activities

Discovery with Number Triangles

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Teacher's Guide

Grade level: 7–12

Materials: One set of worksheets for each student, a set of transparencies (if desired) for class discussion, and calculators (optional).

Objectives: Students will (1) collect and organize data, (2) discover number patterns, and (3) make conjectures.

Directions: Distribute the worksheets one at a time. Have the students work either individually or in small groups of two or three. Develop a complete set of answers on the transparency. Do the same for sheets 2 and 3.

Comments: Circulate among the students as they work on the problems. Emphasize that all computations should be accurate and predictions should be checked. Calculators can be used if desired. Be receptive to partial solutions and incomplete reasoning patterns. Be sure to indicate that the generalizations formed have *not* been proved. Students who have had some work with mathematical induction may try proving their results.

The triangle on sheet 3 may be difficult for some students to extend. Have them consider row 5 with the squares and circles as indicated here:

[5] (1) [6] (2) [7] (3) [8] (4) [9] (5).

The first square has 5, the second square has 6, the third square 7, and so on. The first circle has 1, the second 2, the third 3, and so on. Thus, row 6 is

[6] (1) [7] (2) [8] (3) . . . [11] (6)

Supplementary problems:

Sheet 1

1. What is the result of alternately subtracting and adding the terms in row 100? In row 101?
2. What is the sum of the terms in all the rows down to and including row 50?

Sheet 2

1. In row 100, what is the sum of all the terms?
2. How many times does each prime number occur in the triangle?
3. How many times does 60 occur in the triangle?

The Even Triangle

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Row 1 ———————————→ 0
Row 2 ——————————→ 2   4
Row 3 ————————→ 6   8   10
Row 4 ——————→ 12   14   16   18
Row 5 ————→ 20   __   __   __   28
          __   __   34   __   __   __
        __   __   __   __   __   __   __
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1. Study the part of the even triangle shown. Then fill in the blanks.

2. The first term in row 3 is 6 = 2 x 3.
The first term in row 4 is 12 = 3 x 4.
The first term in row 5 is 20 = 4 x 5.
Complete the table below.
Predict the first term for row 100.

Row number	1	2	3	4	5	6	7	...	100
First term			6	12	20			...	

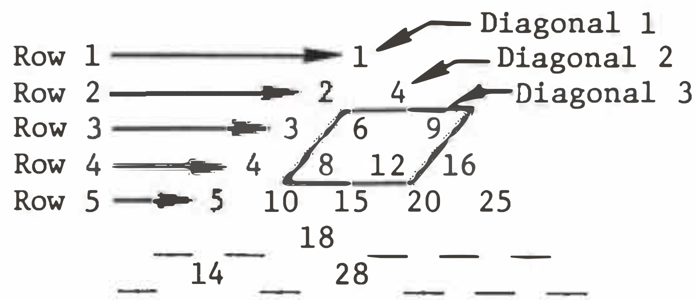
3. The mean or average of the first and last terms in row 3 is (6 + 10)/2, which is 8. The number 8 is 1 less than 9, that is, 8 = 9 - 1. The mean of the first and last terms in row 4 is (12 + 18)/2 = 15 or 16 - 1. Complete the table below. Then predict the result for row 100.

Row number	1	2	3	4	5	6	7	...	100
Mean of first and last term			8	15				...	
Mean, as □ - 1			9-1	16-1				...	

4. The sum of the numbers in row 3 is 6 + 8 + 10 = 24 = 2 x 3 x 4.
The sum of the numbers in row 4 is 12 + 14 + 16 + 18 = 60 = 3 x 4 x 5.
Work similar problems to check the sums for the other rows.
Then predict the sum of the terms in row 100.

Row number	1	2	3	4	5	6	7	...	100
Sum			24	60				...	
Sum, as ⬡ x □ x △			2x3x4	3x4x5	_x5x_			...	

The Multiple Triangle



1. Study the multiple triangle shown. Then fill in the blanks. ______
2. Find the number of terms for each of the first seven rows. Then predict the number of terms for row 100. ______
3. Find the first and last terms for each of the first seven rows. Then predict the first and last terms for row 100.

Row number	1	2	3	4	5	6	7	...	100
First term			3					...	
Last term			9						

4. Notice that 6 is located at the intersection of row 3 and diagonal 2, and 15 is located at the intersection of row 5 and diagonal 3. Find the number located at the intersection of the rows and diagonals indicated in the table below. Then predict the number at the intersection of row 100 and diagonal 71.

Row number	3	5	4	7	6	5	6	7	...	100
Diagonal number	2	3	2	4	5	1	3	5	...	71
Term in the intersection	6	15							...	

Bonus: 5400 is located at the intersection of row 90 and what diagonal?

5. A four-numeral parallelogram has been outlined in the triangle. The sums of pairs of opposite corner numbers are 18 and 17:

$$6 + 12 = 18, \text{ and } 8 + 9 = 17.$$

In each case the product of these opposite corner numbers is 72:

$$6 \times 12 = 72, \text{ and } 8 \times 9 = 72.$$

Find these sums and products for several more four-numeral parallelograms similar to the one shown above. Then state two relationships that appear always to exist for such parallelograms.

The Mixed-Up Number Triangle

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Row 1 ————————→ 1 1
Row 2 ——————→ 2 1 3 2
Row 3 ————→ 3 1 4 2 5 3
Row 4 ——→ 4 1 5 2 6 3 7 4
Row 5 →5 1 6 2 7 3 8 4 9 5
        _ _ 7 _ _ _ _ _ _ _ _ _
      _ _ _ _ _ _ _ _ _ _ _ _ 13 _
```

1. Study the mixed-up number triangle shown. Then extend it by two more rows.
2. Find the number of terms for each of the first seven rows. Then predict the number of terms in row 75.

Row number	1	2	3	4	5	6	7	...	75
Number of terms								...	

3. The sum of the numbers in row 2 is 2 + 1 + 3 + 2 = 8 = 2 x 4. Find the sums of the numbers in the other rows and complete the table below.
Next represent each sum as 2 times a number.
Then predict the sum for row 100.

Row number	1	2	3	4	5	6	7	...	100
Sum		8						...	
Sum, as 2 x □		2x4	2x_					...	

4. Alternately subtracting and adding terms in row 3 gives
3 - 1 + 4 - 2 + 5 - 3 = 6 = 2 x 3.
Similarly, for row 4: 4 - 1 + 5 - 2 + 6 - 3 + 7 - 4 = 12 = 3 x 4.
Do the same for the rest of the first seven rows and predict the result for row 100.

Row number	1	2	3	4	5	6	7	...	100
Alternately subtract and add terms			6	12				...	
Result, as □ x △			2x3	3x4				...	

5. A five-numeral diamond has been outlined in the triangle. The sums of both pairs of opposite corner numbers are 13: 5 + 8 = 13, and 6 + 7 = 13. The products of these opposite pairs of numbers are 40 and 42: 5 x 8 = 40, and 6 x 7 = 42. Find these sums and products for several more five-numeral diamonds. Then state two relationships that appear always to exist for such diamonds.

Sheet 3

1. In row 100, what are the first, second, third, fourth, and sixtieth terms?
2. For any four consecutive terms in a row, compare the sum of the first and fourth terms with the sum of the second and third terms.

Many of the problems on the three activity sheets can be generalized for row *n*. The generalized activities for row *n* could be used as supplementary problems for upper-level students.

Answers:

Sheet 1: (2) 0, 2, 30, 42; 9900; (3) 0, 3, 24, 35, 48; 9999; (4) 0, 6, 120, 210, 336; 999 900. *Sheet 2:* (3) 1, 1; 2, 4; 4, 16; 5, 25; 6, 36; 7, 49; 100; 10 000; (4) 8, 28, 30, 5, 18, 35; 7100; 60; (5) the sums differ by 1; the products are equal. *Sheet 3:* (2) 2, 4, 6, 8, 10, 12, 14; 150; (3) 2, 18, 32, 50, 72, 98; 20 000; (4) 0, 2, 20, 30, 42; 9900; (5) the sums are equal; the products either are equal or differ by 2.

REFERENCE

Ouellette, H. "Number Triangles—a Discovery Lesson." *Mathematics Teacher* 68 (December 1975): 671–74.

A publication featuring math games, tricks, and puzzles for holiday occasions is available from Modern Math Materials. *Let's Celebrate Math* by Geer, Geer, Geer and Gast is a collection of holiday happenings. The following puzzle is found in the book.

HALLOWEEN

George, Geraldine, Gertrude and Gilbert were dressed as a goblin, ghost, gorilla and ghoul. From the clues below, can you find out which costume each person wore?

1. No one is wearing the costume they wore last year.
2. One of the boys is the ghost.
3. Geraldine and the goblin are sisters.
4. The gorilla has no brothers or sisters.
5. George and the goblin are sixth graders.
6. Only the ghoul and Gilbert like bubblegum.
7. Last year Geraldine was the best student in the fifth grade.
8. George isn't the ghost or ghoul.
9. Last Halloween Gertrude was a ghost and Gilbert a gorilla.

Answer: George (gorilla), Gilbert (ghost), Geraldine (ghoul), Gertrude (goblin).

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