

EDITOR'S NOTE: The following "Activities" from the *Mathematics Teacher* and "Ideas" from the *Arithmetic Teacher* are samples of some of the advantages you may have by belonging to the National Council of Teachers of Mathematics. Another advantage is the meeting in Calgary, October 11-13, 1979. Because NCTM is a cosponsor, we will be provided with resources not normally available for our annual meetings. These will include extra personnel from western Canada and western United States in attendance and NCTM financial support for the meeting expenses.

# activities

## Tetrahexes

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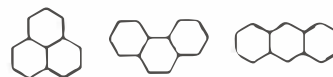
### Teacher's Guide

**Grade level:** 7-10

**Materials:** Copies of worksheets 1-3 and scissors

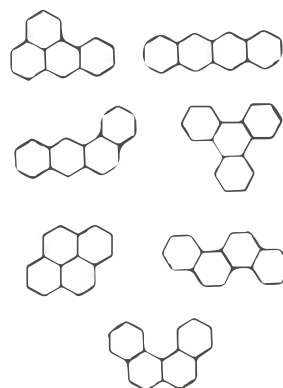
**Objectives:** The students will solve problems in a geometric setting where congruence and symmetry are significant concepts. They will develop problem attack skills such as making conjectures and testing those conjectures.

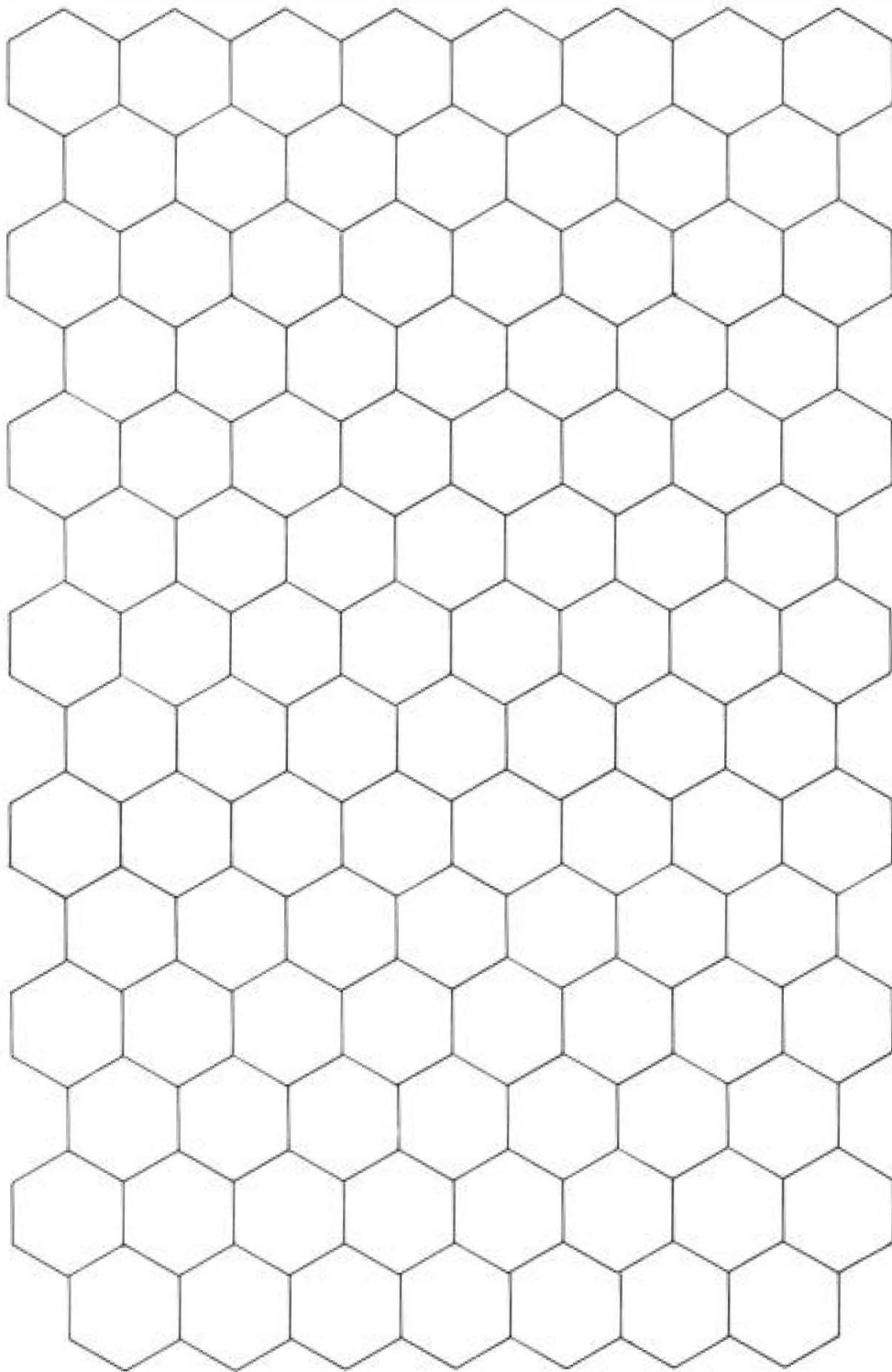
**Sheet 1, Activity 1.** Each student will need one copy of this sheet, which should be made on construction paper, if possible. Ask your students to cut three hexagons out of sheet 1 and then find as many different polygonal shapes as possible that can be made using all three hexagons. One rule in making these shapes is that the hexagons have to match up along a common edge. It should also be noted that two shapes are not considered different if one of these can be flipped, turned, or moved so that it fits on top of the other. Your students should find these three shapes (called trihexes):



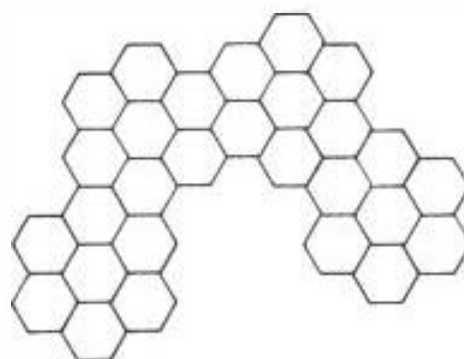
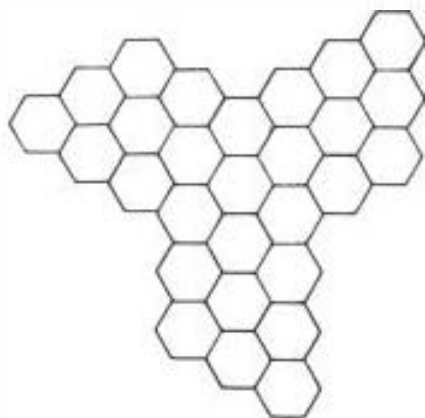
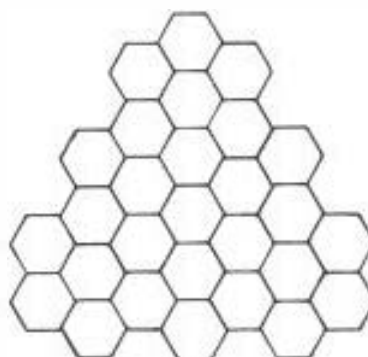
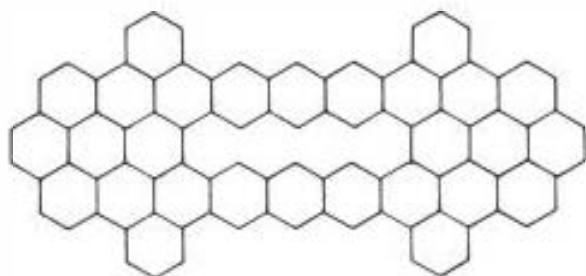
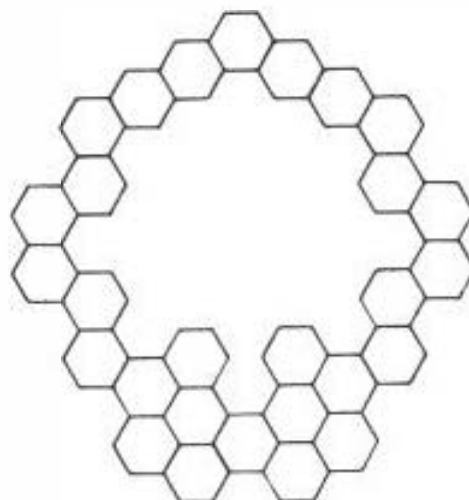
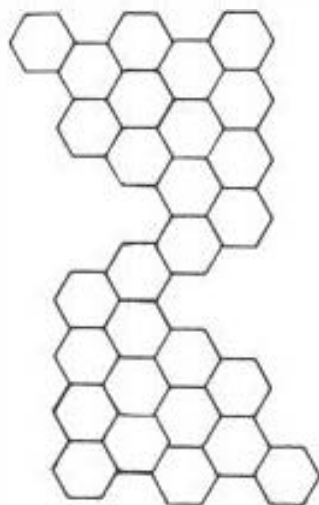
Have your students cut the three different trihexes out of the hexagonal grid on sheet 1.

**Activity 2.** Ask your students to use the trihexes they have just cut out to find all the different polygonal shapes that can be made with *four* hexagons. They should find the following shapes (called tetrahexes):

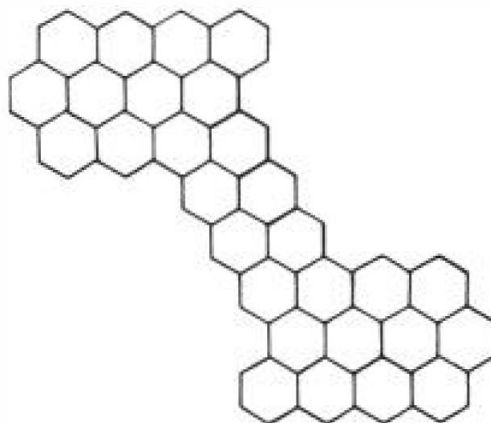
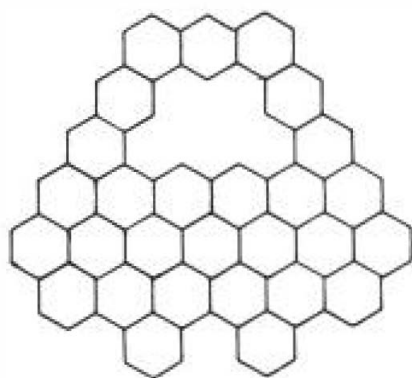
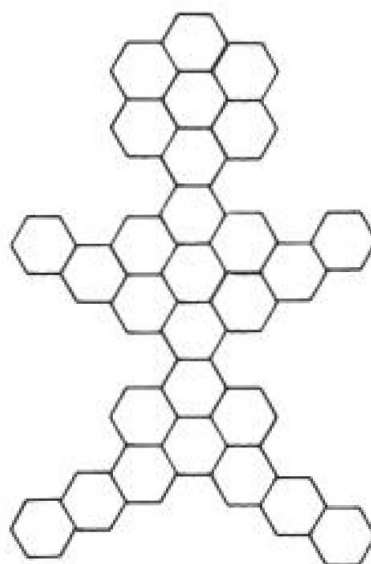
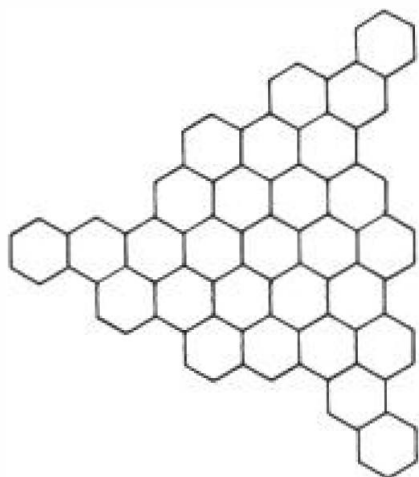
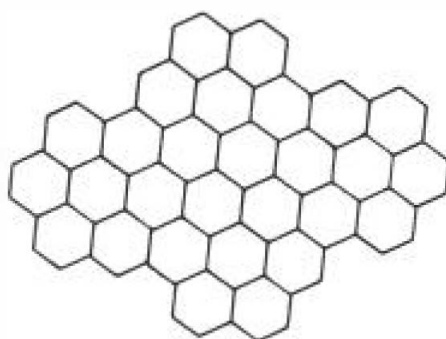
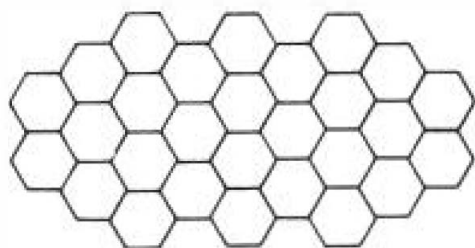




SHEET 2



SHEET 3



Have your students cut the seven tetrahexes out of the hexagonal grid on sheet 1.

*Activity 3.* Ask the students to find all the motions of symmetry for the seven tetrahexes and one of the hexagons. The motions of symmetry for a shape can be found by tracing the shape on a piece of paper and then finding all the ways the shape can be turned and flipped and then placed so that it fits exactly on the original tracing. It might be helpful to label the shape in some manner first. Two of the tetrahexes can be placed on their tracings in only one way. These are said to be asymmetric shapes. Two of the tetrahexes can be placed on their tracings in exactly two ways. One of these has a line of symmetry, which means that you can flip the piece over a line so that it fits on its tracing. The other has a point of symmetry, meaning that a half turn in the plane about a point is a possible symmetry.

*Sheets 2 and 3, Activity 1.* Use all seven tetrahexes to make each of the designs on these sheets. It may be necessary to turn some of the pieces over to find a solution. Your students should find that selecting a key piece and deciding where they think it might go is a good means of getting started.

In many instances the position of the first piece determines the positions of other pieces, leading to a solution or an impossible situation. If the latter occurs, the position of the key piece may be in question.

For some students, it might be helpful to reproduce the designs shown on sheets 2 and 3 in full size. This way they can arrange the seven tetrahexes directly on the design.

*Activity 2.* Have your students determine the motions of symmetry for the shapes on sheets 2 and 3. See if they can do this without cutting out the shapes. Warn them to be careful, since one of the shapes is asymmetric.

#### *Follow-Up*

Seven more designs that can be made with the tetrahexes can be found on page 149 of Martin Gardner's book, *Mathematical Magic Show* (1977). Many more designs exhibiting various kinds of symmetry can be made from the seven tetrahexes.

#### **References**

- Gardner, Martin. *Mathematical Carnival*. New York: Alfred A. Knopf, 1975.  
———. *Mathematical Magic Show*. Alfred A. Knopf, 1977.

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## TAPESTRY OF MATHEMATICS

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