Remedial Program in Mathematics

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The Problem

Each year more students than ever are required to study mathematics at an advanced level. Some of these students never really learned their high school mathematics; others are mature students who have been away from mathematics for many years. Without help, their success in university math courses is doubtful.

This problem is not unique to the University of Alberta. Most universities in North America are facing the same situation.

Identifying the Problem Students

For the past three years the Mathematics Department at the University of Alberta has been giving a diagnostic exam to students entering first year calculus. The exam given in the fall of 1978 is presented here.

In the fall of 1977, the Department identified 230 problem students. Of these, 63 percent failed or withdrew from calculus.

In the fall of 1978, approximately 2,000 students entering first year calculus took the exam. There were 29 questions. We identified the 324 students who scored less than 13 correct as problem students. *This score was selected because, for budgetary reasons, we could not handle more in our remedial program.*

Attempts to Solve the Problem in 1978

The 24 students who scored below six on the exam were told to repeat a high school algebra course.

The students receiving between six and 13 were advised to take a six-week refresher course organized by the Department. Approximately one-third of the students needing the refresher course took it. A high proportion of these were mature students who were highly motivated hard workers. As in the previous year, they were enthusiastic about the benefits of the course. We were not so successful in attracting students to the refresher program who had graduated from high school the previous year.

In the past, the students who took the refresher course had more success in their calculus course. It is not clear if this is because of the refresher course itself or because the students who take it are highly motivated.

Mathematics Advisory Examination

How many years has it been since you completed the equivalent of Mathematics 30?

[a] 0 (spring of 1978) [b] 1 [c] 2

[d] 3 [e] more than 3

Basic Techniques

1. The number 1/3 is:
   [a] a natural number
   [b] an integer
   [c] rational
   [d] irrational
   [e] all of these

2. The multiplicative inverse of 5 is:
   [a] -5
   [b] 1
   [c] .5
   [d] 1/5
   [e] none of these

3. $x^2 - 9$ factors into
   [a] $(x+3)^2$
   [b] $(x-3)^2$
   [c] $x(x-9)$
   [d] $(x+3)(x-3)$
   [e] none of these

4. $(x^3+1)(x^2+x-1)$ is
   [a] $x^6 + x^3 + x^2 - 1$
   [b] $x^5 + x^4 - x^3 + x^2 + x - 1$
   [c] $x^5 - x^3 + x^2 + x - 1$
   [d] $x^5 + x^4 + x^3 - x^2 - 1$
   [e] none of these

5. $\dfrac{\frac{1}{a} + \frac{1}{b}}{\frac{a}{b}}$ simplifies to:
   [a] $\dfrac{1}{ab}$
   [b] $\dfrac{a}{b} + \dfrac{b}{a}$
   [c] $\dfrac{a + b}{a^2}$
   [d] $\dfrac{a + b}{ab}$
   [e] $\dfrac{a + b}{b^2}$

6. $\dfrac{1}{a + b} + \dfrac{1}{a^2 - b^2}$ simplifies to give:
   [a] $\dfrac{a - b + 1}{a^2 - b^2}$
   [b] $\dfrac{a + b - 1}{a^2 - b^2}$
   [c] $\dfrac{a - b - 1}{a^2 - b^2}$
   [d] $\dfrac{a - b}{(a+b)(a^2-b^2)}$
   [e] none of these

Equations, Inequalities, Division

7. The solution set of $\frac{1}{x+1} + \frac{1}{x+2} = \frac{2}{x+3}$ is:

[a] $\frac{5}{3}$ [b] $\frac{-5}{3}$ [c] $\frac{3}{5}$ [d] $\frac{1}{2}$ [e] none of these

8. The solution set for $\frac{x^2 + 2x}{x + 2} = x$ is:

[a] $\{1\}$ [b] $\{x : x \neq 0\}$ [c] $\{x : x \neq -2\}$ [d] $\{x : x \neq 2\}$

[e] none of these

9. If $x^2 - 2bx + c = 0$, then

[a] $x = -b \pm \sqrt{b^2 - c}$ [b] $x = b \pm \sqrt{b^2 - c}$ [c] $x = c \pm \sqrt{c^2 - b}$

[d] $x = -c \pm \sqrt{c^2 - b}$ [e] none of these

10. Written in the form $y = a(x-h)^2 + b$, the equation $y = 2x^2 - 8x + 7$ becomes:

[a] $y = (x-2)^2 + 7$ [b] $y = 2(x-2)^2 + 1$ [c] $y = 2(x-2)^2 + 7$

[d] $y = 2(x-2)^2 - 1$ [e] none of these

11. Which of the following does not satisfy the inequality $|x| - |y| \geq 2$?

[a] $x = -3, y = +1$ [b] $x = -2, y = 0$ [c] $x = -3, y = 0$

[d] $x = -4, y = +2$ [e] $x = 2, y = -1$

12. $1 - 2x > 5$ is equivalent to:

[a] $x > +2$ [b] $x < 1/2$ [c] $x < -2$ [d] $-x < 2$ [e] $x > -2$

13. Given that $x = -2$ is a root of $x^3 + 2x^2 + x + 2$, we can factor this cubic into

[a] $(x-2)$ (polynomial) [b] x(polynomial) [c] x^2 (polynomial)

[d] $(x+2)$ (polynomial) [e] none of these

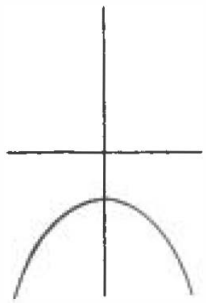
14. $x^5 + 2x^3 - x^2 - x + 1$ divided by $x + 1$ is

$x^4 - x^3 + 3x^2 - 4x + 3$, with remainder

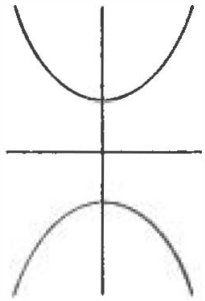
[a] 1 [b] -2 [c] 3 [d] 2 [e] 0

Graphs

The next two questions involve the following 5 possible answers:



[a]



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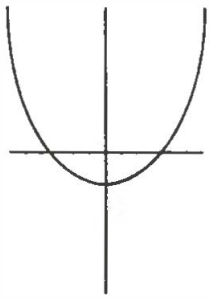

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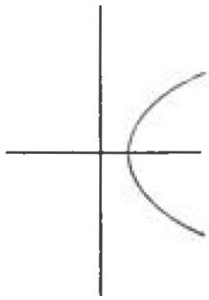
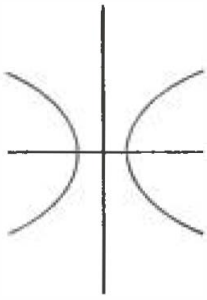


[d]

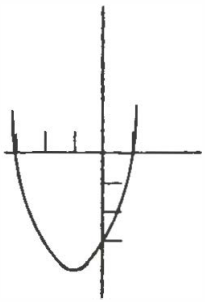


[e]

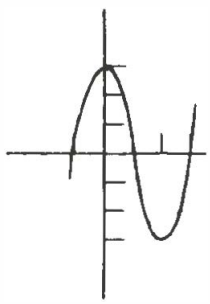
15. The graph of $y = x^2 - 1$ looks like?

16. The graph of $y^2 = x^2 - 1$ looks like?

The next two questions involve the following 5 possible answers:



[a]



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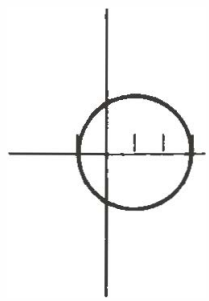
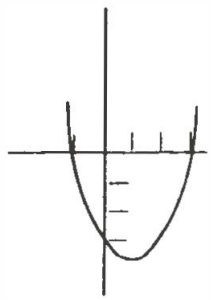
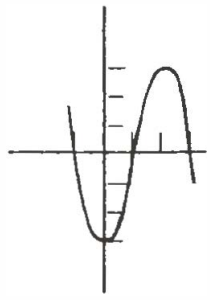


[c]



[d]



[e]

17. The graph of $y = x^2 - 2x - 3$ looks like?

18. The graph of $y = x^3 - 3x^2 - x + 3$ looks like?

Exponents

19. $(3^2)^3$ is:

[a] 729 [b] 27 [c] 3^8 [d] 243 [e] none of these

20. $\frac{2^2}{2^{1/3}}$ is:

[a] $2^{4/3}$ [b] $2^{2/3}$ [c] $2^{5/3}$ [d] $1^{2/3}$ [e] none of these

21. $(3^{-2})^{-3}$ is:

[a] $\frac{1}{243}$ [b] $\frac{1}{3^8}$ [c] $\frac{1}{729}$ [d] 729 [e] none of these

22. a^{x+y} is the same as

[a] $a^x + a^y$ [b] $a^x \div a^y$ [c] $a^x \cdot a^y$ [d] $(a^x)^y$ [e] none of these

Logarithms

23. $\log_{10}100$ is:

[a] 1 [b] 2 [c] 0 [d] 10 [e] 1,000

24. $\log_{10}30$ is:

[a] $3\log_{10}10$ [b] $(\log_{10}15)^2$ [c] $\log_{10}15 + \log_{10}15$ [d] $\log_{10}15 + \log_{10}2$

[e] none of these

25. $\log_{10}1/2$ is:

[a] $\frac{1}{\log_{10}2}$ [b] $-1 + \log_{10}2$ [c] $-\log_{10}2$ [d] $1 + \log_{10}2$

[e] $\frac{-1}{\log_{10}2}$

26. Solve the following equation for y in terms of x; $x = 3^y$.

[a] $y = \sqrt[3]{x}$ [b] $y = 3x$ [c] $y = \frac{\log_{10}3}{\log_{10}x}$ [d] $y = x^3$ [e] $y = \frac{\log_{10}x}{\log_{10}3}$

Trigonometry

27. $\cos(\frac{\pi}{3})$ is equal to:

[a] 2 [b] $\sqrt{\frac{3}{2}}$ [c] -1/2 [d] 1/2 [e] 1

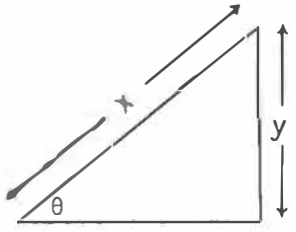
28. 45° measured in radians is the same as:

[a] 1/4 [b] $\pi/4$ [c] π [d] $-\pi/4$ [e] 1/2

29. In the right triangle shown, $\sin(\theta) = 1/5$.
The value of y is:

[a] 5 [b] 5x [c] 1/5 [d] x/5

[e] none of these



30. $(1 - \sin\theta)(1 + \sin\theta)$ is the same as:

[a] $1 + \cos\theta$ [b] $\cos 2\theta$ [c] $-\cos^2\theta$ [d] $\sin 2\theta$ [e] $\cos^2\theta$

Geometrical Theorems - In Slides
by Ved Madan, Red Deer College, Red Deer, Alberta.
Intergalactic Publishing Company, 221 Haddon Avenue,
Westmont, New Jersey 08108.
Price: $30 per set.

Geometrical Theorems - In Slides is a set of 21 fascinating color slides produced by the Canadian author and designed to serve as an innovative instructional aid for the teaching of geometry in secondary schools and colleges and some university level courses. The slides depict some very fundamental theorems in the Euclidean, non-Euclidean, and projective geometry, as well as in topology. The theorems of Pythagoras, Desargues, Pappus, Ceva, and Menelaus, and Euler's formula are some of the topics of study. The slides come with a 12-page supplement which suggests material for involving additional class discussion and project work.
