CONSTRUCTIVE RATIONAL NUMBER TASKS

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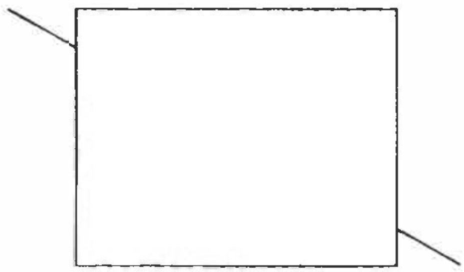
The first of a series of number tasks were published in the March 1979 issue of delta-k. *The following exercises are a continuation of the series.*

FRACTION TASK 3: Operators and Machines

1. Complete the table below:



Input	*Output*
10	6
20	12
50	30
100	60
15	____
5	____
75	____
5000	____
3000	____

The name of this machine is a ________ for ________ machine. For every __5__ that go in, ________ come out.

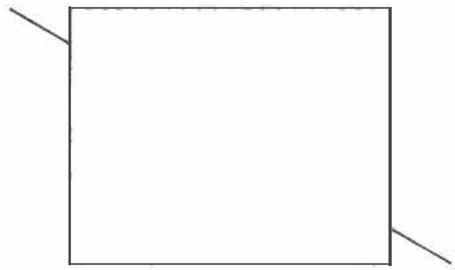
2. For the above machine, complete this table:

Input	*Output*
____	9
____	300
____	600

How did you know these results?

You were using the notion of "inverse." A machine which would do the reverse of the 3 for 5 machine above would be a 5 for 3 machine.

3. Find a partner. Each of you make up a machine with a mixed list of eight inputs and outputs. Make sure you give three complete pairs. Exchange lists and see who can give the most correct answers. Here is a sample game machine:



Input	*Output*
15	10
9	6
60	40
6	___
___	2
24	___
18	___
___	20

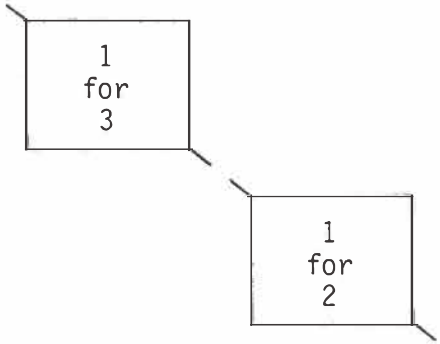
This machine is a ________ for ________ machine. Its "inverse" machine would be a ________ for ________ machine.

4. Here is a mysterious machine's input and output list. Can you complete it?

Input	*Output*
10	10
3	3
727	727
___	46
29	___
11	___
7777	___
___	21

Can you name this machine? ________ for ________. A formal mathematical name for this machine is the *identity machine*. Its inverse machine would be a ________ for ________ machine.

5. Here are two machines.



The *output* from the first machine is the *input* for the second machine. Can you complete the table below?

Machine 1			*Machine 2*	
I	O		I	O
30	10	----------	10	5
12	4	----------	4	2
24	8	----------	_____	_____
60	20	----------	_____	10
90	_____	----------	_____	_____
120	_____	----------	_____	_____
_____	2	----------	_____	1
_____	_____	----------	_____	4
_____	_____	----------	_____	2000

Look at these machines carefully. What fraction could you use to automatically get the final result if 300 were put into the first machine?

We can write this result 1 for 3 followed by 1 for 2 is the same as 1 for 6.

In more mathematical symbols, 1/3 _____ 1/2 = 1/6.

6. Use the machine idea to solve the following:

1 for 2 followed by 1 for 2 is _____ for _____.

3 for 4 followed by 1 for 2 is _____ for _____.

1 for 1 followed by 3 for 7 is _____ for _____.

3 for 5 followed by 5 for 3 is _____ for _____.

4 for 7 followed by _____ for _____ is 1 for _____.

7. What mathematical operations and what ideas are related to this approach to fractions?

FRACTION TASK 4: Part-Whole Equivalence

1. Using the set of 72 objects in front of you, complete the following list of all the ways you can divide 72 objects into subsets of the same size.

   (1) 36 sets of 2

   (2) ____ sets of 36

   (3) 24 sets of ____

   (4)

   (5)

   (6)

   *

   *

   *

   How many ways of partitioning the 72-object set did you get?

   Why is there such an abundance of ways? (Remember Kennedy, pp.268-273)

2. Looking at a list of partitions helps one see ways in which fractions can be expressed. For example, because there are four sets of 18 in 72, 18/72 can be expressed as 1/4.

   Complete the following lists of ways that the partitioning of 72 suggests for expressing various fractions.

   (a) 18/72, 1/4, ____, ____, ____

   (b) 24/72, 2/6, ____, ____, ____

   *(c) 10/72, ____, ____, ____, ____

   *(d) 4/72, ____, ____, ____, ____

   *(e) 17/72, ____, ____, ____, ____

   *Can you fill all the blanks? Why or why not?

   What can you say about the fractions in set (a) above?

   What can you say about all the sets of fractions above?

FRACTION TASK 5: Measurement and Addition

1. Look back to Fraction Task 1 and use your tape from that task or make a new tape.

2. Measure the following objects as precisely as you can, and complete the following table.

Object	A Side 1	B Adjacent Side	C Both Sides in a Single Measure	D Sum of A + B
(a) book				
(b) table				
(c) room or part of room				

3. What appears to be the relationship between columns C and D in the table?

4. If the two sides of the room measure 6 1/2 and 4 3/8 tapes, we can relate these to the total length 10 7/8 with the following sentence:

   $6\ 1/2 + 4\ 3/8 \doteq 10\ 7/8$

   Using your data, write three sentences which describe the relationship between the side lengths and the total.

   Why do we use $\doteq$ instead of =?

   Why, in theory or in elementary school texts, can we write:

   6 1/2 + 4 3/8 = 10 7/8?

5. Use two pieces of mayfair board which contain units marked off in eighths. Label the points starting at 1/8 with appropriate fractional and whole number names (for example, 3/4, 11/8, 3/2, et cetera).

6. Use the two rulers to add 1/2 and 1/4. Result ________. Write a set of directions for Grade VI or VII children which would tell them how to add numbers using these rulers.

7. Use your ruler to add the following numbers:

   (a) 3/8 + 3/4 = ______

   (b) 5/8 + 3/2 = ______

   (c) 7/4 + 1/8 = ______

   (d) 17/8 + 1/2 = ______

8. Re-label your rulers using mixed numerals (for example, 1 1/2, 2 1/4, et cetera) or at least think of the partitions in those terms. Complete the following:

   (a) 3/8 + 3/4 = ______

   (b) 1 3/8 + 1 1/2 = ______

(c) 1 1/4 + 7/8 = ______

(d) 1 3/4 + 1 5/8 = ______

9. Use your ruler to answer the following questions:

   (a) 3/4 = /8?

   (b) 5/2 = /4?

   (c) 5/4 = /8?

   (d) 5/4 + 7/8 = ______

   This is the same question as /8 + 7/8 = ______

   (e) 3/2 + 5/4 = ______

   is the same as /4 + 5/4 = ______

10. The purpose of this task sheet has been to show two things:

    A. Fractions or rational numbers can be added! There is no question of common denominators!

    B. When you are making up a quick algorithm which uses symbols only, equivalence allows you to make use of the common denominator notion to do so.