann (1826-1866). Riemann's elliptic geometry replaced Euclid's postulate of parallelism with a postulate stating that through a point P, which is not on the line l, there are no lines parallel to l.

Differential geometry is the application of calculus to study curves and surfaces. This geometry was developed by Monge and Gauss (1777-1855) in the late eighteenth and early nineteenth centuries. They were able to determine the shortest distance between two points on a curved surface. This was named a geodesic distance.

Topology is the branch of geometry that deals with patterns involving position and relative position. Topology is not concerned with the magnitude of angles, distance, and area. Karl Gauss was one of the main initiators of topology in 1833. Topology is so fundamental that its influence is apparent in almost all branches of math.

The study of geometry began hundreds of years ago. From the beginnings of Euclid several branches of geometry have been formed. As mathematicians further pursue this type of math, new questions will arise and doubts about the validity of previous research will form. New types of geometry will then be formed to account for new findings.

Bibliography

Aldler, Claire F. *Modern Geometry*, 2nd ed. McGraw-Hill, 1967.

Fishback, W. T. *Projective & Euclidean Geometry*. John Wiley & Sons, 1969.

James/James. *Mathematics Dictionary*, 4th ed. New York, N.Y.: Van Nostrand Reinhold, 1976.

Rosenbaum, Robert A. *Introduction to Projective Geometry & Modern Algebra*. Reading, Massachusetts: Addison-Wesley, 1963.

Smart, J. R. *Modern Geometries*. Brooks/Cole Publishing, 1973.

BASEBALL STATISTICS: Examples of the Mean, Median, and Mode

by *Bonnie H. Litwiller* and *David R. Duncan*
Professors of Mathematics
University of Northern Iowa
Cedar Falls, Iowa

When students refer to "the average" of a set of data, they usually are describing either the mean (sum of the scores divided by the number of scores), the median (the middle score), or the mode (the most frequent score). Teachers are always searching for examples of data for which the mean, median, and mode present dissimilar meanings of average. For example, suppose an 80-year-old grandfather and 78-year-old grandmother invite their three grandchildren for lunch. They are the twins (Shawn and Heather, age 3) and Gary (age 6). The mean age of those present at lunch is 34, the median age is 6, and the mode is 3. Which of these measures of central tendency gives the best representation of "average"?

We attempted to find a real world situation in which the mean, median, and mode would be greatly different. We located the names of all major league baseball players whose careers were contained wholly in the years 1920 through 1974. For each of these players, we located the total number of home runs that he hit in his entire *regular season* career. Complete information concerning these and other major league baseball statistics can be found in the publication *The Baseball Encyclopedia* published in 1976 by the Macmillan Publishing Company.

We used 1920 as the starting date because of the common agreement that this was the first year that the "live ball" was used. Prior to that time, the ball that was used was very difficult to hit a great distance. This change was apparently made to make baseball more exciting. Consequently, we did not include a player whose career began before 1920, since he did not have the opportunity to hit the live ball for his entire career. We made one exception - we included Babe Ruth who was able to hit *any* type of ball.

We stopped with those players whose careers ended prior to or in 1974 because their career records were then complete in our reference book.

Also we did not include any player who was primarily a pitcher, since he would not have the same opportunity to bat as would players in other positions.

Table I shows the frequency distribution for all of the 3049 players that we described previously.

TABLE I

Total Number of Home Runs	Number of Players	Cumulative Frequency
0	1138	1138
1	290	1428
2	172	1600
3	95	1695
4	77	1772
5	56	1828
6	71	1899
7	43	1942

Total Number of Home Runs	Number of Players	Cumulative Frequency	Total Number of Home Runs	Number of Players	Cumulative Frequency
8	50	1992	84	3	2799
9	47	2039	85	2	2801
10	22	2061	86	3	2804
11	22	2083	87	4	2808
12	30	2113	88	1	2809
13	32	2145	89	5	2814
14	32	2177	90	3	2817
15	26	2203	91	3	2820
16	28	2231	92	1	2821
17	19	2250	93	7	2828
18	16	2266	94	4	2832
19	26	2292	95	4	2836
20	17	2309	96	2	2838
21	10	2319	97	1	2839
22	27	2346	98	4	2843
23	11	2357	99	1	2844
24	19	2376	100	2	2846
25	10	2386	101	4	2850
26	13	2399	103	5	2855
27	12	2411	104	5	2860
28	19	2430	105	4	2864
29	19	2449	106	6	2870
30	9	2458	107	1	2871
31	17	2475	108	3	2874
32	19	2494	109	1	2875
33	6	2500	110	2	2877
34	6	2506	111	1	2878
35	8	2514	112	4	2882
36	9	2523	113	1	2883
37	8	2531	114	1	2884
38	13	2544	115	3	2887
39	11	2555	116	2	2889
40	11	2566	117	3	2892
41	9	2575	119	3	2895
42	6	2581	120	1	2896
43	6	2587	121	1	2897
44	12	2599	122	1	2898
45	8	2607	124	1	2899
46	10	2617	125	3	2902
47	10	2627	126	4	2906
48	7	2634	127	2	2908
49	5	2639	128	1	2909
50	8	2647	129	2	2911
51	4	2651	130	3	2914
52	6	2657	131	3	2917
53	4	2661	132	2	2919
54	3	2664	133	2	2921
55	4	2668	134	2	2923
56	5	2673	135	3	2926
57	7	2680	136	1	2927
58	10	2690	137	1	2928
59	3	2693	138	1	2929
60	4	2697	139	1	2930
61	12	2709	141	1	2931
62	3	2712	142	1	2932
63	3	2715	147	1	2933
64	4	2719	148	2	2935
65	7	2726	149	1	2936
66	6	2732	151	1	2937
67	4	2736	152	1	2938
68	4	2740	153	1	2939
69	7	2747	154	1	2940
70	1	2748	155	2	2942
71	1	2749	156	1	2943
72	3	2752	159	2	2945
73	7	2759	162	1	2946
74	1	2760	163	2	2948
75	1	2761	164	5	2953
76	2	2763	166	1	2954
77	8	2771	167	2	2956
78	3	2774	169	1	2957
79	7	2781	170	3	2960
80	5	2786	172	2	2962
81	3	2789	173	1	2963
82	3	2792	178	2	2965
83	4	2796	179	3	2968

Total Number of Home Runs	Number of Players	Cumulative Frequency
180	1	2969
181	1	2970
183	2	2972
184	1	2973
186	1	2974
189	1	2975
190	2	2977
192	2	2979
194	1	2980
199	1	2981
200	1	2982
202	3	2985
205	1	2986
206	2	2988
210	1	2989
211	2	2991
213	1	2992
219	2	2994
223	1	2995
226	1	2996
228	2	2998
235	1	2999
236	1	3000
237	1	3001
238	1	3002
239	1	3003
240	1	3004
242	2	3006
244	1	3007
247	1	3008
248	1	3009
253	2	3011
256	1	3012
264	1	3013
266	1	3014
275	1	3015
277	1	3016
279	1	3017
282	1	3018
286	1	3019
288	3	3022
300	1	3023
307	1	3024
318	1	3025
331	1	3026
336	1	3027
342	1	3028
358	1	3029
359	1	3030
361	1	3031
369	1	3032
370	1	3033
374	1	3034
377	1	3035
379	1	3036
382	1	3037
399	1	3038
407	1	3039
475	1	3040
493	1	3041
511	1	3042
512	2	3044
521	1	3045
534	1	3046
536	1	3047
660	1	3048
714	1	3049

Mean = 23.91
Median = 2
Mode = 0
Standard Deviation = 57.98

Observe that the mean, median, and mode differ greatly. The standard deviation is more than twice as large as the mean. Clearly, these data would not describe the normal curve.

In the normal curve, 68 percent (approximately two-thirds) of the data points should be within one standard deviation of the mean. In this baseball example, the data points less than the mean all lie within 41 percent of a standard deviation of the mean, while 2789 of the 3049 data points are within one standard deviation of the mean. This means that 91.5 percent of all the data points are within one standard deviation of the mean.

To better visualize these data, let the height of each player be proportional to the total number of home runs that he hit in his career. Also let the mean number of home runs represent a player whose "height" is 6 feet.

Suppose that all 3049 baseball players line up according to "height" (shortest to tallest) and begin running out of a dugout at the rate of one per second. If the first player runs out of the dugout at 1:00 p.m., the last player will appear at 1:50:48 p.m. (48 seconds after 1:50 p.m.)

We assume that for any given number of home runs, the players in that category run out in alphabetical order. The first player in line is Jimmy Adair (Chicago Cubs, 1931) who hit no homeruns in his brief 18-game career as a shortstop. He did however bat .276 in 76 times at bat. Adair's height is 0 feet.

For several minutes the players are of height 0. The last such player in this category is Frank Zupo (Baltimore Orioles, 1957-61). Zupo was a little used catcher, as he appeared in 16 games and had 18 times at bat in his career. He appears at 1:18:57.

At 1:18:58 (58 seconds after 1:18), the first player whose height is greater than 0 appears. He is Joe Abreu (Cincinnati Reds, 1942). He played in 9 games and hit one home run. His height is .25 feet or 3.0 inches.

Luiz Alcaraz (Dodgers and Royals, 1967-70) who hit 4 home runs appears at 1:28:15. His height is one foot and he follows the "median player" by only 2 minutes and 51 seconds. The median player, Johnny Neun (Tigers and Red Sox, 1925-31) hit 2 home runs and is 6.0 inches tall. Observe that this small height represents the median in a group of baseball players whose "mean height" is 6 feet. Consequently, over half of the baseball players are 6 inches or less in "height" - a short group of baseball players!

The first player whose height is greater than or equal to -

1. 2 feet is Ruben Amaro, who hit 8 home runs and appears at 1:32:22.
2. 3 feet is Babe Barna, who hit 12 home runs and appears at 1:34:43.
3. 6 feet (the mean) is Pete Castiglione, who hit 24 home runs and appears at 1:39:17. With only 11 minutes and 31 seconds remaining of the 50 minutes and 48 seconds that it takes for the group to run from the dugout, the first player taller than the mean height appears.
4. 10 feet is Ed Coleman, who hit 40 home runs and appears at 1:42:35.
5. 20 feet is Bobby Avila, who hit 80 home runs and appears at 1:46:21.
6. 50 feet is Don Mincher, who hit 200 home runs and appears at 1:49:41.

There are 12 players whose heights exceed 100 feet. They are:

1. Al Kaline, who hit 399 home runs. He is 100 feet 2 inches in height and appears at 1:50:37.
2. Duke Snider, who hit 407 home runs. He is 102 feet 2 inches in height and appears at 1:50:38.
3. Stan Musial, who hit 475 home runs. He is 119 feet 2 inches in height and appears at 1:50:39.
4. Lou Gehrig, who hit 493 home runs. He is 123 feet 9 inches in height and appears at 1:50:40.
5. Mel Ott, who hit 511 home runs. He is 128 feet 3 inches in height and appears at 1:50:41.
6. Ernie Banks, who hit 512 home runs. He is 128 feet 6 inches in height and appears at 1:50:42.
7. Eddie Mathews, who hit 512 home runs. He is 128 feet 6 inches in height and appears at 1:50:44.
8. Ted Williams, who hit 521 home runs. He is 130 feet 9 inches in height and appears at 1:50:44.
9. Jimmie Foxx, who hit 534 home runs. He is 134 feet in height and appears at 1:50:45.
10. Mickey Mantle, who hit 536 home runs. He is 134 feet 6 inches in height and appears at 1:50:46.
11. Willie Mays, who hit 660 home runs. He is 165 feet 7 inches in height and appears at 1:50:47.
12. Babe Ruth, who hit 714 home runs. He is 179 feet 2 inches in height and appears at 1:50:48.

(Hank Aaron would be even taller, but he retired after our cut-off date.)

It is interesting to observe the heights of players who appear at specific times. Table II displays this information.

TABLE II

Time	Player	Number of Home Runs	Height
1:00	Jimmy Adair	0	0
1:05	Bobby Floyd	0	0
1:10	Red Lutz	0	0
1:15	Norm Schlueter	0	0
1:20	Ox Eckhardt	1	3.0 in.
1:25	Ty La Forest	2	6.0 in.
1:30	Joe Lafata	5	1 ft. 3 in.
1:35	Lloyd Merriman	12	3 ft.
1:40	Horace Clarke	27	6 ft. 9 in.
1:45	Billy Johnson	61	15 ft. 4 in.
1:46	Sammy West	75	18 ft. 10 in.
1:47	Jim Hegan	92	23 ft. 1 in.
1:48	Paul Waner	112	28 ft. 1 in.
1:49	Ed Bailey	155	38 ft. 11 in.
1:50:00	Gus Zernial	237	59 ft. 6 in.
1:50:10	Joe Gordon	253	63 ft. 7 in.
1:50:20	Bob Johnson	288	72 ft. 3 in.
1:50:30	Joe DiMaggio	361	90 ft. 7 in.
1:50:35	Orlando Cepeda	379	95 ft. 1 in.
1:50:40	Lou Gehrig	493	123 ft. 9 in.
1:50:45	Jimmie Foxx	534	134 ft.
1:50:46	Mickey Mantle	536	134 ft. 6 in.
1:50:47	Willie Mays	660	165 ft. 7 in.
1:50:48	Babe Ruth	714	179 ft. 2 in.

Suggestions for the reader and his/her class:

1. Calculate a similar set of data for basketball players. For example, calculate the "heights" of basketball players using their total numbers of points.

2. Calculate the heights of players on your high school or favorite college baseball team.

3. Calculate the heights of baseball players using other statistics such as total hits or stolen bases.

4. Look for settings other than sports which give rise to "badly skewed" data. (The median is very close to one end of the range.) Published salaries of public institutions may be interesting to investigate for this purpose.

John Janzen Nature Centre Discovery Activities

The Nature Centre offers activity-oriented programs that change through the year with a winter snowshoeing emphasis, spring and early-summer birth and growth-of-life emphasis, and a fall early-winter preparing-for-the-cold-season emphasis.

These programs are designed to integrate science, language arts, outdoor physical education, mathematics and social studies into one all-encompassing fun-filled two-hour session.

The staff of the Nature Centre has prepared pre- and post-visitation kits which include a variety of activities that teachers can use in the classroom and the out-of-doors before and after their visit to the Nature Centre. These kits are sent to each class making a reservation for a program.

Program fees are presently $20 per class for a two-hour session at and around the Nature Centre.

Career Opportunities in Mathematics and Statistics

by *J. G. Timourian*
Department of Mathematics
University of Alberta
Edmonton, Alberta

Students with ability in mathematics are avoiding the subject once they leave high school, the main reason for this being that they do not recognize the wide variety of careers that training in mathematics and statistics can lead to.

The Mathematics Department at the University of Alberta is eager to publicize the opportunities our programs offer. The following sections on "Careers in Mathematics" and "Careers in Statistics" were prepared for the University Student Counselling Service. If you would like a representative of our Department to speak to your students or staff on career opportunities and the programs we offer, please call Myrna Janzen at 432-3396.

Careers in Mathematics

Description of Subject Field

Many people do not realize how widely mathematics is used and, as a result, ignore mathematics courses in their training. Later they find themselves severely handicapped in their careers.

Mathematics is extremely important in modern society. It is as useful as ever in engineering and physics, and has become indispensable in economics, business, biology, medicine and many other fields. For example, law schools find that education in mathematics is good training for their students. Each year the University of Alberta Mathematics Department must accommodate more students whose programs require more mathematics than was required in the past.

Unfortunately, there seems to be an educational and intellectual barrier to learning mathematics after being away from it (very much like the barrier that makes it harder for an adult to learn a foreign language than it is for a child). As a result, people who are trained in mathematics and are knowledgeable in another field are in constant demand. In addition, the number of students with above-average ability in mathematics is not large. Therefore, persons with advanced training in mathematics find themselves even more in demand.

Mathematics can be regarded in three ways:

1. It is a discipline which insists on careful and logical thinking. A mathematics major develops mental habits which are extremely important for success in many fields. This is one reason why students trained in mathematics

do so well in areas far removed from it, like law, for instance.

2. Mathematics is a tool which can be used to recognize and solve problems. Most students are attracted to the subject (or are required to take it) because it has been applied so successfully in so many different areas.

3. Mathematics is an art form as beautiful and creative as painting, chess, or poetry. New forms of mathematics are constantly being created. Often it is only later that the newly invented forms are found to be useful in solving significant problems in the real world.

The University of Alberta offers several different, flexible programs in mathematics designed to appeal to a wide range of students.

Preparation (High School and Other)

The more mathematics you take in high school, the better. To enter any of these programs you must complete Mathematics 30, and you would be wise to take Mathematics 31 also. You should check the requirements for admission to the Faculty of Science in the current University Calendar.

The Department also offers a B.A. degree in the Faculty of Arts. Details on admission requirements to this program can be obtained from the Faculty of Arts section of the Calendar.

University Instruction

A *Specialization* degree in mathematics takes four years and includes at least eight courses in mathematics and statistics. You may choose twelve other courses to satisfy your interests and faculty requirements.

This is the degree program that provides flexibility for the student who is interested in mathematics and has a strong interest in another field as well. It is an excellent program for the student who wants to apply mathematics in business, economics, computing science, statistics, engineering or education, or who wants to attend graduate school in any of these subjects.

The Department also offers a specialization program with concentration in actuarial science designed to train actuaries and to prepare students for graduate work in Faculties of Commerce.

The *Honors* degree in mathematics is an intellectually-rewarding program that usually contains at least ten honors-level courses in mathematics or statistics. These courses are more difficult than those normally taken by non-honors students.

Although it requires more mathematics than the specialization program, this program is still flexible enough for the student to design a program to fit his or her particular career interest. Past honors graduates have been very successful in a wide range of careers. They include professors, doctors, lawyers, economists, computing scientists, statisticians and businessmen.

The three-year B.Sc. degree with concentration in mathematics can be used to obtain a broad liberal arts education, or training for a career in teaching, or it can be used for further studies in another area. Many students who originally enter this program eventually change to the specialization or honors program as they learn how useful it is to take more basic courses in mathematics and statistics.

Career Opportunities

Training in mathematics is useful for a wide variety of careers. Here are some examples:

TEACHING (Secondary)

No matter what your career interest, from agriculture to pharmacy, mathematics is important. This means that there will always be a demand for people with ability in mathematics to teach the subject.

Since most secondary pupils will be learning mathematics for use as a tool, students interested in education can enhance their opportunities for a teaching career by choosing options in a degree program that cover areas in which mathematics is applied. Students in mathematics programs may choose up to four options in the Education Faculty. A student can obtain a teaching certificate by studying in the Education Faculty after receiving a degree in mathematics.

ACTUARIAL PROFESSION

Actuaries are specialists at designing pension and insurance plans. They are employed by governments, labor unions, corporations and other large institutions.

Before becoming an actuary, a person must pass exams set by the Society of Actuaries. The Department offers mathematics of finance courses specially designed to prepare students for the actuarial exams. Since actuaries must be well trained in mathematics and mathematics of finance, they are also in demand for general executive positions.

UNIVERSITY AND COLLEGE TEACHING

A university teaching position usually requires a research-oriented Ph.D., while college teaching requires a teaching-oriented Ph.D. or a strong M.Sc. The proper training for teaching at these levels is a degree in mathematics followed by graduate study in the desired specialty, such as mathematics, statistics, management science, computing science, economics, operations research, physics or engineering.

It is difficult to predict now what job opportunities will be open in college or university teaching eight or more years from now. Currently, mathematicians with Ph.D. degrees in computing science, operations research, or statistics are in very short supply. Those trained in management science, economics, or numerical analysis also have plenty of job opportunities today. A few years ago, mathematicians with research interests in geometry or algebra were in demand.

INDUSTRY

Mathematicians working in industry are either employed in a general executive capacity (hired because of their ability to think logically) or are actually applying what they know to solve problems in such areas as research, design, or marketing. Students interested in an industrial career should concentrate their options in economics and other business-related courses, in statistics and computing science, or in some area of physics and engineering.

GOVERNMENT

Most mathematicians employed by the government are specialists in statistics (especially with computing) or have concentrated their options in economics and management science. There are also opportunities for applied mathematicians trained in physics and engineering with such government organizations as the National Research Council, Alberta Energy Corporation, and the Defense Research Board.

LAW

You may be surprised to find a profession such as this under career opportunities in mathematics. But an actuary who also has a law degree, for example, is in a powerful position when it comes to employment. A person who is able to combine training in mathematics, statistics, and computing science with a law degree is in a position to help solve the new legal problems which arise from our sophisticated technological age.

For more detailed information on programs available in the Mathematics Department, please write to:

Professor Murray Klamkin, Chairman
Department of Mathematics
University of Alberta
Edmonton T6G 2G1

(403) 432-3396

Careers in Statistics

Description of Subject Field

The best way to define statistics is to consider what the statistician does and why.

Originally, statisticians studied characteristics of people in a population, classifying them according to such criteria as age, marital status, and sex. They then summarized these data in the form of tables, charts, and graphs in order to make comparisons. The result was a general *description* of the population.

As statistics developed, many of the words used in population studies came to be used in a much wider context. For example, a *population* can be any collection of objects or ideas about which information is required. Today's statistician may study a population of shoes, ships, or of sealing waxes; cabbages, kings, or even of political opinions.

In a great many experiments, the results are influenced by extraneous factors which cannot be completely eliminated or controlled. The effects of these factors are usually of least importance in the physical sciences where rigid control can be maintained under laboratory conditions. They are of greater importance in the biological sciences, and of greatest importance in the social and behavioral sciences where it is often impossible to impose any control at all. The statistician is concerned with the haphazard or *random* variations resulting from such factors.

EXAMPLES:

- Four observations were made on the electron charge. The results in coulombs were 1.60203×10^{-19}, 1.60206×10^{-19}, 1.60209×10^{-19}, 1.60207×10^{-19}.
- One hundred grains of wheat were planted in each of four plots of earth. The numbers germinating were found to be 76, 82, 79, 80.
- Four students were given an intelligence test. Their I.Q.s were found to be 84, 143, 112, 106.

You will notice in the first example that there was relatively little variability in the measurements of electron charge.

In the second example, where less rigid control was possible, the variability is much larger and you might think about possible reasons for the variation. These could include variations in soil fertility, in light and shade, in moisture or in the genetic material of the wheat grains.

The third example, where virtually no control is possible, shows the greatest variability. Sources of variation here could include difference in sex, genetic background, cultural and physical environment, or even in what the students had for breakfast.

In studying such "chance phenomena," the statistician tries to design experiments to eliminate, minimize, or balance the effects of the extraneous factors. The results of the experiments are used to make inferences about the unknown situation under study.

A report on work done by a statistician would typically include an estimate of the reliability of the conclusions. For example, you might estimate that on the average, 80 percent of a certain type of wheat grain germinate and that the true (unknown) germination rate is almost certainly between 78 and 82 percent.

No summary of modern statistics would be complete without mention of the central role played by the Mathematical Theory of Probability. Without it, statistics would not have developed beyond the descriptive stage. Today's statistician wants to be able to make statements like "I am 95 percent confident that the average height of Slobovians is between 65 and 67 inches." Here's where the Probability Theory comes into play. With it, the statistician can assess the reliability of experimental results.

Statistical procedures have been devised even for measuring the risks of making the wrong conclusions!

University Instruction in Subject Field

Before World War II, most statisticians entered the field because they were interested in solving problems which arose in other areas they were experts in.

Noted examples are the careers of statisticians R.A. Fisher, who started as a schoolmaster with an interest in genetics, H. Hotelling, who began his career as an economist, and J. Tukey, who was originally a mathematician.

In recent years, statistics has developed rapidly as a discipline in its own right.

At the University of Alberta, statisticians find their home in the Department of Mathematics. Applied statistics is also taught by a number of other departments and faculties including agriculture, business, computing science, education, engineering and psychology.

Degree programs offered by the Department of Mathematics include a four-year honors B.Sc. program in mathematical statistics and a four-year B.Sc. program with specialization in statistics, as well as graduate degrees at the master's and doctoral levels.

The honors program stresses the mathematical and theoretical foundations of statistics and is designed mainly for students going into academic or research careers.

The specialization program, while giving a sound theoretical foundation, is directed toward students who would apply statistical methods in various other fields. This program provides students with a wide range of career choices.

To enter either the honors or specialization programs, you must take Physics 30 or Mathematics 31 in high school. You can check other requirements for admission to the Faculty of Science in the current University Calendar.

Career Opportunities

Statistics can be applied in such a wide range of fields that it is impossible to list them all. The following broad categories may offer some guidance:

GOVERNMENT

Since the days of Imperial Rome and probably even before, governments have found it necessary to collect statistical data on population, agricultural production, and trade for the purposes of levying taxes, conscripting armies, ensuring food supply, organizing elections and enforcing laws. With the increasing complexity of government planning, data piles up at an accelerated pace. In this country, Statistics Canada devotes itself exclusively to collecting, summarizing, and interpreting data on an ever increasing range of topics. Just glance at the Canada Year Book and you will be amazed at the great scope of these activities - and that's just on the national level. Provincial and municipal governments also collect, summarize, and publish statistics.

Population and technology are expanding so rapidly that governments must note trends as early as possible so that they can plan efficiently for the future. For example, knowledge of the 1979 birth rate is essential to the planner of elementary educational facilities for 1985, to the planner of secondary school facilities for 1993, and to the university planners for 1997.

In the past, governments have tried to obtain *complete* data on each individual item of interest. Thus, in the federal census, the aim is to obtain information on *every* individual in the population.

In other areas, the enormous cost of such a study is prohibitive. So statisticians must resort to statistical inference from appropriate samples in order to arrive at conclusions about a population. This introduces problems of sampling. How should a sample be selected so that it is representative of the population? How large should it be so that the conclusions have a prescribed reliability? These and other sampling problems are the subject of serious study by statisticians.

As well as the economic advantage of using statistical inference from samples, there is often an important saving in time. This was illustrated during World War II when Allied statisticians had to estimate German industrial output. They based their estimates on studies of the serial numbers of captured German equipment. After the war, detailed study showed that these estimates were as accurate as those made by the Germans themselves. Furthermore, the Allied estimates were available considerably sooner since they were based on sampling methods, while the Germans waited for complete coverage.

BUSINESS

Statistical methods find almost unlimited scope for application in business and finance. Many corporations employ statisticians to study consumer preference, inventory analysis, and quality control and to predict business cycles and trends.

For example, a telephone company needed to determine the value of its capital equipment such as poles, cables, batteries, tools and buildings. It had lists of these things and knew their original and replacement costs. But what percentage of their useful life still remained? This was costly to estimate because many items were in remote areas or inaccessible positions or because the evaluation required highly-paid experts, or dismantling equipment and interrupting service. Furthermore, the number of items was very large.

Instead of examining every piece of equipment, which would have been prohibitively costly, the company chose a relatively small sample of each type of equipment according to statistical principles. The average condition of each kind of item was then estimated from the samples and an allowance was made for possible errors due to sampling. The results were as useful as if a full examination had been made.

SCIENCE

Much of modern statistical theory grew out of attempts to solve certain problems in experimental agriculture. Plant and animal breeding experiments, genetic studies, comparative studies on fertilizers, experiments on animal nutrition and countless other studies have led to highly refined and efficient techniques for designing and analyzing experiments.

In the health sciences, researchers use statistical methods in their search for the causes of disease and in the evaluation of new forms of therapy. The extensive statistics on Salk Polio Vaccine and on smoking and lung cancer are well-known examples. Stochastic (random) models have been designed to describe such phenomena as epidemics, the reproduction of elementary organisms, and the movements of spermatozoa. Statistical methods are regularly used in dose-response studies of drugs.

In the social and behavioral sciences too, the range of application of statistics is very wide. Statistical models have been devised to describe such things as the learning behavior of rats, the formation of cliques in human societies, and dominance behavior in primates. In the fields of intelligence, personality, and aptitude testing, statistical analysis has long played an important role.

These examples show only a few of the applications of statistical methods in science.

The Employment of Statisticians

Currently there is a shortage of qualified statisticians in North America. Statisticians are well aware of the demand for their services and many set themselves up in private practice as consultants to government and industry. In this way, a hard-working statistician in a large centre can easily earn an income of $100,000 a year or more.

In the United States, about 40 percent of all statisticians are employed in business or industry, about 30 percent in government, and about 20 percent in academic posts or research centres and the remainder in miscellaneous positions. Comparable figures are not available for Canada, but almost certainly the number of statisticians in government or in academic posts exceeds the number in business and industry. Thus, statisticians will find even more career opportunities in the world of commerce in Canada.

For more detailed information on programs available in statistics, please write to:

Professor Murray Klamkin, Chairman
Department of Mathematics
University of Alberta
Edmonton T6G 2G1

(403) 432-3396

MATH 20 ASSIGNMENT: A Student's Answer

by *Linda Quon*
Mathematics 20 Student
Stirling High School
Stirling, Alberta

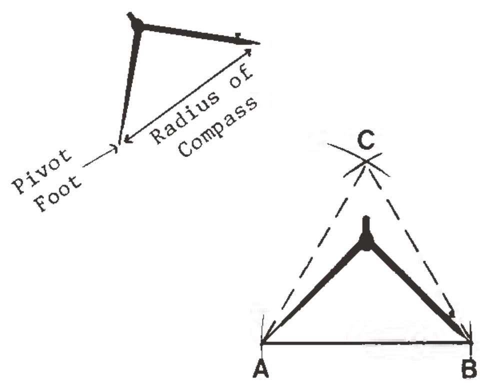
Part of our Math 20 assignment was to trisect a line segment using compass and straight edge only. As a typical teenager, I had plans on Friday evening, so as usual, homework was left until Sunday evening. Since my homework procedure was to do the easiest first, I had left the math assignment for the end.

I carefully thought over the hints given to us in class and tried to examine all the possibilities. Almost instantly I decided that *Postulate 23* was the answer. *Postulate 23* states that if a line is parallel to one side of a triangle and intersects the other sides at interior points, then the measures of one of those sides and the two segments into which it is divided are proportional to the measures of the three corresponding segments in the other side. The problem was how to apply it.

After short deliberation, I realized that *Postulate 23* did not apply. However, I did come up with the following solution as a result, which I applied to various other segments. As closely as I could measure, my method seemed to work and it was not until the next day that I found my answer to be only a close approximation and *not* a trisection.

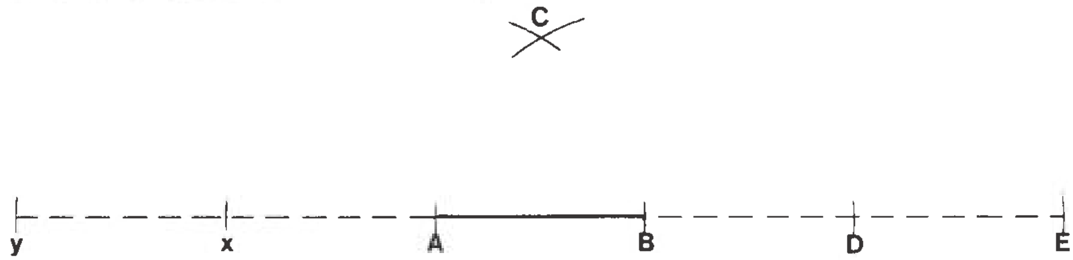
An Approximate Trisection of a Segment

1. Extend your compass so that its radius is equal to $m\overline{AB}$. Place the pivot foot on A and make an arc over AB as in the diagram. Now place the pivot foot on B and make an arc that will intersect the first arc. Call the point of intersection C. Notice, if you join A to C and B to C, you will have an equilateral triangle.
   NOTE: Once your compass has been set on d(A,B), *do not* adjust it throughout this entire process.

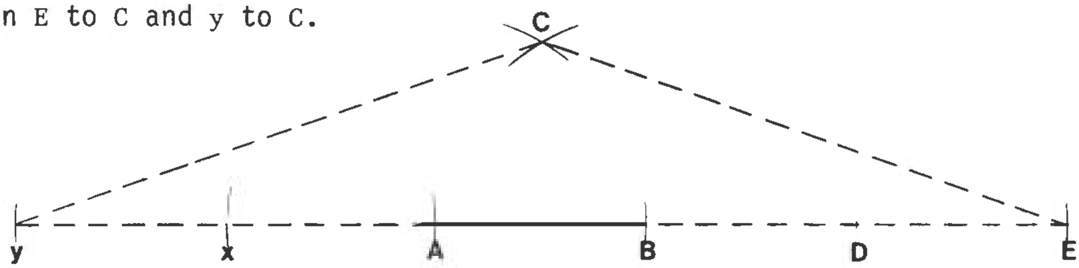


2. Extend $\overline{AB}$. With your compass pivot foot on B, intersect the line $\overleftrightarrow{AB}$ at D with an arc, move your pivot foot to D, and mark off another arc at E. Do the same on the other side.

   Now you have $\overline{yx} \cong \overline{xA} \cong \overline{AB} \cong \overline{BD} \cong DE$.

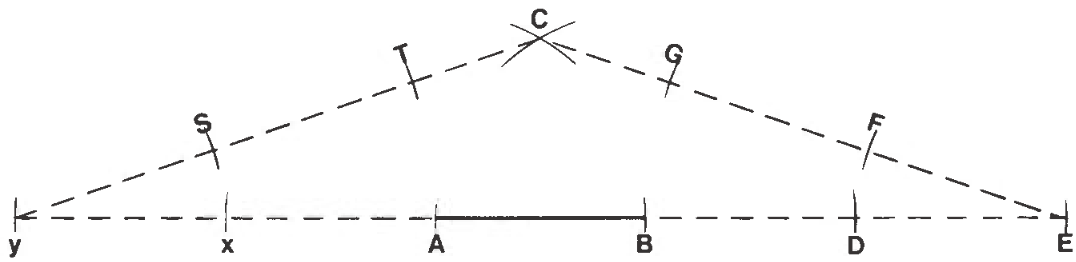


3. Join E to C and y to C.

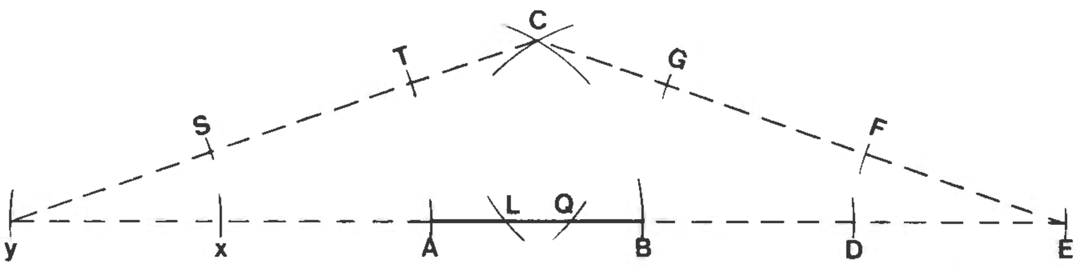


4. With pivot foot on E, make an arc that intersects $\overline{EC}$. Call it F. Now move your pivot foot to F and make an arc that intersects $\overline{FC}$. Call it G. Do the same on the other side.

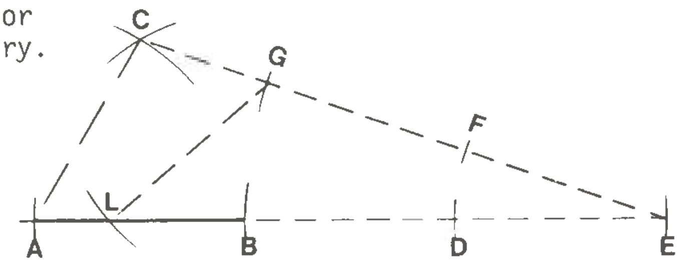
   Now you have $\overline{TS} \cong \overline{Sy} \cong \overline{yx} \cong \overline{xA} \cong \overline{AB} \cong \overline{BD} \cong \overline{DE} \cong \overline{EF} \cong \overline{FG}$.



5. With pivot foot on G, make an arc which intersects $\overline{AB}$. Call it L. Move your pivot foot to T, make another arc which intersects $\overline{AB}$. Call it Q. Your segment is now so close to being trisected that the naked human eye *cannot* detect that it isn't trisected.



Let us now show the percent error of this method using trigonometry.



Calculating Percent Error

Let $m\overline{AB} = 1$

$\therefore m\overline{AB} = 1$	$m\overline{FG} = 1$
$m\overline{BD} = 1$	$m\overline{AC} = 1$
$m\overline{DE} = 1$	$m\overline{EG} = 2$
$m\overline{EF} = 1$	$m\overline{AE} = 3$

1. TO FIND $m\overline{CE}$:

$$a^2 = b^2 + c^2 - 2bc \operatorname{Cos} \cdot A^\circ$$
$$a^2 = (3)^2 + (1)^2 - (2 \cdot 3 \cdot 1 \cdot \tfrac{1}{2})$$
$$a^2 = 9 + 1 - (6 \cdot \tfrac{1}{2})$$
$$a^2 = 10 - 3$$
$$a^2 = 7$$

$$\boxed{m\overline{CE} = \sqrt{7}}$$

2. TO FIND $m\angle E$:

$$\frac{a}{\operatorname{Sin} A^\circ} = \frac{b}{\operatorname{Sin} B^\circ} = \frac{c}{\operatorname{Sin} C^\circ} \text{ etc.}$$

$$\frac{\sqrt{7}}{\frac{\sqrt{3}}{2}} = \frac{1}{\operatorname{Sin} E^\circ}$$

$$\frac{2\sqrt{7}}{\sqrt{3}} = \frac{1}{\operatorname{Sin} E^\circ}$$

$$\operatorname{Sin} E^\circ = \frac{\sqrt{3}}{2\sqrt{7}} = \frac{\sqrt{7} \cdot \sqrt{3}}{2 \cdot 7} = \frac{\sqrt{21}}{14}$$

$$\operatorname{Sin} E^\circ = .32732684$$
$$E^\circ = 19.106605^\circ$$

$$\boxed{m\angle E = 19.106605^\circ}$$

3. TO FIND $m\angle GLB$:

$$\frac{2}{\operatorname{Sin} GLB^\circ} = \frac{1}{\frac{\sqrt{21}}{14}}$$

$$\operatorname{Sin} GLB^\circ = \frac{2\sqrt{21}}{14} = \frac{\sqrt{21}}{7}$$

$$GLB^\circ = 40.893395^\circ$$

$$\boxed{m\angle GLB^\circ = 40.893395^\circ}$$

4. TO FIND $m\angle LGF$:

$$m\angle GLB^\circ + m\angle E = 60^\circ$$
$$180^\circ - 60^\circ = 120^\circ$$

$$\boxed{m\angle LGF = 120^\circ}$$

5. TO FIND IF $m\overline{LE}$ IS REALLY $2\frac{2}{3}$:

$$\frac{m\overline{LE}}{\operatorname{Sin} 120^\circ} = \frac{1}{\frac{\sqrt{21}}{14}}$$

$$m\overline{LE} = \frac{14}{\sqrt{21}} \text{ X } \operatorname{Sin} 120^\circ$$
$$= \frac{14}{\sqrt{21}} \text{ X } \frac{\sqrt{3}}{2}$$
$$= \frac{7}{\sqrt{21}} \text{ X } \frac{\sqrt{3}}{1}$$
$$= \frac{7\sqrt{3}}{\sqrt{21}} = \frac{7\sqrt{3} \cdot \sqrt{21}}{21}$$
$$= \frac{\sqrt{3} \cdot \sqrt{21}}{3} = \frac{\sqrt{63}}{3} = \sqrt{7}$$

$$\boxed{m\overline{LE} = 2.6457513}$$

6. TO FIND PERCENT ERROR:

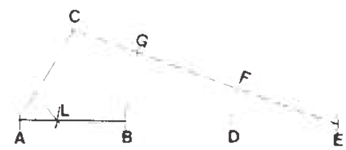
$$\% \text{ error} = \frac{(2.\overline{666} - 2.6457513)}{2.6666666} \times 100$$

$$= \frac{.0209153}{2.6666666} \times 100 = .784324$$

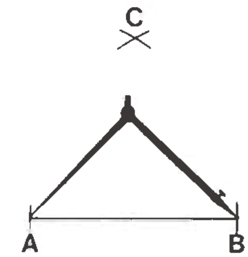
The percent error is .784324% - less than 1% error. L is about 8/1000 of a unit off the 1/3 mark!

A Shortcut

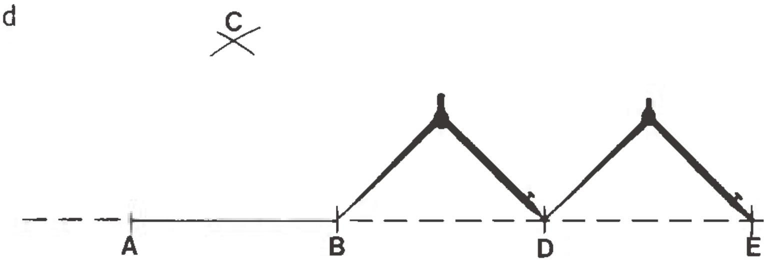
In this diagram, trigonometry has proven $\overline{LE} \cong \overline{CE}$. With that in mind, my method may be condensed to the following.



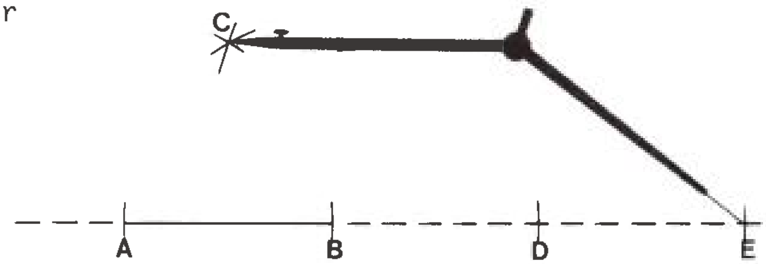
1. Measure d(A,B) with your compass. Mark off the arcs to make C.



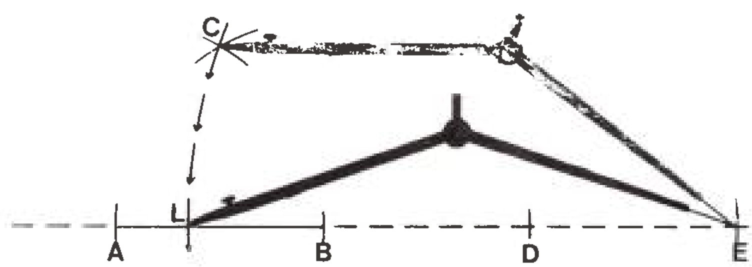
2. Extend $\overleftrightarrow{AB}$. Measure off $\overline{BD}$ and $\overline{DE}$ so that $\overline{AB} \cong \overline{BD} \cong \overline{DE}$.



3. Extend your compass (with your pivot foot on E) to point C.



4. With the pivot foot still on E, move your compass so that it will make an arc that intersects $\overline{AB}$. That point of intersection should be point L as on the original diagram.



Research Council for Diagnostic and Prescriptive Mathematics

by *Bill Speer*
Publications Chairperson, RCDPM
Bowling Green, Ohio

A seed of an idea was planted at the First National Conference on Remedial Mathematics held at Kent State University in 1974. The seed was carefully nurtured during 1975, and at the Third National Conference on Remedial Mathematics held at Kent State University in 1976, a new organization was conceived. Under the guidance of a steering committee, a constitution and bylaws were developed, approved, and implemented for the new organization: *Research Council for Diagnostic and Prescriptive Mathematics*. At the 1977 Maryland meeting, the new organization became a full organization with officers and over 300 members (currently) from the United States and eight other countries. The 1978 Arizona meeting heralded even further growth.

The Research Council for Diagnostic and Prescriptive Mathematics is a relatively new organization that has as its major goals the stimulation, generation, coordination and dissemination of research and development efforts in diagnostic and prescriptive techniques and other techniques which either prevent or correct learning deficiencies in mathematics instruction.

After one full year of life, the RCDPM is gaining momentum and becoming a strong professional organization in American (and perhaps international) mathematics education. A newsletter is distributed four times yearly, several ancillary publications have been issued by the organization, and a national meeting is conducted annually. (This year the annual meeting is in Tampa immediately following NCTM.)

As with any infant organization, there are many plans and decisions to be made. If you are not now a member, we hope you will give serious consideration to becoming an active member and contribute your support to research in diagnostic and prescriptive mathematics (and its dissemination).

A united front needs to be developed to organize and to avoid duplication of research in this area. Diagnostic instruments, techniques of diagnosing, and unique methods of correcting teaching need to be developed. Researchers are needed at all levels - in the clinic and in the classroom - professors, clinicians, supervisors and classroom teachers. There is room for everyone to actively participate in the Research Council for Diagnostic and Prescriptive Mathematics.

For further information, contact:
Research Council for Diagnostic and Prescriptive Mathematics
441 Beryl Drive
Kent, Ohio 44240

MAGIC SQUARES

An Activity for Middle and Upper Grades

by *Elaine V. Alton, Joseph E. Kuczkowski,*
and *Judith L. Gersting*
Department of Mathematical Sciences
Indiana University, Purdue University
Indianapolis

Magic Squares

A *magic square* is a collection of numbers arranged in a square in such a way that the same sum is obtained when the numbers in any row, column, or diagonal are added. This special sum is called the magic sum or magic constant of the square.

We are going to work with 3 X 3 magic squares which have 3 rows and 3 columns. There are other sizes of magic squares. The oldest known magic square is of Chinese origin and is called the Lo-Shu. Its magic constant is 15.

4	9	2	→ 4 + 9 + 2 = 15
3	5	7	→ 3 + 5 + 7 = 15
8	1	6	→ 8 + 1 + 6 = 15

2 + 5 + 8 = 15 ↙ ↓ ↓ ↓ ↘ 4 + 5 + 6 = 15

4	9	2
3	5	7
+8	+1	+6
15	15	15

Find the magic sum for each of the following magic squares.

12	5	10
7	9	11
8	13	6

_____ magic sum

4	4	4
4	4	4
4	4	4

_____ magic sum
(Be careful - the magic sum here isn't 4.)

$\frac{4}{3}$	$\frac{1}{6}$	1
$\frac{1}{2}$	$\frac{5}{6}$	$\frac{7}{6}$
$\frac{2}{3}$	$\frac{3}{2}$	$\frac{1}{3}$

_____ magic sum

Fill in the boxes to get a magic square whose magic sum is 39.

5		15
	7	

magic sum is 39

Adding Magic Squares

We can add two magic squares of the same size to get another magic square by adding the numbers in the matching boxes. For example:

2	7	6
9	5	1
4	3	8

+

6	11	4
5	7	9
10	3	8

=

2 + 6	7 + 11	6 + 4
9 + 5	5 + 7	1 + 9
4 + 10	3 + 3	8 + 8

=

8	18	10
14	12	10
14	6	16

15 magic sum; 21 magic sum; 36 magic sum

For the following two problems, add the given magic squares to find a new magic square. Then find the magic sums for each magic square.

8	1	6
3	5	7
4	9	2

+

3	2	7
8	4	0
1	6	5

=

______ magic sum; ______ magic sum; ______ magic sum

4	$\frac{1}{2}$	3
$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$
2	$\frac{9}{2}$	1

+

3	3	3
3	3	3
3	3	3

=

______ magic sum; ______ magic sum; ______ magic sum

What do you think? When we add two magic squares, the magic sum of the new magic square can be found by adding subtracting multiplying dividing
circle one

the magic sums of the old squares.

Multiples of Magic Squares

We can also multiply a magic square by a number to get another magic square. We do this by multiplying the number in each box of the magic square by this number. For example:

8	1	6
3	5	7
4	9	2

15
magic sum

2 X

8	1	6
3	5	7
4	9	2

=

16	2	12
6	10	14
8	18	4

30
magic sum

For the following two problems, find the indicated multiple of the magic square. Then find the magic sum for each magic square.

9	4	5
2	6	10
7	8	3

______ magic sum

3 X

9	4	5
2	6	10
7	8	3

=

______ magic sum

9	4	5
2	6	10
7	8	3

______ magic sum

$\frac{1}{2}$ X

9	4	5
2	6	10
7	8	3

=

______ magic sum

What do you think? When we multiply a magic square by some number, the magic sum of the new magic square can be found by

adding subtracting multiplying dividing — *circle one* — that given number by the magic sum of the old square.

Doing Your Own Magic with Magic Squares

Let's see if you can create some magic squares. Take the Lo-Shu magic square and multiply it by any number you choose. You should now have a new magic square. Did you find its magic sum? Now, multiply the Lo-Shu magic square by another number. You now have two of your own magic squares. Add these magic squares. You now have another magic square. What is its magic sum? By following these directions, you've found three new magic squares. Now you can create lots of your own magic squares by adding and multiplying.

CONSTRUCTIVE EXPERIENCES WITH DECIMALS

by *T. E. Kieren*
Faculty of Education
University of Alberta
Edmonton, Alberta

The following are the first of a series of decimal exercises to be published in delta-k.

The 10-Slicer

MATERIALS: A number of 10 X 10 grids
1 X 10 strips and unit squares

Suppose you have just received a Pizza Jenie pizza cutter at your house. This machine takes any sized piece of pizza and cuts it into *10 equal pieces.*

You and a friend have just taken 2 pizzas from the oven when a second friend arrives. You want to split the 2 pizzas 3 ways evenly, and you say, "Let's try my Pizza Jenie."

You put the 2 pizzas into the slicer and get 20 equal pieces.
What part of a pizza is each slice?

You distribute these 20 pieces among the 3 of you equally and get *6 each* and 2 left over. "No problem," you say, and put these 2 pieces in the Pizza Jenie and get 20 smaller equal pieces.
What part of a previous slice is each of these pieces?
What part of a whole pizza is each of these?
In distributing these pizza pieces what happens?

Can you repeat this process again with the 2 "leftover" pieces?

Using the Pizza Jenie, each of the three of you get:

	Part of a Whole
6 slices	6 ________ = .6
then 6 small pieces	6 ________ =
then 6 smaller pieces	6 ________ =
then 6 even smaller pieces	6 ________ =
then ____________	6 ________ =

How long can this continue?

We know that each person should get 2/3 of a pizza to be fair. We know the Pizza Jenie process should work out fairly as well. So:

2/3 should equal 6 tenths + 6 hundredths + 6 ________ + 6 ________ + ...
or 2/3 = .666··· = $\overline{6}$.

Use the squares and strips *and* your imagination to solve the following "pizza problems" using the Pizza Jenie.

1. 1 pizza for 2 persons. *Each get* ____________________________.

2. 5 pizzas for 4 persons. *Each get* ____________________________.

3. 2 pizzas for 6 persons. *Each get* ____________________________.

4. 1 pizza for 4 persons. *Each get* ____________________________.

5. 5 pizzas for 15 persons. *Each get* ____________________________.

6. 1 pizza for 10 persons. *Each get* ____________________________.

7. 1 pizza for 100 persons. *Each get* ____________________________.

8. 3 pizzas for 200 persons. *Each get* ____________________________.

9. 3 pizzas for 5 persons. *Each get* ____________________________.

10. 6 pizzas for 8 persons. *Each get* ____________________________.

11. 1 pizza for 7 persons. *Each get* ____________________________.

12. 1 pizza for 9 persons. *Each get* ____________________________.

Write a report which describes what you have found out about repeatedly cutting up objects into 10 parts and distributing these among differing numbers of persons. For example, does it always come out evenly? How do the results relate to decimal notation? For example, what does 5/4 equal? Or suppose at a party each person received .125 or .1 + .02 + .005 parts of a pizza. Can you tell how many poeple and pizzas there were? Is there more than one answer?

Native Speaker

MATERIALS: One calculator

10 X 10 grids

1 X 10 strips and unit squares

It is said that calculators "speak decimal." Try to experiment and see if you can learn something about "decimal" by "talking" with your calculator.

1. Consider 2.17. Represent this with squares and strips. Suppose you were to give 10 persons each such a set of squares and strips. You would need:

   - 20 large squares
   - 10 strips
   - 70 small squares

   You could get these using only large squares and strips. You could get 10 strips from _____ large squares. You could get 70 small squares from _____ strips.

   Thus, 10 X 21.7 = 10 X 2 + 10 X .1 + 10 X .07

   = 20 + 1 + .7

   = 21.7

   Now key 2.17 into your calculator. Multiply by 10. What is the result?
   ____

   Multiply by 10 again. What is the result? __________

   Again. What is the result? __________

   Key in 2.17. Multiply by 100. What is the result? __________

   Key in 2.17. Multiply by 1000. What is the result? __________

   Key in 2.17. Multiply by 100,000. What do you think will result? Check it.

2. Again think of 2.17 in terms of strips and squares. What would happen if you divide these evenly among 10 people?

   Each will get __________ strips → .2

__________ small squares → .01

__________ smaller squares → .007

Key in 2.17 in your calculator. Divide by 10. __________

Divide by 10 again. __________

Again. __________

Again. __________

Key in 2.17. Divide by 100. __________

Key in 2.17. Divide by 1000. __________

3. Explore 3741 by keying it into your calculator.

   Repeatedly divide by 10.

   Key it in again, repeatedly multiply by 10.

   Key 3741 in and divide by 100.

   multiply by 1000.

   divide by 10,000.

   multiply by 100.

   Make up other activities involving powers of 10.

4. Do similar explorations for

   (a) 36.81

   (b) 3.579

   (c) .00036

   (d) 486590 (What happens when you multiply this by 10,000 on your calculator? What does this mean?)

5. Write up what you know about dividing and multiplying decimal fractions by various powers of 10.

6. Suppose you multiply $.\overline{6}$ by 10. What happens to the result? Explore this on your calculator: Divide 2 by 3. What happens? Do you get .66666666 or .66666667? What might each mean? Now multiply your results by 10. What happens? Again. What is the result? What seems to be going on here?

CONSTRUCTIVE RATIONAL NUMBER TASKS

by *T. E. Kieren*
Faculty of Education
University of Alberta
Edmonton, Alberta

The following are the first of a series of number tasks to be published in delta-k.

FRACTION TASK 1:
Measurement and Partitioning

1. Take a piece of calculator tape and "work it" until it lays flat rather than curling up. Cut the ends so that they are perpendicular to its length.

2. Consider your piece of tape as a *unit*. Use your unit to measure the following objects:

   table ________units

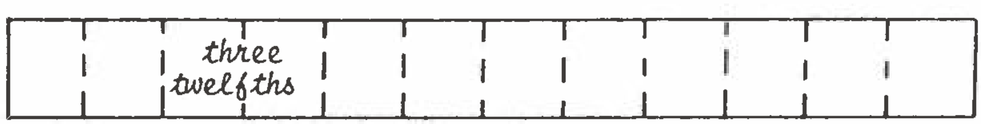
   book ________units

   your partner's height ________units

   your waist ________units

   Because your unit will not usually fit "evenly," you must subdivide your unit into 2, 3, 4, 6, 8, 12, 16 ... parts. You can do this by folding your tape appropriately. (For example, how can you fold "thirds"?) Write the names of the division lines on your tape.

   EXAMPLE:



   Make the measurements using your divided tape.

3. What do you do if your divisions don't give you an even measure? Why can you always find numbers to represent your repeated partitions?

4. This activity is done to answer the following questions.

   (a) Are fractional numbers always less than one?

(b) Counting is a useful mechanism in understanding whole numbers. What mechanism appears useful in understanding fractions?

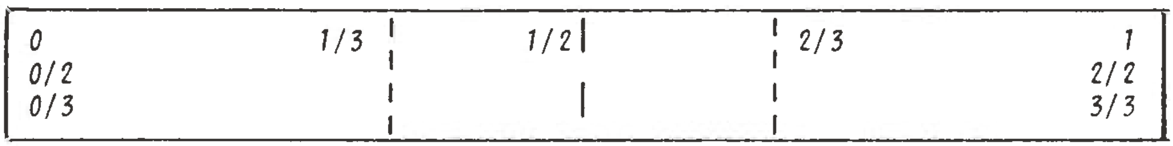
FRACTION TASK 2A: Measurement, Order, and Equivalence

1. Take a piece of calculator tape about 1 metre long and work it until it lays flat. Cut the ends perpendicular to the length and make them straight. Label the ends 0 and 1 right at the top of the tape.

2. Fold the tape lengthwise in two equal parts. Label as follows:

Because of space limitations you will want to use the formal forms 0/2, 1/2, 2/2, but *remember* - as children learn fractions, start with *word names first* and only later use ordered pairs of numbers.

3. Fold the tape lengthwise in three equal parts. Think before you act and *do it carefully*. Label the folds on the tape as follows:



4. Fold the tape into 6 equal parts, label the ends and the "sixths" folds appropriately. (Remember to add the label "2/6" to the "1/3" fold, et cetera.)

5. Fold the tape in 12 equal parts. Label the ends and the "twelfths" folds. (Remember to label the "2/3" fold with "8/12," et cetera.)

6. Fold the tape in 4 equal parts. Label as above.
   Fold the tape in 8 equal parts. Label as above.

7. Is 5/8 greater than 7/12? How can you tell?
   Make up a half-dozen ordering tasks using your tape.

8. List the fractions on the "1/2" fold.

   1/2, 2/4, ____, _____, _____, _____, _____, _____.

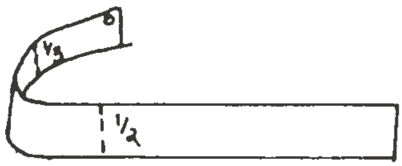
   What can we say about these fractions?
   Why are there no "thirds" in this list?
   Give other sets of equivalent fractions from your tape.

9. How could you generate other fractions to go on the "7/12" fold?

FRACTION TASK 2B: Meaning of Addition and Measurement

1. Take your tape from task 2A. Hold the "1/3" fold directly on the "1/2" fold.

   Where does the "zero" end lie?



   Why?

   A mathematical sentence to describe this is:

   $$1/3 + 1/2 = 5/6$$

2. Repeat 1 above, but fold 1/3 on 7/12. Write the appropriate mathematical sentence.

   $$1/3 + \_\_\_ = \_\_\_.$$

   Do other "additions" using your tape.

   Can you "add" fractions without like denominators?

3. What happens if you lay the "2/3" fold on the "5/6" fold?

   Can you figure out how much beyond 1 the tape extends?

   Complete this mathematical sentence

   $$2/3 + 5/6 = 1\ \_\_\_$$

4. Using the tape, do other additions whose sum is greater than 1. Write the related mathematical sentences.

5. Using your tape (and imagination) to solve the following:

   1/3 + ____ = 5/6 ____ + 1/12 = 11/12 5/8 + 1/2 = ____

6. Think up a way to show subtraction using your tape.

THE PRODUCT OF SIGNED NUMBERS:
Dissection of an Unmotivated Proof

by *Stephen I. Brown*
State University of New York at Buffalo

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Among the many justifications or explanations for why signed numbers behave the way they do under multiplication, the most puzzling is one that relies heavily upon the distributive principle. It looks as if the argument belongs to the domain of legerdemain, for no motivation is provided and we are essentially expected to accept the demonstration on the grounds that the ends justify the means.

The case is quite the opposite in supposedly intuitive demonstrations that make use of patterns rather than the structure of mathematics. Surprisingly enough, it turns out that the legerdemain explanation and a particular intuitive one are linked, and a dissection of their linkage provides illumination for each of them. We turn first to each of the demonstrations.

The Distributive Principle Demonstration

Let us look first at the case of the product of a negative and a positive number. What should (2)(-3) be?

We assume that we know how signed numbers behave under addition, and that we are familiar with basic properties of non-negative reals under multiplication such as:

$ab = ba$
$(ab)c = a(bc)$
$a \times 1 = a$
$a \times 0 = 0$
$a(b + c) = ab + ac$

The demonstration would then be:

$0 = 2(0)$
by the multiplication property of 0 above
$= 2(3 + -3)$
since 0 is 3 + -3
$= 2 \times 3 + 2(-3)$
if the distributive principle is to hold
$= 6 + 2(-3)$
by a fact of arithmetic
$= 6 + -6$
by the additive-inverse property of addition

The case of the product of two negatives is similarly demonstrated as follows [consider (-2)(-3)]:

$0 = (-2)(0)$
if the multiplication property for zero is to hold
$= (-2)(3 + -3)$
since 0 is 3 + -3
$= (-2)(3) + (-2)(-3)$
if the distributive property is to hold

$= (3)(-2) + (-2)(-3)$
if the commutative property is to hold for real numbers
$= -6 + (-2)(-3)$
by the previous result

Therefore $(-2)(-3) = 6$ by additive-inverse property.

Completing a Pattern

Compare the unmotivated demonstration above, with the following two intuitive arguments based upon a pattern:

$2(3) = 6$
$2(2) = 4$
$2(1) = 2$
$2(0) = 0$
$2(-1) = ?$
$2(-2) = ?$
$2(-3) = ?$

It is obvious that if one is committed to the continuation of a pattern (subtracting 2 in each case) established for the familiar cases (positive integers), then the "?" in each of the bottom three cases could be filled in as follows:

$2(-1) = -2$
$2(-2) = -4$
$2(-3) = -6$

Thus $2(-3) = -6$.

Similarly for the case of 2 negatives [for example, $(-2)(-3)$], once we have established the one for the product of a negative and a positive we have:

$(-2)(3) = -6$
$(-2)(2) = -4$
$(-2)(1) = -2$
$(-2)(0) = 0$
$(-2)(-1) = ?$
$(-2)(-2) = ?$
$(-2)(-3) = ?$

Again, if the pattern of adding 2 in each case to get the answer to the one below is to continue we have:

$(-2)(-3) = 6$

A First Approximation in Seeing Linkages

Neither the case of the distributive principle nor the pattern argument provides us with a proof. The reason of course is *not* that we have focused on specific rather than general cases (for we could generalize the arguments without difficulty) but rather that each of the two types of demonstrations shares an important obstacle that could not be overcome by introducing all the variables in the world. The "proofs" are based upon "wishful thinking." That is, there is no God-given reason in the world why the axiomatic structure embedded in the case of non-negatives is *required* to continue as we move to the negatives. It is only *if* we force the distributive law (and others too) to apply in our new set-up that we are led to conventional results. We *cannot* prove that these laws *must* be extended. We merely can investigate the *consequences* of making such extensions.

In the above argument, we have applied a heuristic that is used generally in extending mathematical concepts - the *preservation principle*. The principle asserts that if we wish to extend a mathematical concept beyond its original domain, then that candidate ought to be chosen which leaves as many principles of the old system intact as possible. The preservation principle is, however, an *aesthetic* and not a logical one. The mathematical world would not collapse if we were to modify drastically old principles when we extend to new systems. As a matter of fact, we frequently *must* relinquish some old principles when we extend our domain, for we may be led to contradictions otherwise. (See for example what havoc is played if we try to relate $\sqrt{-1}$ to zero as part of an ordered-field structure as we move from the reals to the complex numbers.)

It is important to see that such an aesthetic argument is made in the case of the pattern demonstration as well. There is no God-given reason why the terms *must* decrease by 2 in the new domain as they do in the old.

2(2) = 4
2(1) = 2
2(0) = 0
2(-1) = ?

We are of course familiar with a function that behaves quite differently - the absolute value function. We *could* force the pattern to revise itself below zero:

2(2) = 4
2(1) = 2
2(0) = 0
2(-1) = 2
2(-2) = 4

It would then be interesting to investigate what principle in the system might have to be modified, based upon this new extension.

Putting a Fine Point to It

Let us now take a closer look at the pattern argument to see just what it is we are attempting to preserve as we continue the pattern. Let us leave the answers on the right hand side in unsimplified form:

2(3) = 2(3)
2(2) = 2(2)
2(1) = 2(1)
2(0) = 2(0)
2(-1) = ?
2(-2) = ?
2(-3) = ?

Notice that as we move upward from 2(0), we add a multiple of two each time. If that is the pattern we want to preserve, then in the case of 2(-1), we want to be able to add 2 in order to get to the next level, 2(0); for 2(-2), we want to be able to add a multiple of 2 two times to get the 2(0) level; for 2(-3), we want to be able to add a multiple of 2 three times to get to the 2(0) level.

Thus, merely to preserve the *pattern* that we already have for multiplication of non-negative signed numbers, we would want:

(1) 2(-3) + 2(3) = 2(0)

But justification of the above equation (coming strictly from the pattern) *is tantamount to extending the distributive principle!* That is, now that the *pattern* has motivated us to strive for (1), how might we achieve it by looking strictly at the axiomatic structure of the number system? It is obvious that we could achieve the equality if we were allowed to distribute the left side of that equation, that is,

if 2(-3) + 2(3) = 2(-3 + 3).

It might be possible to view the desired result slightly differently. Since we want 2(-3) + 2(3) to be 0, we really are requesting that 2(-3) act like the additive inverse of 2(3); that is, we want 2(-3) to act like - [(2)(3)]. But that perspective sends us back immediately to the analysis we have just completed, for to say that 2(-3) = -[(2)(3)] is equivalent to asserting that 2(-3) + 2(3) = 0.

Extension of the distributive principle thus provides the desired missing link and we have accomplished two things at once:

1. We can use the intuitive pattern argument to *motivate* the more axiomatically-based argument.

2. We see that the intuitive pattern argument does - in a disguised way - assume exactly what we felt we could bypass by moving away from an axiomatic approach.[1]

It should be a source of consolation rather than distress that - as Morris Kline has been trying to tell us for a long time - rigorous formulations of a problem and intuitive ones not only do not belong to different moral planes but may in fact have more in common logically than we generally concede.

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[1]See Stephen I. Brown, "Multiplication, Addition and Duality," in *The Mathematics Teacher*, October 1966, pp.543-51, for an analysis of why it is that $a(-n) = -[(a)(n)]$ belongs to the class of equations that *require* the distributive principle in their proofs.

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