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THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

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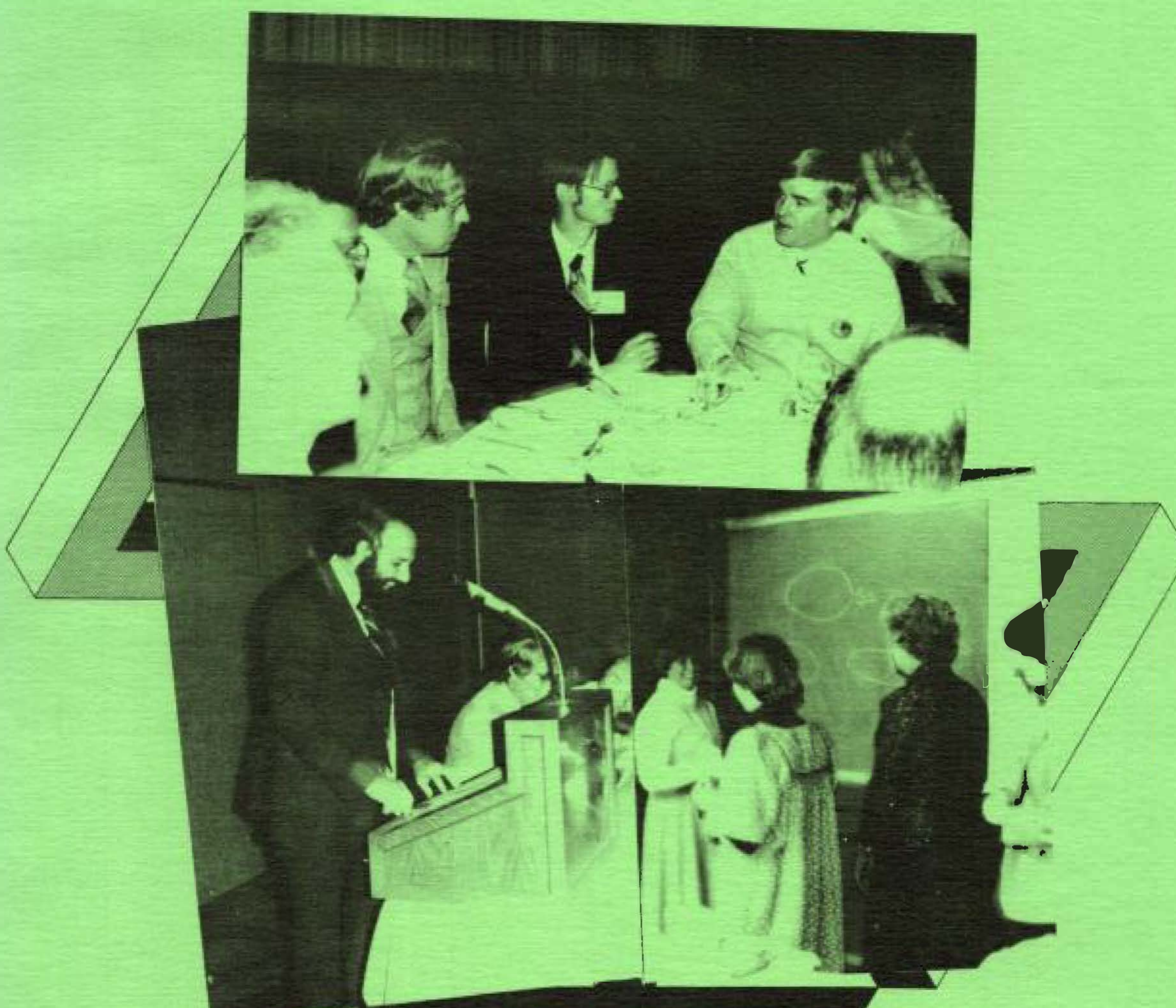

Scenes from the 18th Annual Conference
October 13 and 14, 1978

delta-k

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Volume XVIII, Number 4 **May 1979**

Contents

EDPRESS

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From the Editor's Desk

Your editor, NCTM representative, and NCTM "name of site" chairman were all able to attend the annual NCTM meeting. Resolutions passed at the business session that affect MCATA and other Alberta math teachers expressed the following:

- a need for improvement in the ability to teach reading math and/or improvement in the ability of students to read math properly;
- a need for proper facilities for students with special needs, both handicapped and exceptionally gifted;
- a need for better public awareness of the importance of mathematics in modern society and its future;
- a need for the use of multi-year membership dues;
- a need for the use of particular articles from the *Arithmetic Teacher* and the *Mathematics Teacher* in the classroom, including duplicating thereof without special permission;
- a need for increased publication of material for special education needs; and
- a need for a position on the use of microcomputers.

The publications personnel already accept the policy of using the material in the classroom, including duplicating, so long as only pertinent articles are used and no charge to students is involved.

Almost all of the programs that followed the business session focused on reading in math and/or special needs in a very real way and provided some concrete assistance toward attaining the goals for improvement in teaching in these areas.

Another activity of our group included making contacts to enlarge our Calgary "name-of-site" meeting, October 11-13, 1979. We now have a program that includes one or more activities for every grade level for every session, with local and international speakers in abundance. Programs will be sent to each MCATA and/or NCTM member soon for your early perusal and planning. Whatever needs you may have or may anticipate will be given exposure at this meeting, so you know you will return home with more knowledge, ideas, and inspiration for having participated. Further, there will be NCTM representatives at this meeting who will be ready to discuss your concerns about the above resolutions, about publications, or about other issues.

Ed Carriger
Editor

What's in a Cryptarithm

by *William J. Bruce*

The purpose of this article is to suggest to teachers of mathematics how cryptarithms can be used as a source for design and instruction experiences in arithmetic and algebra.

Cryptarithms have been around for a long time, but for most people they have been little more than a superficial exercise in trial and error problem solving. Invariably these have been additive. By use of an example, the problem may be stated as follows:

Solve on the set {0,1,2,3,4,5,6,7,8,9} the *addition* cryptarithm

$$\begin{array}{r} S\ E\ E \\ \underline{S\ E\ A} \\ E\ B\ B \end{array}$$

in which each different literal number is to be replaced by a different decimal digit and S and E have nonzero replacements.

Examination of this example shows that four different literal numbers are involved. The replacements for these literal numbers must be such that the resulting numbers for S E E and S E A must add up to the number replacement for E B B. Carrying is permitted. By trial and error alone, even an elementary-grade student might find a solution to be one or the other of

$$\begin{array}{r} 255 \\ \underline{256} \\ 511 \end{array} \quad \text{and} \quad \begin{array}{r} 377 \\ \underline{378} \\ 755 \end{array}.$$

In this process, considerable practice in addition with carrying will have been experienced, as well as the thrill of having solved a problem, even though, possibly, only partially. The question should arise as to whether other solutions exist and the student will realize that the method of trial and error is limited.

Junior high school students should be able to pursue the problem of finding the general solution of this cryptarithm. It will be evident that the system

$$\begin{aligned} E + A &= B \\ E + E &= B \\ S + S &= E \end{aligned}$$

must be examined. From the first two relations it follows that E + A = E + E, so that A = E. But this is not possible since the replacements for A and E must be different. Thus, this system does not yield a solution. Note that no carrying has been used.

If carrying is introduced in the first equation, we get

$$\begin{aligned} E + A &= 10 + B \\ E + E + 1 &= B \\ S + S &= E \end{aligned}$$

as another possible system. But from the first two relations it follows that $E + A - 10 = E + E + 1$, so that $A = E + 11$. This implies that $A \geq 12$, since $E > 0$. But none of the replacements can be greater than 9, so the above system cannot have a solution either.

Similarly, if carrying is permitted in the second equation, we have

$$E + A = B$$
$$E + E = 10 + B$$
$$S + S + 1 = E.$$

The first two equations now yield $A = E - 10$ and this implies $A < 0$, which is not admissible, so this system has no solution.

There remains only one other possibility, namely, that carrying might occur in both of the first two equations. The system then will be

$$E + A = 10 + B$$
$$E + E + 1 = 10 + B$$
$$S + S + 1 = E.$$

From the first two equations, we get $A = E + 1$, which is possible. Since $E + A \geq 10$, and since $2S = E - 1$ from the third equation, the only possibilities are found by using these relations and choosing E first, say, to obtain

E = 5	6	7	8
A = 6	7	8	9
B = 1	3	5	7
S = 2	-	3	-

Therefore, this system yields only two solutions, namely,

$$\begin{array}{r} 255 \\ \underline{256} \\ 511 \end{array} \quad \text{and} \quad \begin{array}{r} 377 \\ \underline{378} \\ 755 \end{array} .$$

Since there are no other systems to solve, these are the only solutions of the cryptarithm. The procedure just described is applicable, with only minor differences, to the problem of finding the general solution of any given cryptarithm.

Students should be encouraged to experience the fun of designing and trying to solve their own addition cryptarithms. They should restrict their choice at first to three-letter words with few unknowns. The ones with three unknowns are not numerous. For example, the silly ones,

$$\begin{array}{c} B\ A\ A \\ \underline{B\ A\ A} \\ E\ B\ B \end{array} \quad \text{and} \quad \begin{array}{c} A\ B\ A \\ \underline{A\ B\ A} \\ B\ E\ B \end{array} ,$$

have three unknowns and two solutions each. Those with four unknowns are quite easily designed. These could be

$$\begin{array}{c} T\ A\ R \\ \underline{F\ A\ T} \\ R\ A\ T \\ \text{(no solution)} \end{array} \quad \begin{array}{c} F\ A\ T \\ \underline{R\ A\ T} \\ T\ A\ R \\ \text{(3 solutions)} \end{array} \quad \begin{array}{c} A\ N\ N \\ \underline{E\ A\ T} \\ T\ E\ A \\ \text{(2 solutions)} \end{array} .$$

As more and more unknowns are included, an even wider variety is possible and in some cases the number of solutions becomes quite great. The following have five unknowns:

```
  A N N          A N N          T O M
  T O O          N O T          T O O
  -----          -----          -----
  F A T          F A T          F A T
(10 solutions)  (no solution)  (11 solutions) .
```

For those with six unknowns, try

```
   P A T          S A M        T E N     S U N     D A N     E A T
   S I P          S I P        P I N     N O T     N O T     B A D
   -----          -----        -----     -----     -----     -----
   P O P          P O P        B E T     H O T     B A D     D O G .
(no solution) (many solutions)
```

For seven unknowns, attempt

```
   E A T       J A N       T E A       D A N       P A T
   H O T       N O T       F O R       E A T       N O T
   -----       -----       -----       -----       -----
   D O G       B A D       T W O       D O G       B A D
                                              (320 solutions) .
```

When cryptarithms are extended to four-letter words, the solutions involve solving systems of four equations but the procedure is the same as before. Consider the addition cryptarithm

```
  J E A N
  B A R R
  -------
  N I C E .
```

The systems that must be considered are

CASE I. No carrying

N + R = E
A + R = C
E + A = I
J + B = N (no solution)

CASE II. Carrying in one equation

(a) N + R = 10 + E
A + R + 1 = C
E + A = I
J + B = N
(4 solutions)

(b) N + R = E
A + R = 10 + C
E + A + 1 = I
J + B = N
(no solution)

(c) N + R = E
A + R = C
E + A = 10 + I
J + B + 1 = N
(10 solutions)

CASE III. Carrying in two equations

(a) N + R = 10 + E
A + R + 1 = 10 + C
E + A + 1 = I
J + B = N
(26 solutions)

(b) N + R = 10 + E
A + R + 1 = C
E + A = 10 + I
J + B + 1 = N
(no solution)

(c) N + R = E
A + R = 10 + C
E + A + 1 = 10 + I
J + B + 1 = N
(2 solutions)

CASE IV. Carrying in three equations

N + R = 10 + E
A + R + 1 = 10 + C
E + A + 1 = 10 + I
J + B + 1 = N
(10 solutions)

A total of 52 solutions were found for this addition cryptarithm, but an independent check has not been done.

The four-letter word addition cryptarithms, namely,

$$\begin{array}{c} \text{D E A R} \\ \underline{\text{E D N A}} \\ \text{M E A N} \end{array} \quad \text{and} \quad \begin{array}{c} \text{C A S H} \\ \underline{\text{B U Y S}} \\ \text{C A R S} \end{array},$$

have no solutions, but

$$\begin{array}{c} \text{D E A R} \\ \underline{\text{E D N A}} \\ \text{N I C E} \end{array} \quad \text{and} \quad \begin{array}{c} \text{L I V E} \\ \underline{\text{V I L E}} \\ \text{E V I L} \end{array}$$

do have solutions. The last one is special in that it has only one solution.

Five-letter ones are very common. Try

$$\begin{array}{cccccc} \text{W R O N G} & \text{R I G H T} & \text{P O L A R} & \text{T A R T Y} & \text{A P P L E} & \text{T I T H E} \\ \underline{\text{W R O N G}} & \underline{\text{R I G H T}} & \underline{\text{B E A R S}} & \underline{\text{P A R T Y}} & \underline{\text{M A P L E}} & \underline{\text{T I T H E}} \\ \text{R I G H T} & \text{W R O N G} & \text{L O O S E} & \text{F O R A Y} & \text{B E E C H} & \text{F I F T H} \\ \text{(20 solutions)} & & & & & \text{(one solution)} \end{array}$$

in which the number of solutions is given only for the first and the last ones.

Subtraction cryptarithms can be designed and solved in much the same way as addition ones. Consider the *subtraction* cryptarithm

$$\begin{array}{c} \text{E B B} \\ \underline{\text{S E A}} \\ \text{S E E} \end{array}$$

in which each different literal number is to be replaced by a different decimal digit and S and E have nonzero replacements. The systems that must be considered are as follows:

CASE I. No borrowing

$B - A = E$
$B - E = E$
$E - S = S$

No solution is possible for this case because the first two equations imply $A = E$.

CASE II. Borrowing in one equation

(a) $B + 10 - A = E$
$(B - 1) - E = E$
$E - S = S$
(no solution)

(b) $B - A = E$
$B + 10 - E = E$
$(E - 1) - S = S$
(no solution)

CASE III. Borrowing in two equations

$B + 10 - A = E$
$(B - 1) + 10 - E = E$
$(E - 1) - S = S$

This system yields two solutions, namely,

511		755
256	and	378
255		377

Note that the borrowing of 1 in the tens and hundreds places is indicated by B - 1 and E - 1, respectively, followed by the addition of 10 to each of the left members of the previous equations. Once the system is formed, the solution sets are determined as in the addition cases.

Rearrangement of each system for the *subtraction* cryptarithm will yield the corresponding system for the equivalent *addition* cryptarithm

```
 S E E
 S E A
 E B B .
```

This will be true always, so all *subtraction* cryptarithms can be solved first as *addition* cryptarithms to obtain the solution sets.

Teachers are urged to attempt a school project on the design and solution of both addition and subtraction cryptarithms. Unusual ones and original ones that are found are usually accepted for publication in both mathematics and science journals.

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CONSTRUCTIVE EXPERIENCES WITH DECIMALS

by *T. E. Kieren*
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Edmonton, Alberta

The first of a series of decimal exercises were published in the March 1979 issue of delta-k. *The following exercises are a continuation of the series.*

DECIMAL TASK SET 1:
Tenths, Hundredths, Thousandths

On your table you should have two flats divided into 100 congruent smaller squares, a number of longs, and a number of smaller cubes. You should also have one large cube.

1. If a long is considered as one unit, what fractional name would describe one of the small squares?

   Why?

2. Using longs and small cubes, illustrate the following:

   (a) 7/10

   (b) 3 6/10

3. Using longs and cubes, find the following:

   (a) 3 6/10 + 4 1/10

   (b) 2 5/10 + 1 7/10

   Explain your result:

   (c) 4 5/10 - 2 1/10

   (d) 1 7/10 - 9/10

4. Write a set of directions for children and have them answer the following using longs and squares:

   (a) 2 7/10 = /10

   (b) 13 4/10 = /10

   (c) 43/10 = 4 /10 = 3 /10

5. Sketch a cork board display which shows how to find the following results using longs and cubes:

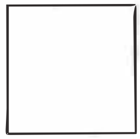
   11 ÷ 10

6. Change gears! Suppose a *flat* is now considered as a *unit*. What fractional name is now assigned to:

   (a) longs ______________?

   (b) small cubes ______________?

   (c) large cubes ______________?



7. Using the materials, illustrate each of the following at least two ways:

   (a)

	flats (1)	longs (1/10)	cubes (1/100)
25/100	______	______	______
	______	______	______

   (b)

	flats (1)	longs (1/10)	cubes (1/100)
1 13/100	1	1	1
	______	______	______
	______	______	______

8. Develop a short *demonstration* for children to show them how to use the blocks to solve the following:

   (a) 1 3/10 + 27/100 =

   (b) 36/100 + 29/100 =

   (c) 4/10 + 3/100 + 49/100 =

   (d) 1/2 + 1/5 + 3/20 =

9. Using the materials, show two fractions equivalent to the following:

   (a) 1/2 5 longs (5/10) 50 cubes (50/100)

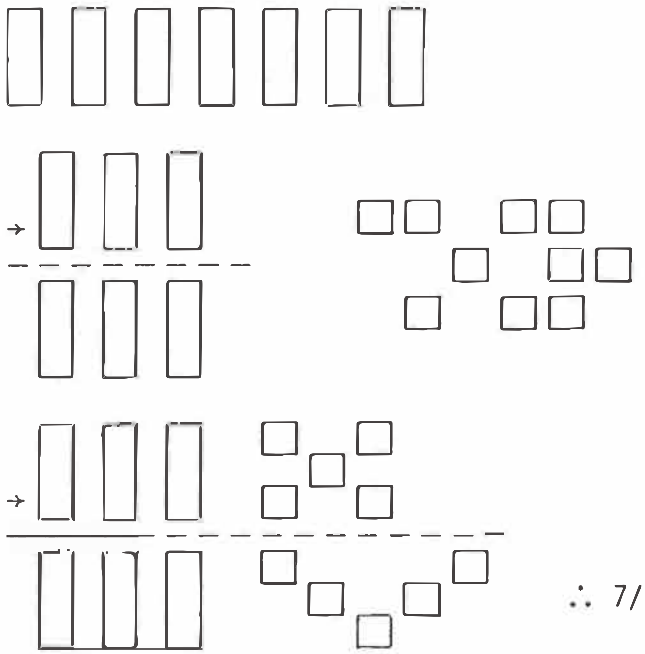
   (b) 1/4

   (c) 2/5

   (d) 3/4

   (e) 7/20

10. The diagram below shows 7/10 ÷ 2.



∴ 7/10 ÷ 2 = 35/100.

Sketch diagrams for the following:

(a) 8/10 ÷ 5

(b) 1 8/10 ÷ 5

(c) 25/100 ÷ 5/10
(Can you think of an equivalent division question?)

11. Change gears one more time. Suppose the large cube is a unit. What is a sequence of activities for children which will lead up to their being able to do the following:

(a) 37/100 + 25/1000 + 5/10

(b) 2 ÷ 3

The purpose of this task set has been to show a way of providing meaning to fractions which relate to decimals, to show physically the simplicity of the decimal operation of adding, and to show experiences relating decimal fractions to other fractions.

CONSTRUCTIVE RATIONAL NUMBER TASKS

by *T. E. Kieren*
Faculty of Education
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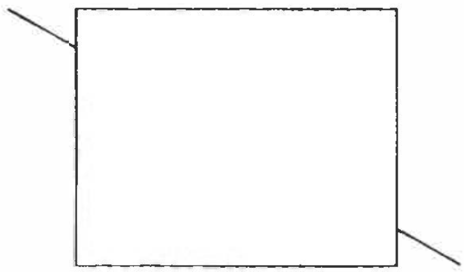
Edmonton, Alberta

The first of a series of number tasks were published in the March 1979 issue of delta-k. *The following exercises are a continuation of the series.*

FRACTION TASK 3: Operators and Machines

1. Complete the table below:



Input	*Output*
10	6
20	12
50	30
100	60
15	____
5	____
75	____
5000	____
3000	____

The name of this machine is a ________ for ________ machine. For every __5__ that go in, ________ come out.

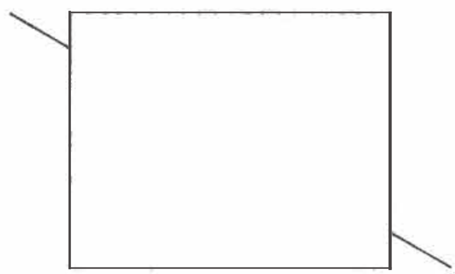
2. For the above machine, complete this table:

Input	*Output*
____	9
____	300
____	600

How did you know these results?

You were using the notion of "inverse." A machine which would do the reverse of the 3 for 5 machine above would be a 5 for 3 machine.

3. Find a partner. Each of you make up a machine with a mixed list of eight inputs and outputs. Make sure you give three complete pairs. Exchange lists and see who can give the most correct answers. Here is a sample game machine:



Input	*Output*
15	10
9	6
60	40
6	____
____	2
24	____
18	____
____	20

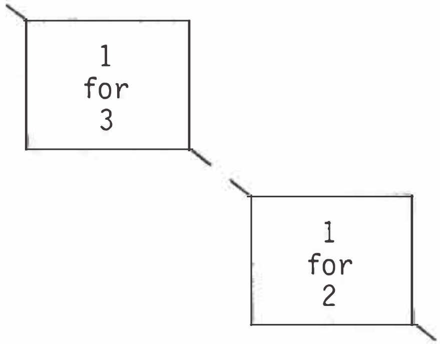
This machine is a __________ for __________ machine. Its "inverse" machine would be a __________ for __________ machine.

4. Here is a mysterious machine's input and output list. Can you complete it?

Input	*Output*
10	10
3	3
727	727
____	46
29	____
11	____
7777	____
____	21

Can you name this machine? __________ for __________. A formal mathematical name for this machine is the *identity machine*. Its inverse machine would be a __________ for __________ machine.

5. Here are two machines.



The *output* from the first machine is the *input* for the second machine. Can you complete the table below?

Machine 1			*Machine 2*	
I	0		I	0
30	10	----------	10	5
12	4	----------	4	2
24	8	----------	_____	_____
60	20	----------	_____	10
90	_____	----------	_____	_____
120	_____	----------	_____	_____
_____	2	----------	_____	1
_____	_____	----------	_____	4
_____	_____	----------	_____	2000

Look at these machines carefully. What fraction could you use to automatically get the final result if 300 were put into the first machine?

We can write this result 1 for 3 followed by 1 for 2 is the same as 1 for 6.

In more mathematical symbols, 1/3 ______ 1/2 = 1/6.

6. Use the machine idea to solve the following:

1 for 2 followed by 1 for 2 is ______ for ______.

3 for 4 followed by 1 for 2 is ______ for ______.

1 for 1 followed by 3 for 7 is ______ for ______.

3 for 5 followed by 5 for 3 is ______ for ______.

4 for 7 followed by ______ for ______ is 1 for ______.

7. What mathematical operations and what ideas are related to this approach to fractions?

FRACTION TASK 4:
Part-Whole Equivalence

1. Using the set of 72 objects in front of you, complete the following list of all the ways you can divide 72 objects into subsets of the same size.

   (1) 36 sets of 2

   (2) ____ sets of 36

   (3) 24 sets of ____

   (4)

   (5)

   (6)

   *

   *

   *

   How many ways of partitioning the 72-object set did you get?

   Why is there such an abundance of ways? (Remember Kennedy, pp.268-273)

2. Looking at a list of partitions helps one see ways in which fractions can be expressed. For example, because there are four sets of 18 in 72, 18/72 can be expressed as 1/4.

   Complete the following lists of ways that the partitioning of 72 suggests for expressing various fractions.

   (a) 18/72, 1/4, ____, ____, ____

   (b) 24/72, 2/6, ____, ____, ____

   *(c) 10/72, ____, ____, ____, ____

   *(d) 4/72, ____, ____, ____, ____

   *(e) 17/72, ____, ____, ____, ____

   *Can you fill all the blanks? Why or why not?

   What can you say about the fractions in set (a) above?

   What can you say about all the sets of fractions above?

FRACTION TASK 5:
Measurement and Addition

1. Look back to Fraction Task 1 and use your tape from that task or make a new tape.

2. Measure the following objects as precisely as you can, and complete the following table.

Object	A Side 1	B Adjacent Side	C Both Sides in a Single Measure	D Sum of A + B
(a) book				
(b) table				
(c) room or part of room				

3. What appears to be the relationship between columns C and D in the table?

4. If the two sides of the room measure 6 1/2 and 4 3/8 tapes, we can relate these to the total length 10 7/8 with the following sentence:

   $6\ 1/2 + 4\ 3/8 \doteq 10\ 7/8$

   Using your data, write three sentences which describe the relationship between the side lengths and the total.

   Why do we use $\doteq$ instead of =?

   Why, in theory or in elementary school texts, can we write:

   6 1/2 + 4 3/8 = 10 7/8?

5. Use two pieces of mayfair board which contain units marked off in eighths. Label the points starting at 1/8 with appropriate fractional and whole number names (for example, 3/4, 11/8, 3/2, et cetera).

6. Use the two rulers to add 1/2 and 1/4. Result ________. Write a set of directions for Grade VI or VII children which would tell them how to add numbers using these rulers.

7. Use your ruler to add the following numbers:

   (a) 3/8 + 3/4 = ______

   (b) 5/8 + 3/2 = ______

   (c) 7/4 + 1/8 = ______

   (d) 17/8 + 1/2 = ______

8. Re-label your rulers using mixed numerals (for example, 1 1/2, 2 1/4, et cetera) or at least think of the partitions in those terms. Complete the following:

   (a) 3/8 + 3/4 = ______

   (b) 1 3/8 + 1 1/2 = ______

(c) 1 1/4 + 7/8 = ______

(d) 1 3/4 + 1 5/8 = ______

9. Use your ruler to answer the following questions:

(a) 3/4 = /8?

(b) 5/2 = /4?

(c) 5/4 = /8?

(d) 5/4 + 7/8 = ______

This is the same question as /8 + 7/8 = ______

(e) 3/2 + 5/4 = ______

is the same as /4 + 5/4 = ______

10. The purpose of this task sheet has been to show two things:

A. Fractions or rational numbers can be added! There is no question of common denominators!

B. When you are making up a quick algorithm which uses symbols only, equivalence allows you to make use of the common denominator notion to do so.

AMUSEMENTS IN DEVELOPING GEOMETRY SKILLS

(Clack and Leitch)

In BOOK 1 (informal geometry), exercises are on Mixed Polygons, Squares, Rectangles, Parallelograms, Trapezoids and Triangles, and deal with perimeter, sides, and area of the latter five. Exercises on Circles deal with lengths only. Word searches and scrambled words are found in Miscellaneous Puzzles.

BOOK 2 (informal geometry) deals with Mixed Polygons (perimeter and sides), Squares and Rectangles (perimeter, area, and sides), Parallelograms (perimeter, area, sides, altitude), Trapezoids (perimeter, area, bases, sides, altitude), Triangles (perimeter, area, sides, base, altitude), Circles (diameter, radius, circumference, area), Mixed Polygons and Circles (perimeter, area), Miscellaneous Puzzles (word search, scrambled words, crosswords).

BOOK 3 (formal geometry) deals with Real Number Properties and Logic, Angle Relationships and Perpendicular Lines, Parallel Lines and Planes, Congruent Triangles, Applying Congruent Triangles, Similar Polygons, Similar Right Triangles, Circles, Constructions - Coordinate Geometry, Areas of Polygons and Circles, Areas and Volumes of Solids.

Obtainable from:	*Prices:*		Duplicator Masters
Western Educational Activities Ltd.,	BOOK 1 (elementary)	$7.15	$15.50
Box 3806,	BOOK 2 (junior high)	7.15	15.50
Edmonton T5L 4J8	BOOK 3 (senior high)	8.35	16.75

1979 Alberta High School Prize Examination Results

Prize	$ Amt.	Student	School
Canadian Mathematical Society Scholarship	400	PATON, Gregory	Lindsay Thurber Composite High Red Deer, Alberta
Nickel Foundation Scholarship	400	Not awarded	
First runner-up	150	WONG, Eric	Ross Sheppard Composite High Edmonton, Alberta
Second runner-up	150	MOREWOOD, Robert	Medicine Hat High School Medicine Hat, Alberta

Special Provincial Prizes

Highest Grade 12 student (below first 3)	75	ANTOLAK, John	St. Joseph's Sr. High School Grande Prairie, Alberta
Highest Grade 10/11 student (below first 3)	75	BARAGAR, Arthur	Old Scona Academic High Edmonton, Alberta

District Prizes

District No.	$ Amt.	Student	School
1	50	SLAVEN, Robert	Sir John Franklin High School Yellowknife, N.W.T.
2	50	KLEM, Raymond	H.A. Kostash High School Smoky Lake, Alberta
3	50	FINLAY, Warren	Salisbury Composite High Sherwood Park, Alberta
4	50	PENNER, Robert	Lindsay Thurber Composite Red Deer, Alberta
5	50	SHEPHERD, Douglas	Hugh Sutherland Sr. High Carstairs, Alberta
6	50	NEUFELDT, David	Kate Andrews High School Coaldale, Alberta
7 (1)	50	BOWMAN, John	Old Scona Academic High Edmonton, Alberta
7 (2)	50	WELSH, Michael	Old Scona Academic High Edmonton, Alberta
8 (1)	50	KRYCZKA, John	Bishop Carroll High School Calgary, Alberta
8 (2)	50	KOŁODINSKY, Steve	Queen Elizabeth High School Calgary, Alberta

267 students from 58 schools in Alberta and the Northwest Territories wrote the 1979 examination. The following students took the first 16 places and are nominated for the Canadian Mathematical Olympiad:

Student	School
ANTOLAK, John	St. Joseph's Sr. High School, Grande Prairie
BARAGAR, Arthur	Old Scona Academic High School, Edmonton
BARAGAR, Henry	Old Scona Academic High School, Edmonton
BOWMAN, John	Old Scona Academic High School, Edmonton
KAY, Lewis	Ross Sheppard Composite High School, Edmonton
KOLODINSKY, Steve	Queen Elizabeth Jr.-Sr. High School, Calgary
KRYCZKA, John	Bishop Carroll High School, Calgary
LEUNG, David	Harry Ainlay Composite High School, Edmonton
LOVE, Nathan	J.G. Diefenbaker High School, Calgary
MA, Raymond	Victoria Composite High School, Edmonton
MOREWOOD, Robert	Medicine Hat High School, Medicine Hat
PATON, Gregory	Lindsay Thurber Composite High School, Red Deer
SHEPHERD, Douglas	Hugh Sutherland Sr. High School, Carstairs
WELSH, Michael	Old Scona Academic High School, Edmonton
WINQUIST, Eric	Bonnie Doon Composite High School, Edmonton
WONG, Eric	Ross Sheppard Composite High School, Edmonton

The following students placed 17-30:

Victoria Cho (McNally Comp. High School, Edmonton), Jocelyn Coates (Old Scona Academic High School, Edmonton), Dean Elhard (William Aberhart High School, Calgary), Warren Finlay (Salisbury Composite High School, Sherwood Park), Brian Hart (Prairie High School, Three Hills), Raymond Klem (H.A. Kostash High School, Smoky Lake), Richard Maisonneuve (Salisbury Composite High School, Sherwood Park), Andrew McIntosh (Bishop Carroll High School, Calgary), David Neufeldt (Kate Andrews High School, Coaldale), William Olsen (Old Scona Academic High School, Edmonton), Robert Penner (Lindsay Thurber Composite High School, Red Deer), Gautam Rao (Eastglen Composite High School, Edmonton), Charles Ursenbach (Dr. E.P. Scarlett High School, Calgary), Carl Wilting (M.E. Lazerte Composite High School, Edmonton).

The following students placed 31-50:

Douglas Allen (William Aberhart High School, Calgary), Mike Blum (Lord Beaverbrook High School, Calgary), Kim Chernowski (Old Scona Academic High School, Edmonton), Edward deBeaudrap (Didsbury High School, Didsbury), Ken deCock (St. Michael's, Pincher Creek), David Filipchuk (Harry Ainlay Composite High School, Edmonton), Donald Haigh (Lethbridge Collegiate Institute, Lethbridge), Joe Kasper (Louis St. Laurent High School, Edmonton), Andrew Kennett (Lord Beaverbrook High School, Calgary), Paul Moret (St. Joseph's Composite High School, Edmonton), Srinath Mutyala (Harry Ainlay Composite High School, Edmonton), David Muzyka (Bonnie Doon Composite High School, Edmonton), Robert Nordal (Dr. E.P. Scarlett High School, Calgary), Robert Proudfoot (Didsbury High School, Didsbury), Michele Reeves (William Aberhart High School, Calgary), Dave Salahub (Sir Winston Churchill High School, Calgary), Robert Slaven (Sir John Franklin High School, Yellowknife), Angelee Wahl (Medicine Hat High School, Medicine Hat), Jeffery Westbrook (Sir Winston Churchill High School, Calgary), Gary Wiens (Didsbury High School, Didsbury).

ANSWER SHEET

To be filled in by the Candidate.

PRINT:

____
Last Name First Name Initial

____
Candidate's Street Address Town/City POSTAL CODE

____
Name of Candidate's School

____
Grade

ANSWERS:

1	2	3	4	5	6	7	8	9	10

11	12	13	14	15	16	17	18	19	20

To be completed by the Department of Mathematics, University of Alberta:

Points		Points Correct	Number Wrong
1-20	5	5 × =	1 × =
TOTALS		C = ______	W = ______

SCORE = Correct - Wrong = ________

PART I **Time: 60 Minutes**

Instructions to Candidates

1. *Please do not open this booklet beyond Page 2 until instructed to do so by the supervisor.*

2. *Please turn now to Page 2 (the next page) and fill in the top four lines - Page 2 is your answer sheet.*

3. *This exam is multiple choice. Each question will be followed by 5 possible answers, labelled A, B, C, D, E. For each question, list your choice of answer in the box on the answer sheet directly above the question number. For example, if you decide the correct answer to question 3 is labelled C, then enter C in the box above 3 on the answer sheet.*

4. *To discourage random guessing, there is a penalty for each incorrect answer; there is no penalty for unanswered questions. Filling in more than one letter in any box counts as a wrong answer for that question.*

5. *At the signal from your supervisor, detach both this page and page 2 (the answer sheet) and begin the examination. Pencil, graph paper, scratch paper, ruler, compass, and eraser are allowed.*

   Calculators are not allowed.

Do all problems. Each problem is worth five points.

1. A triangle has sides of lengths 1, 2 and $\sqrt{3}$.

   Its area is

   (A) 1/2 (B) $\sqrt{3}/2$

   (C) 2 (D) $2\sqrt{3}$

   (E) none of these.

2. $P(x) = 4x^4 - kx^2 + 1$ has two double roots if

   (A) k = 1 (B) k = 2

   (C) k = 3 (D) k = 4

   (E) none of these is true.

3. For what values of a,b is the equation $\left(a^{\log_{10}b}\right)^{ab} = \left(b^{\log_{10}a}\right)^{ba}$ true?

(A) All values of a and b (B) No values of a and b

(C) All positive values of a and b

(D) All negative values of a and b

(E) none of these is correct.

4. The equation of the line that is perpendicular to the line $x + 2y = 3$ and passes through the point $(4,5)$ is

(A) $x - 2y = 3$ (B) $2x - y = 3$

(C) $2x + y = 3$ (D) $-2x + y = 3$

(E) none of these.

5. Let S be the set of points defined by the inequalities $x + y \le 1$, $x - y \le 1$, $y \le 1/2$. The area of the region determined by S is

(A) 3/8 (B) 1/2 (C) 5/8 (D) 1 (E) none of these.

6. X is a square of diagonal 1, Y is an equilateral triangle of side 1 and Z is a right-angled isosceles triangle whose equal sides have length 1 . Comparing the areas of these figures,

(A) X is larger than both Y and Z (B) Y is larger than both Z and X

(C) Z is larger than both X and Y (D) X,Y,Z all have the same area

(E) none of these is correct.

7. A positive integer is squarefree if it cannot be divided exactly by the square of an integer larger than 1 . The number of positive squarefree integers less than 20 is

(A) 0 (B) 3 (C) 9 (D) 13 (E) none of these is correct.

8. The solutions of the equation $(\sin\theta + \cos\theta)^2 = 1$ are

(A) all multiples of $\pi/2$ (B) all multiples of π

(C) all odd multiples of $\pi/2$ (D) all even multiples of π

(E) none of these is correct.

9. The sum of the first 27 odd positive integers is

(A) 153 (B) 196 (C) 144 (D) 216 (E) none of these.

10. $$\left(\left((\sqrt{2})^{\sqrt{2}}\right)^2\right)^{\sqrt{2}}$$ is equal to

(A) 2 (B) $\sqrt{2}$ (C) 4 (D) 8 (E) none of these.

11. If α, β are the roots of $x^2 + 7x - 5 = 0$, then $\alpha^2 + \beta^2$ is equal to

(A) 59 (B) 47 (C) -15 (D) 35 (E) none of these.

12. In the figure, AOB and COD are straight lines and O is the center of the circle, while PD, PA, QB, QC are tangents to the circle. The distance from P to Q is

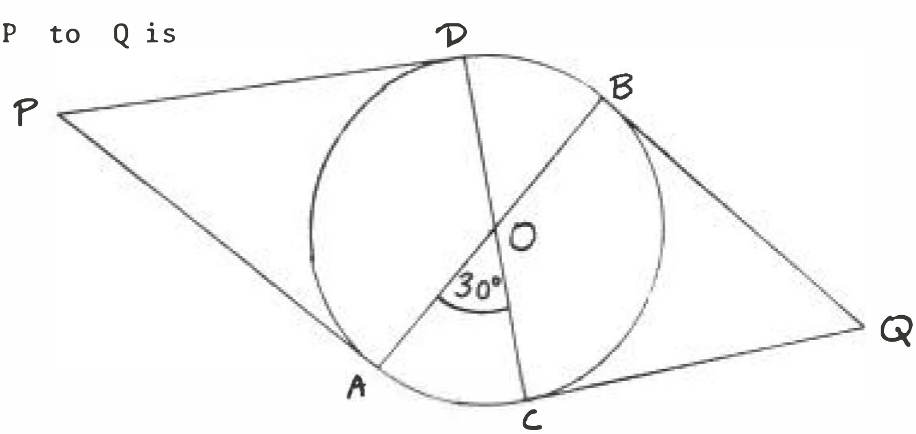
(A) 2

(B) 3

(C) 4

(D) 1/2

(E) none of these.



13. For which values of x is a triangle with sides x, $x+1$, $x+2$ an acute-angled triangle?

(A) $x = 1$

(B) $x > 2$

(C) $x < 4$

(D) $x > 3$

(E) none of these is true.

14. Which of the following inequalities are always true for any pair of real numbers x,y?

(A) $x + y \leq xy$

(B) $(x+y)^2 \geq xy$

(C) $(x+y) \geq xy$

(D) $(x+y)^2 \geq x + y$

(E) none of the preceding is always true.

15. A twelve-hour digital watch displays the hours, minutes and seconds. During one complete day it registers at least one figure 3 for a total time of

(A) 1 hour and 5 seconds

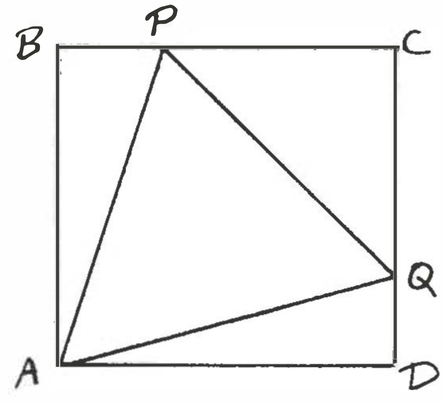
(B) 1 hour, 15 minutes and 15 seconds

(C) 2 hours and 24 minutes

(D) 3 hours

(E) none of these.

16. In the diagram, ABCD is a square of side 1 and APQ is an equilateral triangle. The length DQ is equal to



(A) $1/2$

(B) $\sqrt{2} - 1$

(C) $1/3$

(D) $2 - \sqrt{3}$

(E) none of these.

17. The product of John and Mary's ages is five more than four times the sum of their ages. If Mary is 4 years younger than John, John's age is

(A) 13 (B) 11 (C) 9 (D) 7 (E) none of these.

18. In a poll of 1000 coffee drinkers, 40% preferred their coffee with neither cream nor sugar and 60% of the remainder preferred their coffee with cream only. After deducting both of these groups, 40% of those left preferred their coffee with sugar only. The rest preferred coffee with cream and sugar and their number was

(A) 144 (B) 216 (C) 96 (D) 172 (E) none of these is correct.

19. A sphere and a triangle <u>cannot</u> intersect in exactly

(A) 1 point (B) 2 points

(C) 3 points (D) 4 points

(E) all of the preceding are possible.

20. The picture cards are removed from a pack of 52 playing cards. The number of ways of drawing 2 cards from the remaining 40 so that the sum of the numerical values is 10 is

(A) 100 (B) 10

(C) 50 (D) 70

(E) none of these.

PART II **Time: 110 Minutes**

INSTRUCTIONS TO CANDIDATES

Do each of questions 1, 2 and 3 and any two of questions 4, 5 and 6. All questions are weighted equally and you may attempt them in any order you wish.

Your paper should be concise and complete and each step of your answer should be clearly justified and presented in a legible and intelligible form. Extra credit will be given for particularly elegant solutions as well as for non-trivial generalizations with proof. If there is any doubt about an interpretation of a problem, make a note of that on your paper and state and solve what you consider to be a valid non-trivial interpretation of the problem.

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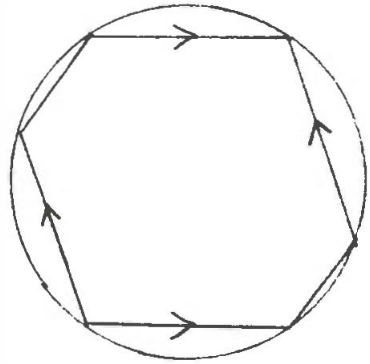
1. A hexagon is inscribed in a circle as shown. If two pairs of opposite sides are parallel, prove that the third pair of sides are also parallel.



2. Determine all real values of A for which the line $x - y = 1$ intersects the curve given by $x^3 + y^3 - 2x^2 + Ay^2 + x + y = 0$ in exactly two distinct points.

3. Prove that for any integer n , $n^5 - n$ is exactly divisible by 5 .

4. If a single die is thrown, each of the numbers 1 through 6 has an equal probability of occurring, namely one-sixth. Now suppose that four such dice are thrown together. Find the most probable value of the total number obtained, and the probability of obtaining this number.

5. Starting with a given positive integer n , the following procedure is used for obtaining a sequence of integers:

Suppose the previous number obtained is m . If m is even, the next number is $\frac{m}{2}$; if m is odd, the next number is the smallest square number larger than m . If 1 is obtained, the sequence terminates. For example, if we started with 20 , the sequence would proceed 20, 10, 5, 9, 16, 8, 4, 2, 1.

Prove that for any starting number n , the number 1 will be obtained after a finite number of steps.

6. In a round-robin tennis tournament, 16 men played each other once with no tie games allowed. Show that it is possible to label the men $M_1, M_2, \ldots, M_{16}$ so that M_1 beat M_2, M_2 beat M_3, M_3 beat M_4, $\ldots$, M_{15} beat M_{16} .

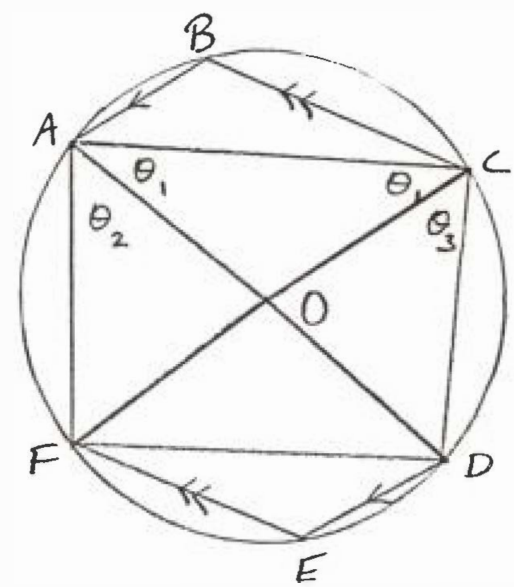
Solutions to Part I

B	D	C	B	E	E	D	A	E	C
1	2	3	4	5	6	7	8	9	10

A	X	D	B	E	D	B	A	E	D
11	12	13	14	15	16	17	18	19	20

X = The diagram associated with this question was incomplete and any answer given was marked as correct.

Solutions to Part II



1. Suppose $AB||DE$, $BC||EF$. Let O be the center of the circle. We have $\widehat{ABC} = \widehat{FED}$ (angles formed by parallel pairs of lines) and so $AC = DF$ (chords of a circle subtending equal angles). Since AO, CO, DO, FO are all equal (radii of the circle), we find that ΔAOC, ΔFOD are conguent isosceles triangles (corresponding sides are equal). Therefore $\widehat{OAC} = \widehat{OCA} = \widehat{OFD} = \widehat{ODF} = \theta_1$, say. Now ΔAOF, ΔCOD are isosceles triangles, and so $\widehat{OAF} = \widehat{OFA} = \theta_2$, $\widehat{OCD} = \widehat{ODC} = \theta_3$, say.

Thus $\widehat{DCA} + \widehat{CAF} = \widehat{OCD} + \widehat{OCA} + \widehat{OAC} + \widehat{OAF} = \theta_3 + \theta_1 + \theta_1 + \theta_2 = 2\theta_1 + \theta_2 + \theta_3$. Similarly, $\widehat{CDF} + \widehat{DFA} = \widehat{CDO} + \widehat{ODF} + \widehat{DFO} + \widehat{OFA} = 2\theta_1 + \theta_2 + \theta_3$, and so $\widehat{DCA} + \widehat{CAF} = \widehat{CDF} + \widehat{DFA}$. But these four angles are the interior angles of a convex quadrilateral, and so add up to 360^o . It follows that $\widehat{DCA} + \widehat{CAF} = 180^o$, and so $CD||AF$

2. Put $x = 1+y$ and substitute into the equation of the curve to get

$$1 + 3y + 3y^2 + y^3 + y^2 - 2 - 4y - 2y^2 + Ay^2 + 1 + y + y = 0 ,$$

which simplifies to

$$y[2y^2 + (A+1)y + 1] = 0 .$$

One root (and hence one point of intersection of the line and the curve) is given by $y = 0$. To obtain exactly one more intersection point, the quadratic $2y^2 + (A+1)y + 1 = 0$ must have a double root. The condition for a double root is that the discriminant $(A+1)^2 - 8 = 0$, that is,

$A = -1 \pm \sqrt{8}$.

3. $$n^5 - n = n(n^4-1) = n(n^2-1)(n^2+1)$$
$$= n(n-1)(n+1)(n^2+1) .$$

Any integer n leaves a remainder of 0, 1, 2, 3 or 4 when divided by 5 . If the remainder is 0, 1 or 4 , respectively, then n, $n-1$, $n+1$, respectively are divisible by 5 . If the remainder is 2 or 3, respectively, then n is of the form $5k \pm 2$, respectively, and so $n^2 + 1 = (5k\pm2)^2$ $= 25k^2 \pm 20k + 5$, which is again divisible by 5 . So in all cases, $n^5 - n$ is divisible by 5 .

4. Throwing a single pair of dice yields the numbers 2 through 12 with the following probabilities:

PROBABILITY TABLE

number:	2	3	4	5	6	7	8	9	10	11	12
prob.	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

Throwing four dice is equivalent to throwing two pairs and the most probable total number obtained is 14. 14 is obtained if the numbers thrown for the two pairs of dice are (2,12), (3,11), ..., or (12,2), and we may use the probability table above to compute the likelihood of throwing 14 as

$$\frac{2(1^2+2^2+3^2+4^2+5^2) + 6^2}{36^2} = \frac{146}{36^2} = \frac{73}{748}$$

5. If n is even, the next number in the sequence is $\frac{n}{2}$, which is less than n If n is an odd number greater than 1, then it lies between the squares of two consecutive positive even integers, say $(2m)^2 < n < (2m+2)^2$, where $m \geq 1$.

The next number in the sequence is $(2m+2)^2$, or it is $(2m+1)^2$ in which case the following number in the sequence is $(2m+2)^2$; in either case $(2m+2)^2 = 4(m+1)^2$ is obtained in the sequence. Then the next two numbers obtained are $2(m+1)^2$, $(m+1)^2$.
Now $m \geq 1$ imples that $m+1 \leq 2m$ and so $(m+1)^2 \leq (2m)^2 < n$.
Thus in all cases, if we start with $n \geq 2$, we eventually obtain a number less than n in the sequence. Repeating this argument inductively, we must eventually obtain 1.

6. Let n be the maximum number of men that can be labelled $A_1, A_2, \ldots, A_n$, say, so that A_1 beat A_2, A_2 beat A_3, ..., A_{n-1} beat A_n. Obviously $n \geq 2$, and we would like to show that $n = 16$. Suppose that $n < 16$.

Let B be any man remaining after we have labelled $A_1, A_2, \ldots, A_n$. If B lost to each of the A_i's , then we may relabel each A_i as B_i , $i = 1, 2, \ldots, n$, and label B as B_{n+1}

Otherwise B must have beaten some A_j and we choose the smallest index j such that B beat A_j . Then B lost to all A_i with $i < j$. Now label B as B_j , relabel A_i as B_i for all $i < j$ and relabel A_i as B_{i+1} for all $i \geq j$.

In both cases we have managed to label $(n+1)$ men as $B_1, B_2, \ldots, B_{n+1}$, so that B_1 beat B_2, B_2 beat B_3, ..., B_n beat B_{n+1} , which violates the assumption that n is the maximum number that can be labelled in this fashion. Hence it must be false that $n < 16$, and so $n = 16$, that is, all the man may be labelled in the appropriate way.

EDITOR'S NOTE: The following "Activities" from the *Mathematics Teacher* and "Ideas" from the *Arithmetic Teacher* are samples of some of the advantages you may have by belonging to the National Council of Teachers of Mathematics. Another advantage is the meeting in Calgary, October 11-13, 1979. Because NCTM is a cosponsor, we will be provided with resources not normally available for our annual meetings. These will include extra personnel from western Canada and western United States in attendance and NCTM financial support for the meeting expenses.

activities

Tetrahexes

by *Raymond E. Spaulding*
Radford College, Radford, Va. 24142

Reprinted from the Mathematics Teacher, *October 1978 (Vol. 7, No. 7, pp.598-602),*

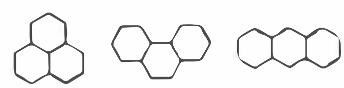
Teacher's Guide

Grade level: 7-10

Materials: Copies of worksheets 1-3 and scissors

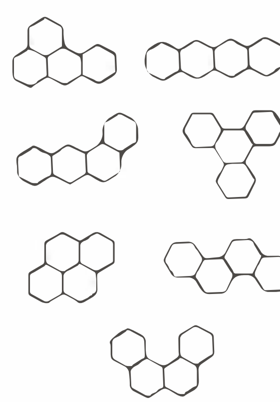
Objectives: The students will solve problems in a geometric setting where congruence and symmetry are significant concepts. They will develop problem attack skills such as making conjectures and testing those conjectures.

Sheet 1, Activity 1. Each student will need one copy of this sheet, which should be made on construction paper, if possible. Ask your students to cut three hexagons out of sheet 1 and then find as many different polygonal shapes as possible that can be made using all three hexagons. One rule in making these shapes is that the hexagons have to match up along a common edge. It should also be noted that two shapes are not considered different if one of these can be flipped, turned, or moved so that it fits on top of the other. Your students should find these three shapes (called trihexes):

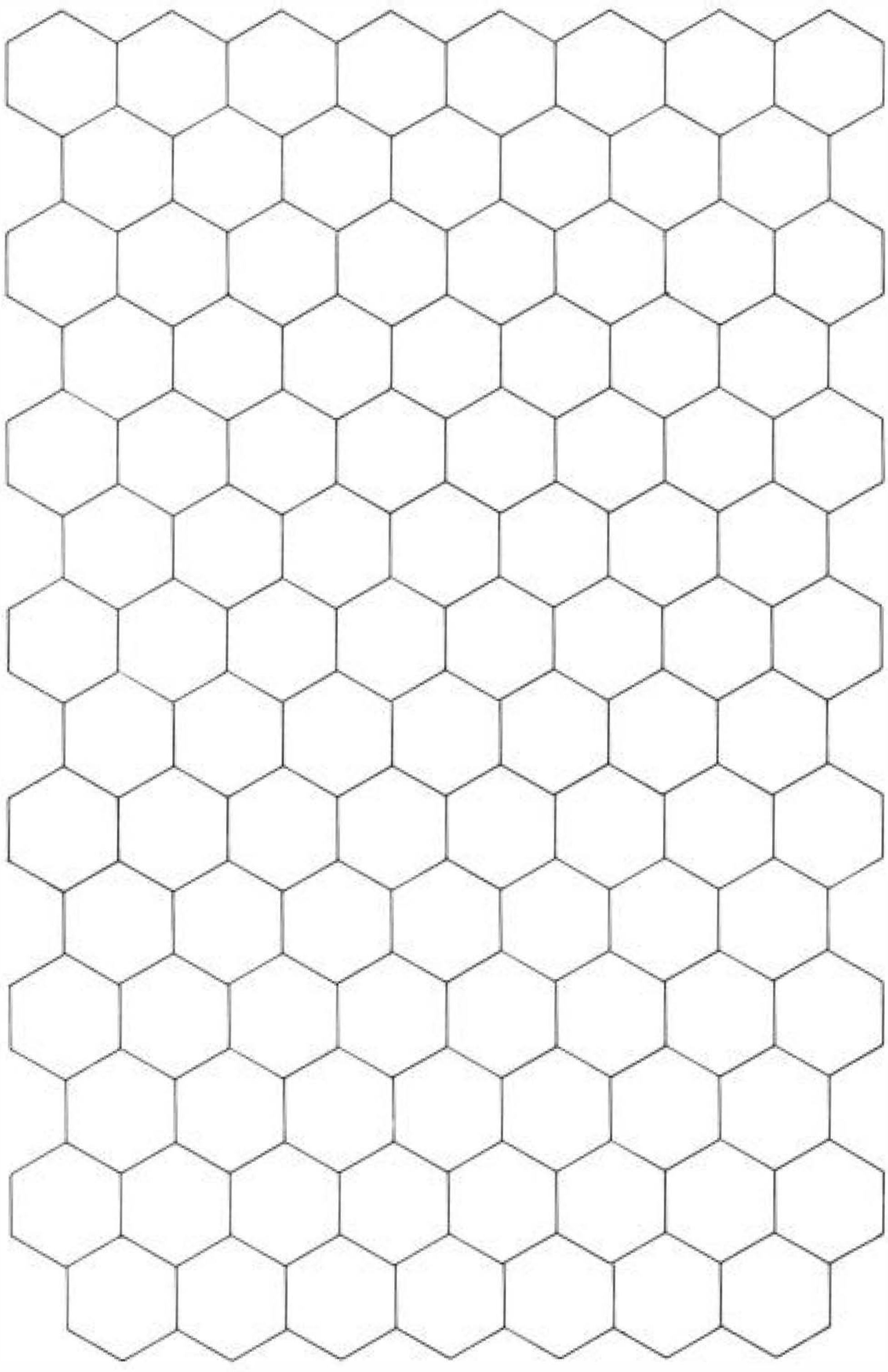


Have your students cut the three different trihexes out of the hexagonal grid on sheet 1.

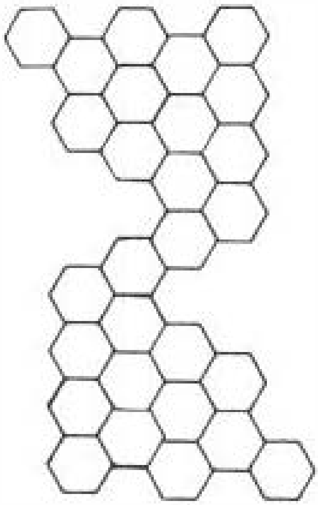
Activity 2. Ask your students to use the trihexes they have just cut out to find all the different polygonal shapes that can be made with ***four*** hexagons. They should find the following shapes (called tetrahexes):

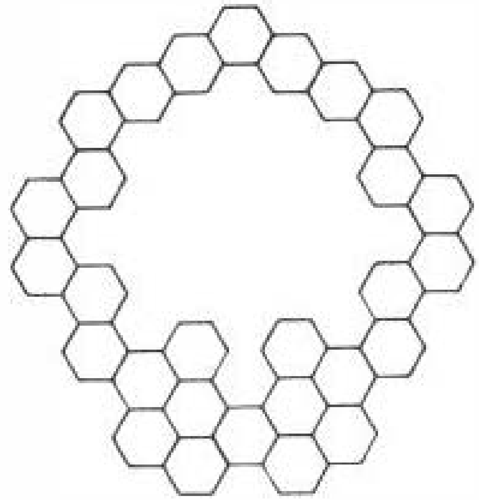


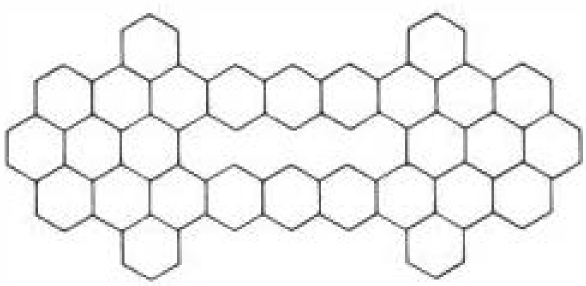
SHEET 1

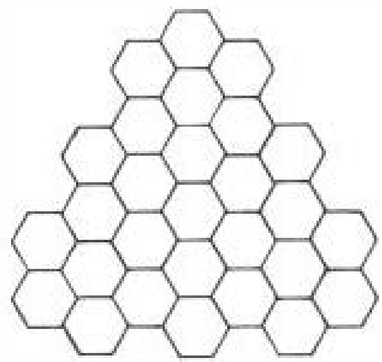


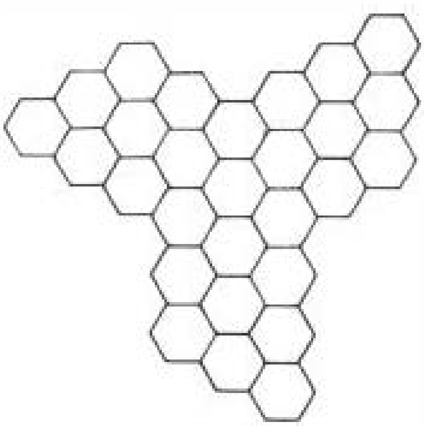
SHEET 2

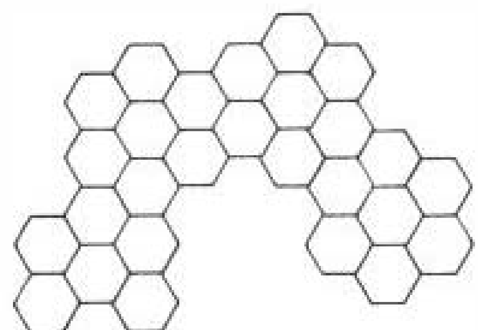




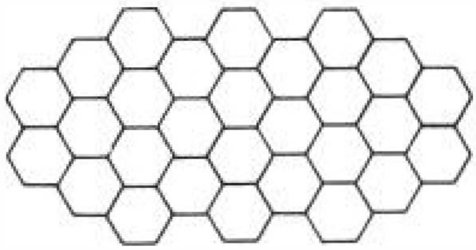


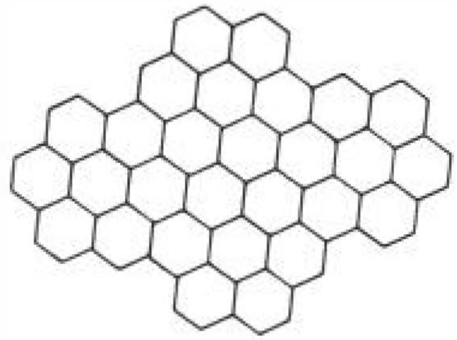


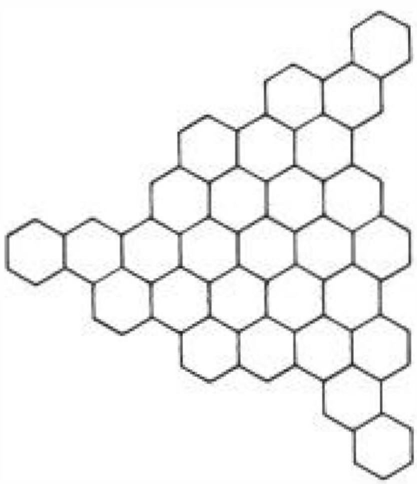


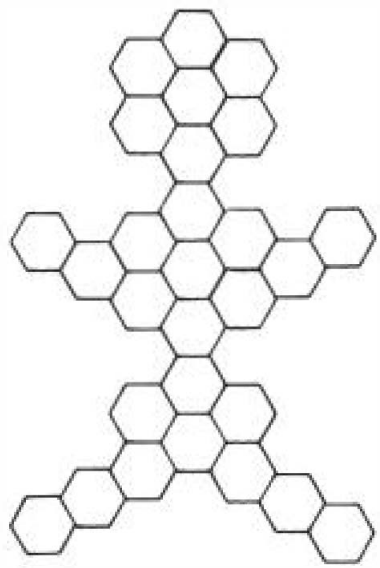


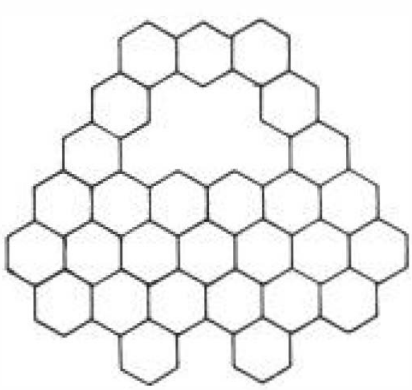
SHEET 3

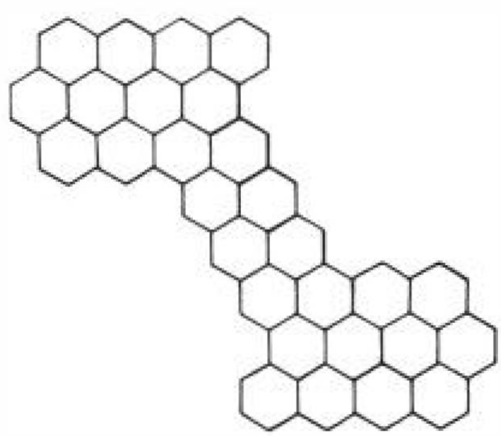












Have your students cut the seven tetrahexes out of the hexagonal grid on sheet 1.

Activity 3. Ask the students to find all the motions of symmetry for the seven tetrahexes and one of the hexagons. The motions of symmetry for a shape can be found by tracing the shape on a piece of paper and then finding all the ways the shape can be turned and flipped and then placed so that it fits exactly on the original tracing. It might be helpful to label the shape in some manner first. Two of the tetrahexes can be placed on their tracings in only one way. These are said to be asymmetric shapes. Two of the tetrahexes can be placed on their tracings in exactly two ways. One of these has a line of symmetry, which means that you can flip the piece over a line so that it fits on its tracing. The other has a point of symmetry, meaning that a half turn in the plane about a point is a possible symmetry.

Sheets 2 and 3, Activity 1. Use all seven tetrahexes to make each of the designs on these sheets. It may be necessary to turn some of the pieces over to find a solution. Your students should find that selecting a key piece and deciding where they think it might go is a good means of getting started. In many instances the position of the first piece determines the positions of other pieces, leading to a solution or an impossible situation. If the latter occurs, the position of the key piece may be in question.

For some students, it might be helpful to reproduce the designs shown on sheets 2 and 3 in full size. This way they can arrange the seven tetrahexes directly on the design.

Activity 2. Have your students determine the motions of symmetry for the shapes on sheets 2 and 3. See if they can do this without cutting out the shapes. Warn them to be careful, since one of the shapes is asymmetric.

Follow-Up

Seven more designs that can be made with the tetrahexes can be found on page 149 of Martin Gardner's book, *Mathematical Magic Show* (1977). Many more designs exhibiting various kinds of symmetry can be made from the seven tetrahexes.

References

Gardner, Martin. *Mathematical Carnival.* New York: Alfred A. Knopf, 1975.

——. *Mathematical Magic Show.* Alfred A. Knopf, 1977.

Ideas

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IDEAS for this month consists of two posters that provide counting problems from geometry. Students will need several days to consider the questions. Supplementary activities can be found in the NCTM publication *Boxes, Squares, and Other Things*, by Walters.

The posters are arranged so that you can make copies for the students to use along with the posters.

IDEAS For Teachers

"Squares, Squares, and More Squares"

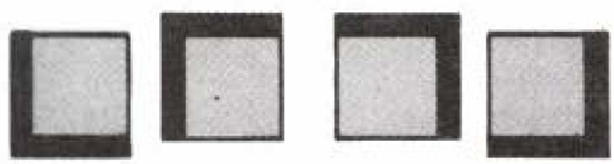
Objective:

To develop two-dimensional visualization and to encourage students to search for patterns as a way to solve problems. Fundamental concepts of area are useful.

Levels: 2,3,4,5

Directions for teachers:

1. Put the poster on the bulletin board and let students think about the questions for several days. Provide space next to the poster for students to write their answers.
2. Be sure students understand that orientation of the small squares does not matter. The following are all counted the same way:



That is,

 means "squares with two colored sides."

Similarly,

 means "squares with one colored side," and

 means "squares with no colored sides."

3. After students have had several days to think about the poster, discuss the answers with the class. Help students identify patterns in the data. (See tables 1 and 2.)

Table 1

Figure	How many? 	
2 by 2	4	0
2 by 3	4	2
2 by 4	4	4
2 by 5	4	6
2 by 6	4	8

Table 2

Figure		How many? 	
3 by 3	4	4	1
4 by 4	4	8	4
5 by 5	4	12	9
6 by 6	4	16	16

Note in table 2 that the numbers in the second column are successive multiples of four, and the numbers in the third column are perfect squares.

Going further:

1. Extend each series of figures. Let students predict the numbers of each type of square, then have them check the answers by drawing the figures and counting.
2. Create a new series of figures; for example, 3 by 3, 3 by 4, 3 by 5, 3 by 6, and so on. Have students predict the correct numbers and then check the answers by drawing the figures and counting.

IDEAS For Teachers

"Cubes, Cubes, and More Cubes"

Objective:

To develop three-dimensional visualization and to encourage students to search for patterns as a way to solve problems. Concepts of area and volume are used.

Levels: 5,6,7,8

Directions for teachers:

1. Put the poster on the bulletin board and let students struggle with the questions for several days. Provide space next to the poster for students to write their answers.
2. You may want to provide small cubes so that students can build models of some of the figures. Be sure that students understand that each small cube has six faces, each of which is in the shape of a square.
3. After students have had several days to think about the poster, discuss the answers with the class. Help students identify patterns in the data. (See tables 3 and 4.)

Table 3

Figure	Number of painted faces 3	2
2 by 2 by 2	8	0
2 by 2 by 3	8	4
2 by 2 by 4	8	8
2 by 2 by 5	8	12

Table 4

Figure	Number of painted faces 3	2	1	0
2 by 2 by 2	8	0	0	0
3 by 3 by 3	8	12	6	1
4 by 4 by 4	8	24	24	8

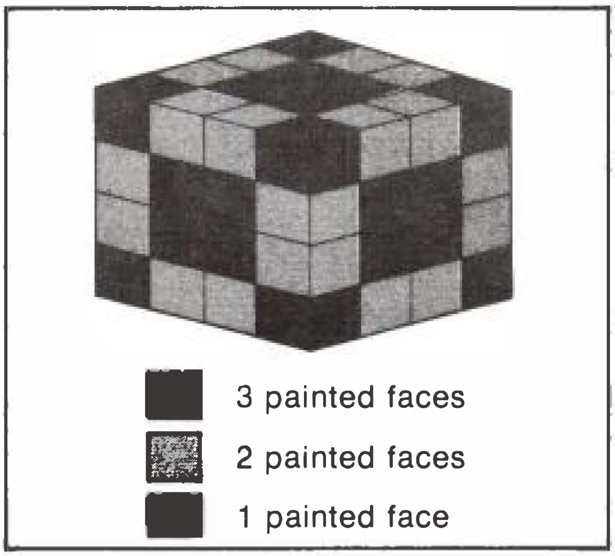
4. If students have trouble finding the answers, check to be sure that they understand that only the outside layers of cubes have painted faces. The 3-by-3-by-3 and 4-by-4-by-4 cubes have smaller inner cubes, two units smaller on an edge, that do not get painted. Similarly, the cubes with one, two, and three painted faces form patterns. (See fig. 1.)

Note that for the cubes (table 4), the numbers in the second column are multiples of 12; the numbers in the third column are six times the perfect squares (6 X 0, 6 X 1, 6 X 4), and the numbers in the fourth column are perfect cubes.

Going further:

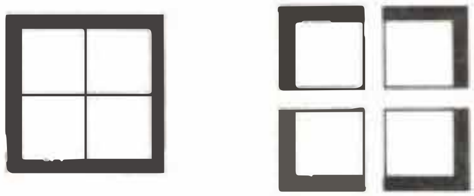
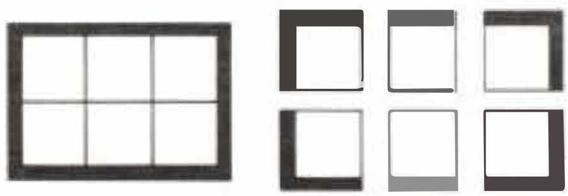
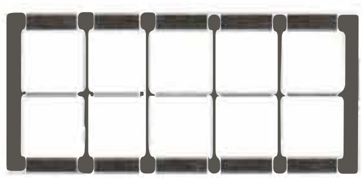
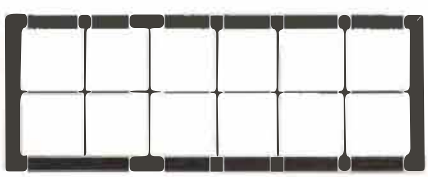
1. Extend each series of figures. Let students predict the numbers of each type of cube and then check the answers by drawing pictures of the figures and counting.

Fig. 1



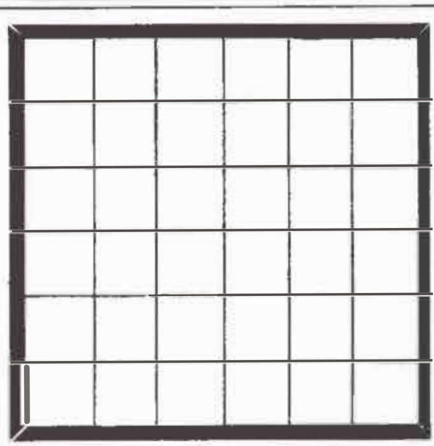
Squares, Squares

How many?

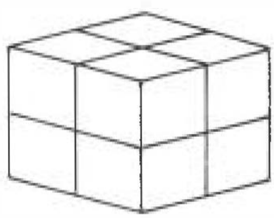
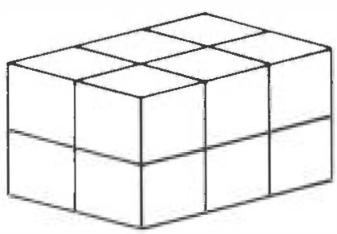
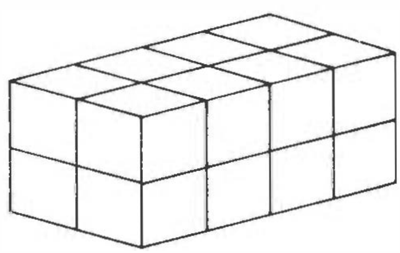
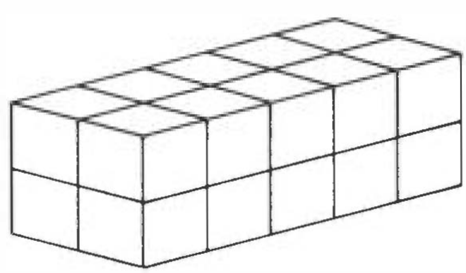
and More Squares

How many?

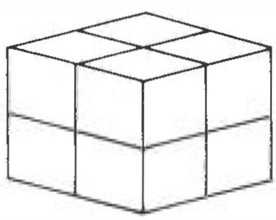
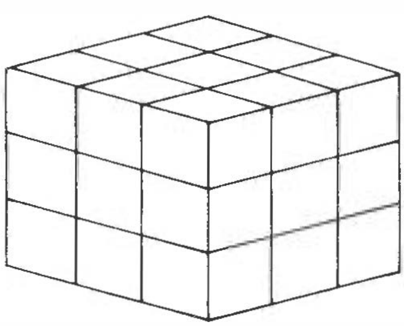
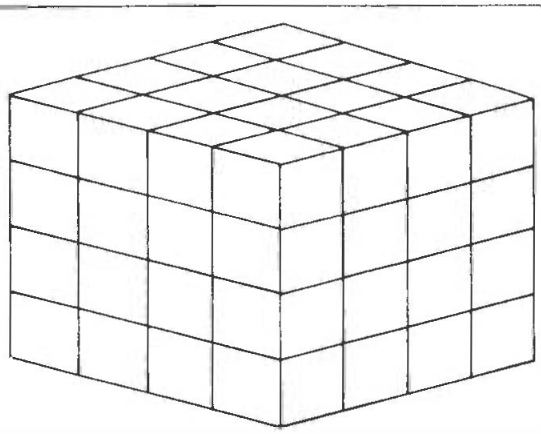
			
			
			
			
			

Cubes, Cubes

Each figure is made from small cubes and only the outside surface of the figure is painted.

	How many small cubes have	
	3 yellow faces	2 yellow faces
		
		
		
		

and More Cubes

	How many small cubes have			
	3 yellow faces	2 yellow faces	1 yellow face	0 yellow faces
				
				
				

Calculators in the Classroom

The NCTM, recognizing the potential contribution of the calculator as a valuable instructional aid in the classroom, has adopted the following position statement:

> The National Council of Teachers of Mathematics encourages the use of calculators in the classroom as instructional aids and computational tools. Calculators give mathematics educators new opportunities to help their students learn mathematics and solve contemporary problems. The use of calculators, however, will not replace the necessity for learning computational skills.
>
> As instructional aids, calculators can support the development and discovery of mathematical concepts. As computational tools, they reduce the time needed to solve problems, thereby allowing the consideration of a wider variety of applications. Furthermore, the use of calculators requires students to focus on the analysis of problems and the selection of appropriate operations. The effective use of calculators can improve student attitudes toward, and increase interest in, mathematics.
>
> Other electronic devices, programmed to generate questions and activities, that provide immediate feedback to students, are not to be confused with calculators or computers. These devices can be used to reinforce computational skills through drill.

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