1979 Alberta High School Prize Examination Results

Prize	$ Amt.	Student	School
Canadian Mathematical Society Scholarship	400	PATON, Gregory	Lindsay Thurber Composite High Red Deer, Alberta
Nickel Foundation Scholarship	400	Not awarded	
First runner-up	150	WONG, Eric	Ross Sheppard Composite High Edmonton, Alberta
Second runner-up	150	MOREWOOD, Robert	Medicine Hat High School Medicine Hat, Alberta

Special Provincial Prizes

Prize	$ Amt.	Student	School
Highest Grade 12 student (below first 3)	75	ANTOLAK, John	St. Joseph's Sr. High School Grande Prairie, Alberta
Highest Grade 10/11 student (below first 3)	75	BARAGAR, Arthur	Old Scona Academic High Edmonton, Alberta

District Prizes

District No.	$ Amt.	Student	School
1	50	SLAVEN, Robert	Sir John Franklin High School Yellowknife, N.W.T.
2	50	KLEM, Raymond	H.A. Kostash High School Smoky Lake, Alberta
3	50	FINLAY, Warren	Salisbury Composite High Sherwood Park, Alberta
4	50	PENNER, Robert	Lindsay Thurber Composite Red Deer, Alberta
5	50	SHEPHERD, Douglas	Hugh Sutherland Sr. High Carstairs, Alberta
6	50	NEUFELDT, David	Kate Andrews High School Coaldale, Alberta
7 (1)	50	BOWMAN, John	Old Scona Academic High Edmonton, Alberta
7 (2)	50	WELSH, Michael	Old Scona Academic High Edmonton, Alberta
8 (1)	50	KRYCZKA, John	Bishop Carroll High School Calgary, Alberta
8 (2)	50	KOŁODINSKY, Steve	Queen Elizabeth High School Calgary, Alberta

267 students from 58 schools in Alberta and the Northwest Territories wrote the 1979 examination. The following students took the first 16 places and are nominated for the Canadian Mathematical Olympiad:

Student	School
ANTOLAK, John	St. Joseph's Sr. High School, Grande Prairie
BARAGAR, Arthur	Old Scona Academic High School, Edmonton
BARAGAR, Henry	Old Scona Academic High School, Edmonton
BOWMAN, John	Old Scona Academic High School, Edmonton
KAY, Lewis	Ross Sheppard Composite High School, Edmonton
KOLODINSKY, Steve	Queen Elizabeth Jr.-Sr. High School, Calgary
KRYCZKA, John	Bishop Carroll High School, Calgary
LEUNG, David	Harry Ainlay Composite High School, Edmonton
LOVE, Nathan	J.G. Diefenbaker High School, Calgary
MA, Raymond	Victoria Composite High School, Edmonton
MOREWOOD, Robert	Medicine Hat High School, Medicine Hat
PATON, Gregory	Lindsay Thurber Composite High School, Red Deer
SHEPHERD, Douglas	Hugh Sutherland Sr. High School, Carstairs
WELSH, Michael	Old Scona Academic High School, Edmonton
WINQUIST, Eric	Bonnie Doon Composite High School, Edmonton
WONG, Eric	Ross Sheppard Composite High School, Edmonton

The following students placed 17-30:

Victoria Cho (McNally Comp. High School, Edmonton), Jocelyn Coates (Old Scona Academic High School, Edmonton), Dean Elhard (William Aberhart High School, Calgary), Warren Finlay (Salisbury Composite High School, Sherwood Park), Brian Hart (Prairie High School, Three Hills), Raymond Klem (H.A. Kostash High School, Smoky Lake), Richard Maisonneuve (Salisbury Composite High School, Sherwood Park), Andrew McIntosh (Bishop Carroll High School, Calgary), David Neufeldt (Kate Andrews High School, Coaldale), William Olsen (Old Scona Academic High School, Edmonton), Robert Penner (Lindsay Thurber Composite High School, Red Deer), Gautam Rao (Eastglen Composite High School, Edmonton), Charles Ursenbach (Dr. E.P. Scarlett High School, Calgary), Carl Wilting (M.E. Lazerte Composite High School, Edmonton).

The following students placed 31-50:

Douglas Allen (William Aberhart High School, Calgary), Mike Blum (Lord Beaverbrook High School, Calgary), Kim Chernowski (Old Scona Academic High School, Edmonton), Edward deBeaudrap (Didsbury High School, Didsbury), Ken deCock (St. Michael's, Pincher Creek), David Filipchuk (Harry Ainlay Composite High School, Edmonton), Donald Haigh (Lethbridge Collegiate Institute, Lethbridge), Joe Kasper (Louis St. Laurent High School, Edmonton), Andrew Kennett (Lord Beaverbrook High School, Calgary), Paul Moret (St. Joseph's Composite High School, Edmonton), Srinath Mutyala (Harry Ainlay Composite High School, Edmonton), David Muzyka (Bonnie Doon Composite High School, Edmonton), Robert Nordal (Dr. E.P. Scarlett High School, Calgary), Robert Proudfoot (Didsbury High School, Didsbury), Michele Reeves (William Aberhart High School, Calgary), Dave Salahub (Sir Winston Churchill High School, Calgary), Robert Slaven (Sir John Franklin High School, Yellowknife), Angelee Wahl (Medicine Hat High School, Medicine Hat), Jeffery Westbrook (Sir Winston Churchill High School, Calgary), Gary Wiens (Didsbury High School, Didsbury).

ANSWER SHEET

To be filled in by the Candidate.

PRINT:

Last Name First Name Initial

Candidate's Street Address Town/City POSTAL CODE

Name of Candidate's School

Grade

ANSWERS:

1	2	3	4	5	6	7	8	9	10

11	12	13	14	15	16	17	18	19	20

To be completed by the Department of Mathematics, University of Alberta:

Points		Points Correct	Number Wrong
1-20	5	5 × =	1 × =
TOTALS		C = ____	W = ____

SCORE = Correct - Wrong = ________

PART I **Time: 60 Minutes**

Instructions to Candidates

1. *Please do not open this booklet beyond Page 2 until instructed to do so by the supervisor.*

2. *Please turn now to Page 2 (the next page) and fill in the top four lines - Page 2 is your answer sheet.*

3. *This exam is multiple choice. Each question will be followed by 5 possible answers, labelled A, B, C, D, E. For each question, list your choice of answer in the box on the answer sheet directly above the question number. For example, if you decide the correct answer to question 3 is labelled C, then enter C in the box above 3 on the answer sheet.*

4. *To discourage random guessing, there is a penalty for each incorrect answer; there is no penalty for unanswered questions. Filling in more than one letter in any box counts as a wrong answer for that question.*

5. *At the signal from your supervisor, detach both this page and page 2 (the answer sheet) and begin the examination. Pencil, graph paper, scratch paper, ruler, compass, and eraser are allowed.*

   Calculators are not allowed.

Do all problems. Each problem is worth five points.

1. A triangle has sides of lengths 1, 2 and $\sqrt{3}$.

   Its area is

   (A) 1/2 (B) $\sqrt{3}/2$

   (C) 2 (D) $2\sqrt{3}$

   (E) none of these.

2. $P(x) = 4x^4 - kx^2 + 1$ has two double roots if

   (A) $k = 1$ (B) $k = 2$

   (C) $k = 3$ (D) $k = 4$

   (E) none of these is true.

3. For what values of a, b is the equation $\left(a^{\log_{10} b}\right)^{ab} = \left(b^{\log_{10} a}\right)^{ba}$ true?

(A) All values of a and b (B) No values of a and b

(C) All positive values of a and b

(D) All negative values of a and b

(E) none of these is correct.

4. The equation of the line that is perpendicular to the line $x + 2y = 3$ and passes through the point $(4,5)$ is

(A) $x - 2y = 3$ (B) $2x - y = 3$

(C) $2x + y = 3$ (D) $-2x + y = 3$

(E) none of these.

5. Let S be the set of points defined by the inequalities $x + y \leq 1$, $x - y \leq 1$, $y \leq 1/2$. The area of the region determined by S is

(A) 3/8 (B) 1/2 (C) 5/8 (D) 1 (E) none of these.

6. X is a square of diagonal 1, Y is an equilateral triangle of side 1 and Z is a right-angled isosceles triangle whose equal sides have length 1. Comparing the areas of these figures,

(A) X is larger than both Y and Z (B) Y is larger than both Z and X

(C) Z is larger than both X and Y (D) X,Y,Z all have the same area

(E) none of these is correct.

7. A positive integer is squarefree if it cannot be divided exactly by the square of an integer larger than 1. The number of positive squarefree integers less than 20 is

(A) 0 (B) 3 (C) 9 (D) 13 (E) none of these is correct.

8. The solutions of the equation $(\sin\theta + \cos\theta)^2 = 1$ are

(A) all multiples of $\pi/2$ (B) all multiples of π

(C) all odd multiples of $\pi/2$ (D) all even multiples of π

(E) none of these is correct.

9. The sum of the first 27 odd positive integers is

(A) 153 (B) 196 (C) 144 (D) 216 (E) none of these.

10. $\left(\left((\sqrt{2})^{\sqrt{2}}\right)^2\right)^{\sqrt{2}}$ is equal to

(A) 2 (B) $\sqrt{2}$ (C) 4 (D) 8 (E) none of these.

11. If α,β are the roots of $x^2 + 7x - 5 = 0$, then $\alpha^2 + \beta^2$ is equal to

(A) 59 (B) 47 (C) -15 (D) 35 (E) none of these.

12. In the figure, AOB and COD are straight lines and O is the center of the circle, while PD, PA, QB, QC are tangents to the circle. The distance from P to Q is

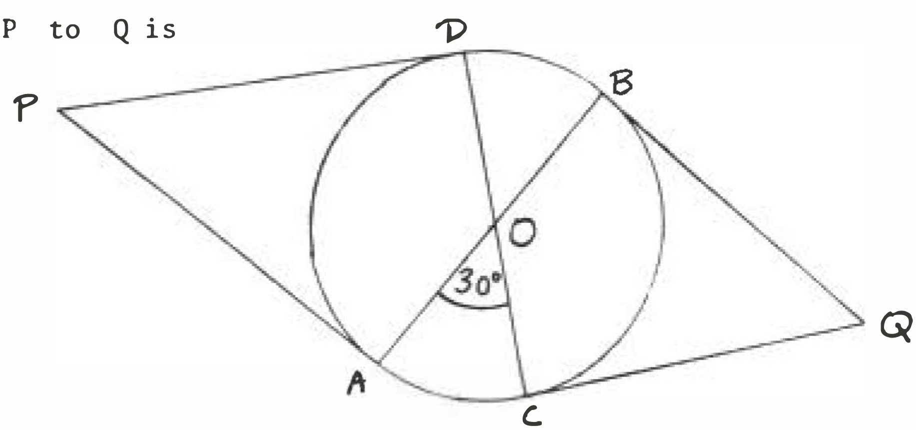
(A) 2

(B) 3

(C) 4

(D) 1/2

(E) none of these.



13. For which values of x is a triangle with sides x, $x+1$, $x+2$ an acute-angled triangle?

(A) $x = 1$ (B) $x > 2$

(C) $x < 4$ (D) $x > 3$

(E) none of these is true.

14. Which of the following inequalities are always true for any pair of real numbers x,y?

(A) $x + y \leq xy$ (B) $(x+y)^2 \geq xy$

(C) $(x+y) \geq xy$ (D) $(x+y)^2 \geq x + y$

(E) none of the preceding is always true.

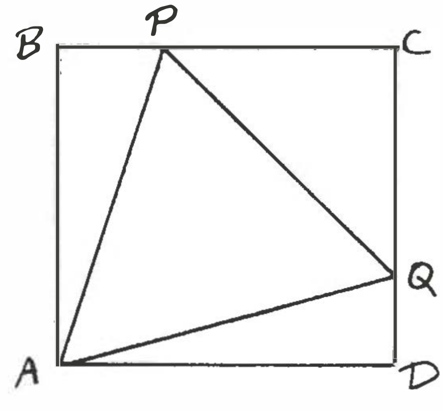
15. A twelve-hour digital watch displays the hours, minutes and seconds. During one complete day it registers at least one figure 3 for a total time of

(A) 1 hour and 5 seconds (B) 1 hour, 15 minutes and 15 seconds

(C) 2 hours and 24 minutes (D) 3 hours

(E) none of these.

16. In the diagram, ABCD is a square of side 1 and APQ is an equilateral triangle. The length DQ is equal to



(A) 1/2

(B) $\sqrt{2} - 1$

(C) 1/3

(D) $2 - \sqrt{3}$

(E) none of these.

17. The product of John and Mary's ages is five more than four times the sum of their ages. If Mary is 4 years younger than John, John's age is

(A) 13 (B) 11 (C) 9 (D) 7 (E) none of these.

18. In a poll of 1000 coffee drinkers, 40% preferred their coffee with neither cream nor sugar and 60% of the remainder preferred their coffee with cream only. After deducting both of these groups, 40% of those left preferred their coffee with sugar only. The rest preferred coffee with cream and sugar and their number was

(A) 144 (B) 216 (C) 96 (D) 172 (E) none of these is correct.

19. A sphere and a triangle <u>cannot</u> intersect in exactly

(A) 1 point (B) 2 points

(C) 3 points (D) 4 points

(E) all of the preceding are possible.

20. The picture cards are removed from a pack of 52 playing cards. The number of ways of drawing 2 cards from the remaining 40 so that the sum of the numerical values is 10 is

(A) 100 (B) 10

(C) 50 (D) 70

(E) none of these.

PART II **Time: 110 Minutes**

INSTRUCTIONS TO CANDIDATES

Do each of questions 1, 2 and 3 and any two of questions 4, 5 and 6. All questions are weighted equally and you may attempt them in any order you wish.

Your paper should be concise and complete and each step of your answer should be clearly justified and presented in a legible and intelligible form. Extra credit will be given for particularly elegant solutions as well as for non-trivial generalizations with proof. If there is any doubt about an interpretation of a problem, make a note of that on your paper and state and solve what you consider to be a valid non-trivial interpretation of the problem.

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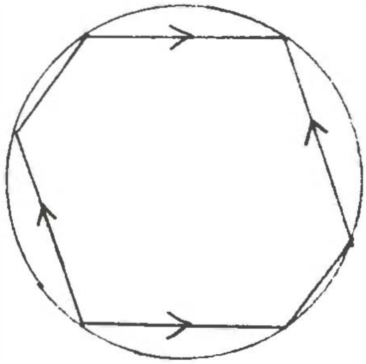
1. A hexagon is inscribed in a circle as shown. If two pairs of opposite sides are parallel, prove that the third pair of sides are also parallel.



2. Determine all real values of A for which the line $x - y = 1$ intersects the curve given by $x^3 + y^3 - 2x^2 + Ay^2 + x + y = 0$ in exactly two distinct points.

3. Prove that for any integer n , $n^5 - n$ is exactly divisible by 5 .

4. If a single die is thrown, each of the numbers 1 through 6 has an equal probability of occurring, namely one-sixth. Now suppose that four such dice are thrown together. Find the most probable value of the total number obtained, and the probability of obtaining this number.

5. Starting with a given positive integer n , the following procedure is used for obtaining a sequence of integers:

Suppose the previous number obtained is m . If m is even, the next number is $\frac{m}{2}$; if m is odd, the next number is the smallest square number larger than m . If 1 is obtained, the sequence terminates. For example, if we started with 20 , the sequence would proceed 20, 10, 5, 9, 16, 8, 4, 2, 1.

Prove that for any starting number n , the number 1 will be obtained after a finite number of steps.

6. In a round-robin tennis tournament, 16 men played each other once with no tie games allowed. Show that it is possible to label the men $M_1, M_2, \ldots, M_{16}$ so that M_1 beat M_2, M_2 beat M_3, M_3 beat M_4, ..., M_{15} beat M_{16} .

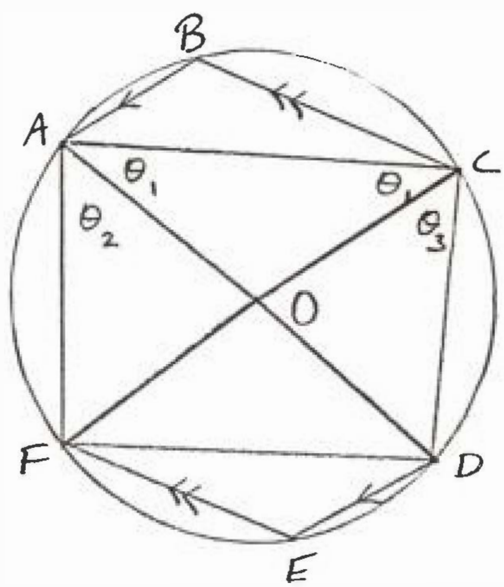
Solutions to Part I

1	2	3	4	5	6	7	8	9	10
B	D	C	B	E	E	D	A	E	C

11	12	13	14	15	16	17	18	19	20
A	X	D	B	E	D	B	A	E	D

X = The diagram associated with this question was incomplete and any answer given was marked as correct.

Solutions to Part II



1. Suppose $AB \| DE$, $BC \| EF$. Let O be the center of the circle. We have $\widehat{ABC} = \widehat{FED}$ (angles formed by parallel pairs of lines) and so AC = DF (chords of a circle subtending equal angles). Since AO, CO, DO, FO are all equal (radii of the circle), we find that ΔAOC, ΔFOD are conguent isosceles triangles (corresponding sides are equal). Therefore $\widehat{OAC} = \widehat{OCA} = \widehat{OFD} = \widehat{ODF} = \theta_1$, say. Now ΔAOF, ΔCOD are isosceles triangles, and so $\widehat{OAF} = \widehat{OFA} = \theta_2$, $\widehat{OCD} = \widehat{ODC} = \theta_3$, say.

Thus $\widehat{DCA} + \widehat{CAF} = \widehat{OCD} + \widehat{OCA} + \widehat{OAC} + \widehat{OAF} = \theta_3 + \theta_1 + \theta_1 + \theta_2 = 2\theta_1 + \theta_2 + \theta_3$. Similarly, $\widehat{CDF} + \widehat{DFA} = \widehat{CDO} + \widehat{ODF} + \widehat{DFO} + \widehat{OFA} = 2\theta_1 + \theta_2 + \theta_3$, and so $\widehat{DCA} + \widehat{CAF} = \widehat{CDF} + \widehat{DFA}$. But these four angles are the interior angles of a convex quadrilateral, and so add up to 360^o . It follows that $\widehat{DCA} + \widehat{CAF} = 180^o$, and so $CD || AF$

2. Put $x = 1+y$ and substitute into the equation of the curve to get

$$1 + 3y + 3y^2 + y^3 + y^2 - 2 - 4y - 2y^2 + Ay^2 + 1 + y + y = 0 ,$$

which simplifies to

$$y[2y^2 + (A+1)y + 1] = 0 .$$

One root (and hence one point of intersection of the line and the curve) is given by $y = 0$. To obtain exactly one more intersection point, the quadratic $2y^2 + (A+1)y + 1 = 0$ must have a double root. The condition for a double root is that the discriminant $(A+1)^2 - 8 = 0$, that is, $\underline{A = -1 \pm \sqrt{8}}$.

3.
$$n^5 - n = n(n^4-1) = n(n^2-1)(n^2+1)$$
$$= n(n-1)(n+1)(n^2+1) .$$

Any integer n leaves a remainder of 0, 1, 2, 3 or 4 when divided by 5 . If the remainder is 0, 1 or 4 , respectively, then n, $n-1$, $n+1$, respectively are divisible by 5 . If the remainder is 2 or 3, respectively, then n is of the form $5k \pm 2$, respectively, and so $n^2 + 1 = (5k\pm 2)^2 = 25k^2 \pm 20k + 5$, which is again divisible by 5 . So in all cases, $n^5 - n$ is divisible by 5 .

4. Throwing a single pair of dice yields the numbers 2 through 12 with the following probabilities:

PROBABILITY TABLE

number:	2	3	4	5	6	7	8	9	10	11	12
prob.	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

Throwing four dice is equivalent to throwing two pairs and the most probable total number obtained is 14. 14 is obtained if the numbers thrown for the two pairs of dice are (2,12), (3,11), ..., or (12,2), and we may use the probability table above to compute the likelihood of throwing 14 as

$$\frac{2(1^2+2^2+3^2+4^2+5^2) + 6^2}{36^2} = \frac{146}{36^2} = \frac{73}{748}$$

5. If n is even, the next number in the sequence is $\frac{n}{2}$, which is less than n. If n is an odd number greater than 1, then it lies between the squares of two consecutive positive even integers, say $(2m)^2 < n < (2m+2)^2$, where $m \geq 1$.

The next number in the sequence is $(2m+2)^2$, or it is $(2m+1)^2$ in which case the following number in the sequence is $(2m+2)^2$; in either case $(2m+2)^2 = 4(m+1)^2$ is obtained in the sequence. Then the next two numbers obtained are $2(m+1)^2$, $(m+1)^2$.
Now $m \geq 1$ imples that $m+1 \leq 2m$ and so $(m+1)^2 \leq (2m)^2 < n$.
Thus in all cases, if we start with $n \geq 2$, we eventually obtain a number less than n in the sequence. Repeating this argument inductively, we must eventually obtain 1.

6. Let n be the maximum number of men that can be labelled $A_1, A_2, \ldots, A_n$, say, so that A_1 beat A_2, A_2 beat A_3, ..., A_{n-1} beat A_n. Obviously $n \geq 2$, and we would like to show that $n = 16$. Suppose that $n < 16$.

Let B be any man remaining after we have labelled $A_1, A_2, \ldots, A_n$.
If B lost to each of the A_i's , then we may relabel each A_i as B_i , $i = 1, 2, \ldots, n$, and label B as B_{n+1}
Otherwise B must have beaten some A_j and we choose the smallest index j such that B beat A_j . Then B lost to all A_i with $i < j$.
Now label B as B_j , relabel A_i as B_i for all $i < j$ and relabel A_i as B_{i+1} for all $i \geq j$.

In both cases we have managed to label (n+1) men as $B_1, B_2, \ldots, B_{n+1}$, so that B_1 beat B_2, B_2 beat B_3, ..., B_n beat B_{n+1} , which violates the assumption that n is the maximum number that can be labelled in this fashion. Hence it must be false that $n < 16$, and so $n = 16$, that is, all the man may be labelled in the appropriate way.

MATHLAB

(K through 8)

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