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delta-k

THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

Volume XIX, Number 1 September 1979

MATHEMATICS COUNCIL, ATA

announces

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and MCATA Annual Meeting

October 11-13, 1979

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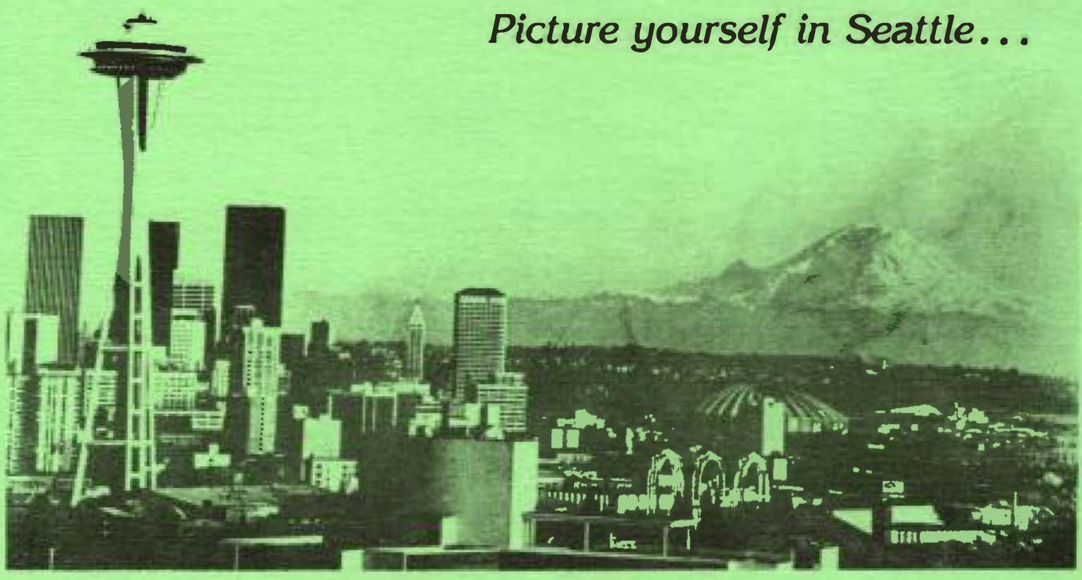

The National Council of
Teachers of Mathematics
Unveils

The Mathematics Curriculum of the 1980's

CONTENT • METHODS • TECHNOLOGY

58th ANNUAL MEETING
APRIL 16-19, 1980
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From the Editor's Desk

Our sincere thanks to Bob Holt for his years of service as president of the Mathematics Council. It is comforting to know that he is continuing as an executive member of the Council.

Our new president is Lyle Pagnucco, a former executive officer and this year's chairman for the "Name-of-Site" MCATA/NCTM joint conference in October. The other table officers were given a vote of confidence and were returned to office: Secretary Art Jorgensen and Treasurer Don Hinde.

We are again looking for new workers to serve as Council directors and to become our future leaders as some of our present members reach the plateau of a well-deserved rest.

Remember our "Name-of-Site" meeting (see cover). This is one of the best ways to avail ourselves of the services offered by the National Council of Teachers of Mathematics. Make this your year to expand your horizon with an NCTM membership and its many benefits for your professional portfolio. We will have an NCTM booth at the meeting. Best wishes for 1979-80.

Ed Carriger

Instructional Problems and Teaching Strategies

by *Joanne Newman* and
Ved Madan
Red Deer College, Alberta

The following is an edited version of the authors' paper.

The teaching of mathematics courses includes many problems. It is our intent to identify and analyze some of these problems as well as to propose some suggestions to stimulate interest in mathematics.

In any classroom the variety of students can be a problem. A group of students implies an assortment of backgrounds. The assortment of backgrounds can mean nationalities and/or the educational backgrounds of the students enrolled in the course. A variety of nationalities can be a problem to the teacher and all the students in the class if some of the students are not fluent in the language of instruction. The educational background of pupils can be a problem if it does not include the basic knowledge that is required for the course. A student may have the necessary prerequisite courses but may not have learned the material he was expected to know. At the other extreme is the student who has previously learned the material being covered. In this case, it is very difficult for the teacher to maintain the interest of this person.

Then, too, there are the different learning abilities of a group of students which poses a problem in determining the pace at which to teach the course. The teacher must cover the material quickly enough to maintain the interest of the fast learners and yet teach slowly enough for the slower learners to grasp the concepts being covered.

The interest level of the pupils is a problem area. Students who are required to take the course will not have as much enthusiasm about the material as those students who have chosen to take it. Without student cooperation, it is almost impossible to teach the knowledge required by the course objectives. The diversity of programs a student is taking is a problem too. Although the teacher has some leeway in the selection of topics to be covered, his selection must be based somewhat on the programs his students are in. For example, the students in geometry courses for the faculty of education do not need to know the depth of the material that mathematics majors require.

Different nationalities and programs can cause another problem in the classroom. Students may not have a great deal in common and therefore there is not much comradeship among them. This deprives the students of the opportunity to learn from each other outside the classroom, which can be as good a learning experience as time spent in class. It is unfortunate when students are not friends with one another and with the instructor, because there is a much better atmosphere for learning when they are.

Another problem that arises in teaching mathematics is determining whether or not the students understand the concepts being presented. For a successful course, the instructor has to ensure the complete comprehension by the student of the topics being covered. It is imperative that the instructor makes clear the meaning and applications of the subject matter. As well, the instructor should try to relate the concepts being taught to real life situations when possible. By doing this, the student is able to visualize abstract ideas and is therefore more likely to retain the knowledge gained.

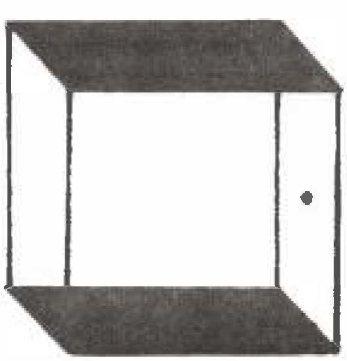
It is helpful for the students to have a few exercises to be done after each day's lecture, rather than after each section which may take two or more classes. If he finds that he has difficulties, he is able to get help right away rather than let the problems get larger and more complex, class after class. As well, by practicing the newly gained knowledge, the student will more likely retain the concepts he has learned.

To account for the different learning abilities in a class of students, the instructor could offer to help the slower learners in an extra class, or individually. If this were done, the slow learners would be able to learn at their own speed without slowing the rest of the class. If the instructor is unable to provide this extra help for the slow learners, he will be confusing some students while boring others.

If there is a lack of comradeship in a class of students, the teacher might initiate a class project where all of the class members are involved as one group (possible in a small class). Usually this proves to be beneficial to all students as well as the instructor.

Each instructor must realize that different instructional approaches work best for different topics. By varying the instructional approach, the students may learn more easily. Another possibility is that it may generate greater interest among the students which, in turn, creates enthusiasm.

A final suggestion to teachers is to share their experiences. By doing so, they can learn teaching techniques from each other.



Is the dot on the inside or the outside of the box?

Reprinted from *The Math Post*.

CONSTRUCTIVE EXPERIENCES WITH DECIMALS

by *T. E. Kieren*
Faculty of Education
University of Alberta
Edmonton, Alberta

The following exercises are the last of a series published in the March 1979 and May 1979 issues of delta-k.

DECIMAL TASK SET 2:
Decimal Numeration and Fractions

1. 3 3 3
   a b c

   The value of the digits indicated by a, b, and c above are:

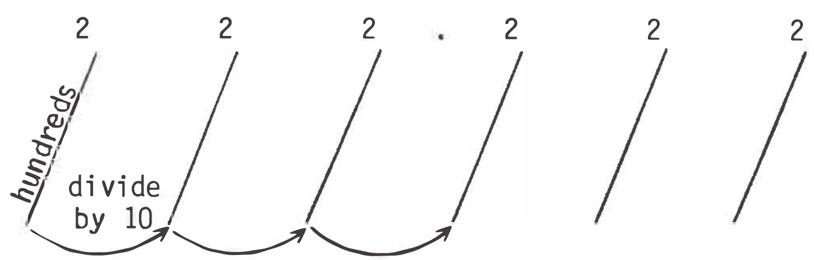
   a) 300
   b)
   c)

   Why?

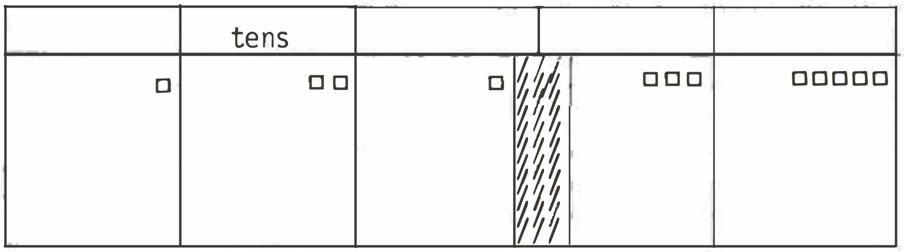
   To get the value of b) from a), one can ______________________

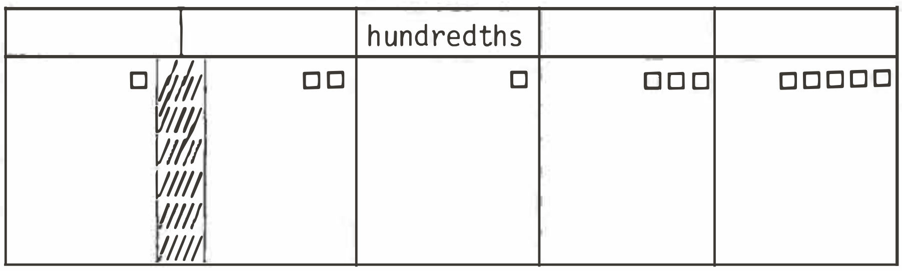
   To get the value of c) from b), one can ______________________

2. Complete the following demonstration for children:



3. a) If the bar represents the "decimal point," give the number represented by the chart in decimal form.





b) Represent 2.3012

c) Represent (2 x 10) + (0 x 1) + 3/10 + 0/100 + 5/1000

4. Sketch a place value pocket chart you would use with your class. (What is the value of a "moveable" decimal point?)

a) Write up a set of 6 exercises for children using the chart.

b) Explain how the chart could be used for addition.

c) Explain how the chart could be used for division.

DECIMAL TASK SET 3:
Addition, Meaning, and Equivalence

Your table should have at least 2 metre sticks divided into decimetres, centimetres, and millimetres. It should also have a long piece of string and calculator tape.

1. Complete the following table.

OBJECT	A length of side 1	B length of side 2	C measure of string combining sides 1 & 2	D A + B
Book				
Table				
Bookcase				

Give length in decimal fractions of a metre. That is, use the metre as your ______________________________.

2. Carefully cut a piece of calculator tape 1 metre long with ends cut perpendicular to the length. Label the ends 0 and 1.

   a) Fold the tape in two. Label the fold and ends in 1/2's.

   b) Fold the tape into 4 congruent parts. Label the folds and ends in 1/4's.

   c) Repeat b) for 1/3's, 1/6's, 1/8's, 1/12's.

   d) (Key exercise!) Use a metre stick to add a decimal fraction to the list of equivalent fractions on each fold.

DECIMAL TASK 4:
Homework

During the next week collect as many different observed uses of decimals as you can find. Make a display which you could use to motivate the study of decimals in your classroom.

CONSTRUCTIVE RATIONAL NUMBER TASKS

by *T. E. Kieren*
Faculty of Education
University of Alberta
Edmonton, Alberta

The following number tasks are the last of a series which first appeared in the March 1979 issue of delta-k *and again in the May 1979 issue.*

FRACTION TASK 6:
Units

1. Take a set of 10 different rods. Choose one rod to be your unit. Write fractional number names for all the other 9 rods in terms of your unit.

   Have your partner choose a longer rod as a unit and do the same task.

   Compare your green rods. Do they have the same name in both systems?

   Why or why not?

2. Could the following sentences ever be true? Explain.

   $3/4 < 1/2$

   $5/4 = 1$

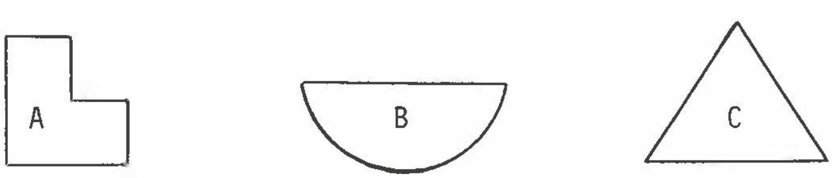
   $2/3 > 3/4$

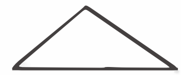
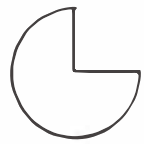
3. What kinds of learning problems are posed by the aspect of fractions as described above?

4. Find the piece of yarn at your table. If that piece is represented by 4/5, cut a piece of yarn from the ball which would represent 1. Describe how you did this.

5. Make up similar problems for children of age 10 or 11 to help them focus on the notion of unit.

6. On the table, find shapes labelled A, B, and C. Below draw the shapes of the figures represented by the given fractions if A, B, or C were considered as units.



Unit	*Number*	*Shape*
C	1	
A	2/3	
B	1/2	
C	3/4	
B	___	

7. Make up interesting and motivating exercises, similar to those in 6, but appropriate for Division II students. Use humor or fantasy.

FRACTION TASK 7: Teaching

1. Make up fraction representation problems and fraction addition problems using a) Cuisenaire rods
   b) Graduated beakers or cans (what is the problem here?).

*FRACTION TASK 8: Ratio Numbers**

1. From the box of rods, select a set, one of each color and order them. Associate a number from 1 to 9 with each rod.

2. Take a red and a light green rod. Describe the relationship between them in as many ways as you can.

red	=	_____	light green	(a)
light green	=	_____	red	(b)
_____ red	=	_____	green	(c)

The ratio of red to light green is _____ to _____. (d)

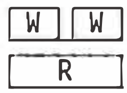
The ratio of light green to red is _____ to _____. (e)

**The idea for this task sheet was taken from the work of Alan Bell and the South Nottingham Project in England.*

How are the numbers used to describe the relationships in (a) and (b) and (d) and (e) related?

3. Select two other pairs of rods and write mathematical sentences which describe the relationship between them.

4. a) How are the red and white rods related? We can picture this relationship as follows:



Find all the other rod pairs which have the same relationship.

List the set of ratio numbers used to describe these rod pairs:

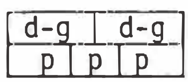
{white to red, ____, ____, . . .}
{½, ____, ____}

How do you know physically that the rod pairs are in the relationship?

What can you say about the ratio numbers used to describe these rod pairs?

b) Test two ratio numbers used to describe the relationships between pink and dark green rods.

Write an equation which describes this picture.



Find all the other rod pairs which share the relationship pictured above.

What can you say about the ratio numbers which describe these rod pairs?

c) Suppose there were a silver rod and a violet rod which were related in the following way:

$$3\ S = 7\ V$$

Write two sets of ratio numbers which would describe rod pairs having the same relationship.

{3/7, ____, ____, ____, ____, ____}
{7/3, ____, ____, ____, ____, ____}

How many ratio numbers would fall in such sets if one included all possible ones?

d) How would you know physically that rod pairs would be represented by equivalent ratio numbers?

Extra for experts: Suppose we have a rod pair of colors x and y, such that

$$ax = by$$

Give a ratio number which relates x and y and give five equivalent ratio numbers.

Remedial Program in Mathematics

by *J. G. Timourian*
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The Problem

Each year more students than ever are required to study mathematics at an advanced level. Some of these students never really learned their high school mathematics; others are mature students who have been away from mathematics for many years. Without help, their success in university math courses is doubtful.

This problem is not unique to the University of Alberta. Most universities in North America are facing the same situation.

Identifying the Problem Students

For the past three years the Mathematics Department at the University of Alberta has been giving a diagnostic exam to students entering first year calculus. The exam given in the fall of 1978 is presented here.

In the fall of 1977, the Department identified 230 problem students. Of these, 63 percent failed or withdrew from calculus.

In the fall of 1978, approximately 2,000 students entering first year calculus took the exam. There were 29 questions. We identified the 324 students who scored less than 13 correct as problem students. *This score was selected because, for budgetary reasons, we could not handle more in our remedial program.*

Attempts to Solve the Problem in 1978

The 24 students who scored below six on the exam were told to repeat a high school algebra course.

The students receiving between six and 13 were advised to take a six-week refresher course organized by the Department. Approximately one-third of the students needing the refresher course took it. A high proportion of these were mature students who were highly motivated hard workers. As in the previous year, they were enthusiastic about the benefits of the course. We were not so successful in attracting students to the refresher program who had graduated from high school the previous year.

In the past, the students who took the refresher course had more success in their calculus course. It is not clear if this is because of the refresher course itself or because the students who take it are highly motivated.

Mathematics Advisory Examination

How many years has it been since you completed the equivalent of Mathematics 30?

[a] 0 (spring of 1978) [b] 1 [c] 2

[d] 3 [e] more than 3

Basic Techniques

1. The number 1/3 is:
   - [a] a natural number
   - [b] an integer
   - [c] rational
   - [d] irrational
   - [e] all of these

2. The multiplicative inverse of 5 is:
   - [a] -5
   - [b] 1
   - [c] .5
   - [d] 1/5
   - [e] none of these

3. $x^2 - 9$ factors into
   - [a] $(x+3)^2$
   - [b] $(x-3)^2$
   - [c] $x(x-9)$
   - [d] $(x+3)(x-3)$
   - [e] none of these

4. $(x^3+1)(x^2+x-1)$ is
   - [a] $x^6 + x^3 + x^2 - 1$
   - [b] $x^5 + x^4 - x^3 + x^2 + x - 1$
   - [c] $x^5 - x^3 + x^2 + x - 1$
   - [d] $x^5 + x^4 + x^3 - x^2 - 1$
   - [e] none of these

5. $\dfrac{\frac{1}{a} + \frac{1}{b}}{\frac{a}{b}}$ simplifies to:
   - [a] $\dfrac{1}{ab}$
   - [b] $\dfrac{a}{b} + \dfrac{b}{a}$
   - [c] $\dfrac{a + b}{a^2}$
   - [d] $\dfrac{a + b}{ab}$
   - [e] $\dfrac{a + b}{b^2}$

6. $\dfrac{1}{a + b} + \dfrac{1}{a^2 - b^2}$ simplifies to give:
   - [a] $\dfrac{a - b + 1}{a^2 - b^2}$
   - [b] $\dfrac{a + b - 1}{a^2 - b^2}$
   - [c] $\dfrac{a - b - 1}{a^2 - b^2}$
   - [d] $\dfrac{a - b}{(a+b)(a^2-b^2)}$
   - [e] none of these

Equations, Inequalities, Division

7. The solution set of $\frac{1}{x+1} + \frac{1}{x+2} = \frac{2}{x+3}$ is:

   [a] $\frac{5}{3}$ [b] $\frac{-5}{3}$ [c] $\frac{3}{5}$ [d] $\frac{1}{2}$ [e] none of these

8. The solution set for $\frac{x^2 + 2x}{x + 2} = x$ is:

   [a] $\{1\}$ [b] $\{x : x \neq 0\}$ [c] $\{x : x \neq -2\}$ [d] $\{x : x \neq 2\}$

   [e] none of these

9. If $x^2 - 2bx + c = 0$, then

   [a] $x = -b \pm \sqrt{b^2 - c}$ [b] $x = b \pm \sqrt{b^2 - c}$ [c] $x = c \pm \sqrt{c^2 - b}$

   [d] $x = -c \pm \sqrt{c^2 - b}$ [e] none of these

10. Written in the form $y = a(x-h)^2 + b$, the equation $y = 2x^2 - 8x + 7$ becomes:

    [a] $y = (x-2)^2 + 7$ [b] $y = 2(x-2)^2 + 1$ [c] $y = 2(x-2)^2 + 7$

    [d] $y = 2(x-2)^2 - 1$ [e] none of these

11. Which of the following does not satisfy the inequality $|x| - |y| \geq 2$?

    [a] $x = -3, y = +1$ [b] $x = -2, y = 0$ [c] $x = -3, y = 0$

    [d] $x = -4, y = +2$ [e] $x = 2, y = -1$

12. $1 - 2x > 5$ is equivalent to:

    [a] $x > +2$ [b] $x < 1/2$ [c] $x < -2$ [d] $-x < 2$ [e] $x > -2$

13. Given that $x = -2$ is a root of $x^3 + 2x^2 + x + 2$, we can factor this cubic into

    [a] $(x-2)$ (polynomial) [b] x(polynomial) [c] x^2 (polynomial)

    [d] $(x+2)$ (polynomial) [e] none of these

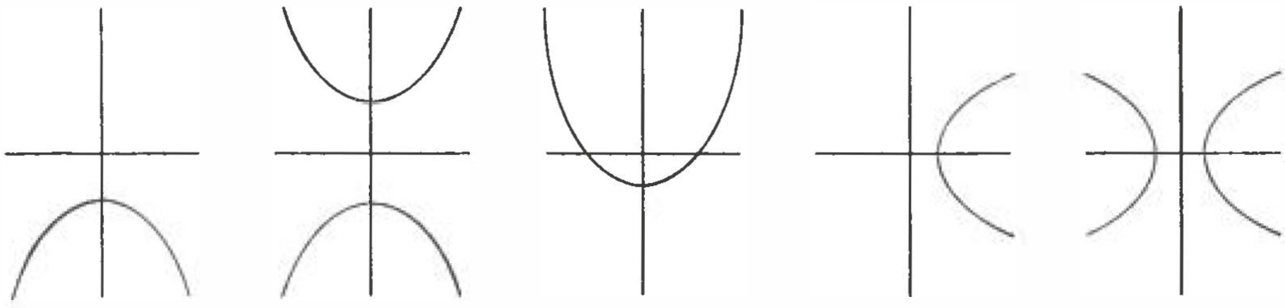
14. $x^5 + 2x^3 - x^2 - x + 1$ divided by $x + 1$ is

$x^4 - x^3 + 3x^2 - 4x + 3$, with remainder

[a] 1 [b] -2 [c] 3 [d] 2 [e] 0

Graphs

The next two questions involve the following 5 possible answers:

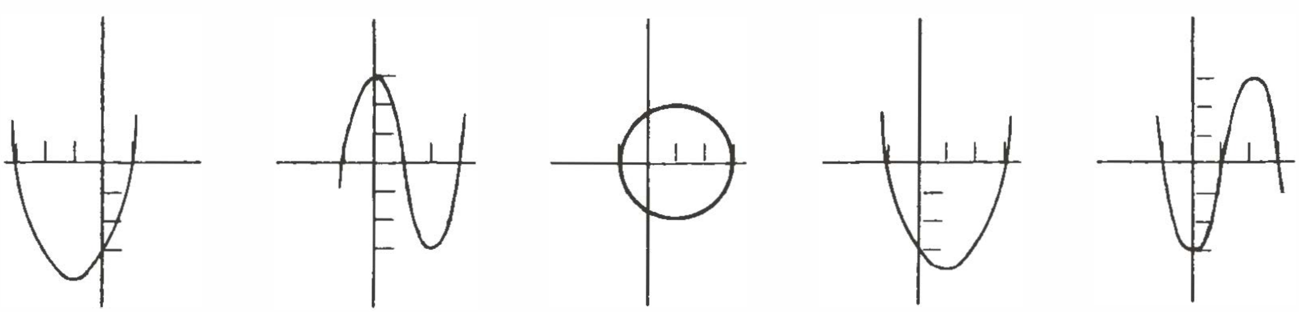


[a] [b] [c] [d] [e]

15. The graph of $y = x^2 - 1$ looks like?

16. The graph of $y^2 = x^2 - 1$ looks like?

The next two questions involve the following 5 possible answers:



[a] [b] [c] [d] [e]

17. The graph of $y = x^2 - 2x - 3$ looks like?

18. The graph of $y = x^3 - 3x^2 - x + 3$ looks like?

Exponents

19. $(3^2)^3$ is:

[a] 729 [b] 27 [c] 3^8 [d] 243 [e] none of these

20. $\frac{2^2}{2^{1/3}}$ is:

[a] $2^{4/3}$ [b] $2^{2/3}$ [c] $2^{5/3}$ [d] $1^{2/3}$ [e] none of these

21. $(3^{-2})^{-3}$ is:

[a] $\frac{1}{243}$ [b] $\frac{1}{3^8}$ [c] $\frac{1}{729}$ [d] 729 [e] none of these

22. a^{x+y} is the same as

[a] $a^x + a^y$ [b] $a^x \div a^y$ [c] $a^x \cdot a^y$ [d] $(a^x)^y$ [e] none of these

Logarithms

23. $\log_{10}100$ is:

[a] 1 [b] 2 [c] 0 [d] 10 [e] 1,000

24. $\log_{10}30$ is:

[a] $3\log_{10}10$ [b] $(\log_{10}15)^2$ [c] $\log_{10}15 + \log_{10}15$ [d] $\log_{10}15 + \log_{10}2$

[e] none of these

25. $\log_{10}1/2$ is:

[a] $\frac{1}{\log_{10}2}$ [b] $-1 + \log_{10}2$ [c] $-\log_{10}2$ [d] $1 + \log_{10}2$

[e] $\frac{-1}{\log_{10}2}$

26. Solve the following equation for y in terms of x; $x = 3^y$.

[a] $y = \sqrt[3]{x}$ [b] $y = 3x$ [c] $y = \frac{\log_{10}3}{\log_{10}x}$ [d] $y = x^3$ [e] $y = \frac{\log_{10}x}{\log_{10}3}$

Trigonometry

27. $\cos(\frac{\pi}{3})$ is equal to:

    [a] 2   [b] $\sqrt{\frac{3}{2}}$ [c] -1/2   [d] 1/2   [e] 1

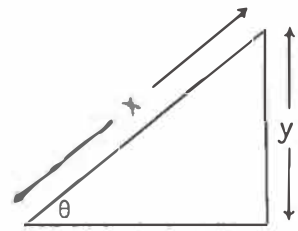
28. 45° measured in radians is the same as:

    [a] 1/4   [b] $\pi/4$ [c] π [d] $-\pi/4$ [e] 1/2

29. In the right triangle shown, $\sin(\theta) = 1/5$.
    The value of y is:

    [a] 5   [b] 5x   [c] 1/5   [d] x/5

    [e] none of these



30. $(1 - \sin\theta)(1 + \sin\theta)$ is the same as:

    [a] $1 + \cos\theta$ [b] $\cos 2\theta$ [c] $-\cos^2\theta$ [d] $\sin 2\theta$ [e] $\cos^2\theta$

Geometrical Theorems - In Slides
by Ved Madan, Red Deer College, Red Deer, Alberta.
Intergalactic Publishing Company, 221 Haddon Avenue,
Westmont, New Jersey 08108.
Price: $30 per set.

Geometrical Theorems - In Slides is a set of 21 fascinating color slides produced by the Canadian author and designed to serve as an innovative instructional aid for the teaching of geometry in secondary schools and colleges and some university level courses. The slides depict some very fundamental theorems in the Euclidean, non-Euclidean, and projective geometry, as well as in topology. The theorems of Pythagoras, Desargues, Pappus, Ceva, and Menelaus, and Euler's formula are some of the topics of study. The slides come with a 12-page supplement which suggests material for involving additional class discussion and project work.

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activities

Discovery with Number Triangles

by *Hugh Ouellette*
Winona State University
Winona, Maine

Teacher's Guide

Grade level: 7–12

Materials: One set of worksheets for each student, a set of transparencies (if desired) for class discussion, and calculators (optional).

Objectives: Students will (1) collect and organize data, (2) discover number patterns, and (3) make conjectures.

Directions: Distribute the worksheets one at a time. Have the students work either individually or in small groups of two or three. Develop a complete set of answers on the transparency. Do the same for sheets 2 and 3.

Comments: Circulate among the students as they work on the problems. Emphasize that all computations should be accurate and predictions should be checked. Calculators can be used if desired. Be receptive to partial solutions and incomplete reasoning patterns. Be sure to indicate that the generalizations formed have *not* been proved. Students who have had some work with mathematical induction may try proving their results.

The triangle on sheet 3 may be difficult for some students to extend. Have them consider row 5 with the squares and circles as indicated here:

$$\boxed{5}\ 1\ \boxed{6}\ 2\ \boxed{7}\ 3\ \boxed{8}\ 4\ \boxed{9}\ 5.$$

The first square has 5, the second square has 6, the third square 7, and so on. The first circle has 1, the second 2, the third 3, and so on. Thus, row 6 is

$$\boxed{6}\ 1\ \boxed{7}\ 2\ \boxed{8}\ 3 \ldots \boxed{11}\ 6$$

Supplementary problems:

Sheet 1

1. What is the result of alternately subtracting and adding the terms in row 100? In row 101?
2. What is the sum of the terms in all the rows down to and including row 50?

Sheet 2

1. In row 100, what is the sum of all the terms?
2. How many times does each prime number occur in the triangle?
3. How many times does 60 occur in the triangle?

The Even Triangle

Row 1 → 0
Row 2 → 2 4
Row 3 → 6 8 10
Row 4 → 12 14 16 18
Row 5 → 20 __ __ __ 28
__ __ 34 __ __ __
__ __ __ __ __ __ __

1. Study the part of the even triangle shown. Then fill in the blanks.

2. The first term in row 3 is 6 = 2 x 3.
The first term in row 4 is 12 = 3 x 4.
The first term in row 5 is 20 = 4 x 5.
Complete the table below.
Predict the first term for row 100.

Row number	1	2	3	4	5	6	7	...	100
First term			6	12	20			...	

3. The mean or average of the first and last terms in row 3 is (6 + 10)/2, which is 8. The number 8 is 1 less than 9, that is, 8 = 9 - 1. The mean of the first and last terms in row 4 is (12 + 18)/2 = 15 or 16 - 1. Complete the table below. Then predict the result for row 100.

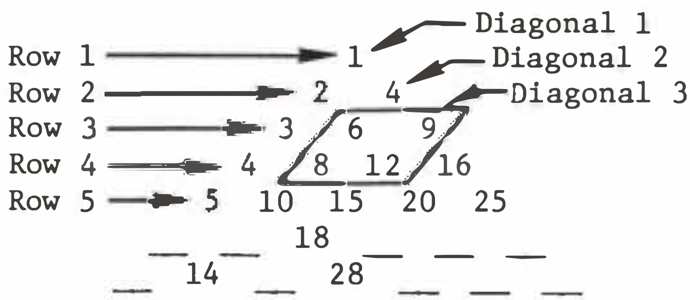
Row number	1	2	3	4	5	6	7	...	100
Mean of first and last term			8	15				...	
Mean, as □ - 1			9-1	16-1				...	

4. The sum of the numbers in row 3 is 6 + 8 + 10 = 24 = 2 x 3 x 4.
The sum of the numbers in row 4 is 12 + 14 + 16 + 18 = 60 = 3 x 4 x 5.
Work similar problems to check the sums for the other rows.
Then predict the sum of the terms in row 100.

Row number	1	2	3	4	5	6	7	...	100
Sum			24	60				...	
Sum, as ⬡ x □ x △			2x3x4	3x4x5	_x5x_			...	

SHEET 2

The Multiple Triangle



1. Study the multiple triangle shown. Then fill in the blanks. ______

2. Find the number of terms for each of the first seven rows. Then predict the number of terms for row 100. ______

3. Find the first and last terms for each of the first seven rows. Then predict the first and last terms for row 100.

Row number	1	2	3	4	5	6	7	...	100
First term			3					...	
Last term			9						

4. Notice that 6 is located at the intersection of row 3 and diagonal 2, and 15 is located at the intersection of row 5 and diagonal 3. Find the number located at the intersection of the rows and diagonals indicated in the table below. Then predict the number at the intersection of row 100 and diagonal 71.

Row number	3	5	4	7	6	5	6	7	...	100
Diagonal number	2	3	2	4	5	1	3	5	...	71
Term in the intersection	6	15							...	

Bonus: 5400 is located at the intersection of row 90 and what diagonal?

5. A four-numeral parallelogram has been outlined in the triangle. The sums of pairs of opposite corner numbers are 18 and 17:

$$6 + 12 = 18, \text{ and } 8 + 9 = 17.$$

In each case the product of these opposite corner numbers is 72:

$$6 \times 12 = 72, \text{ and } 8 \times 9 = 72.$$

Find these sums and products for several more four-numeral parallelograms similar to the one shown above. Then state two relationships that appear always to exist for such parallelograms.

The Mixed-Up Number Triangle

Row 1 → 1 1
Row 2 → 2 1 3 2
Row 3 → 3 1 4 2 5 3
Row 4 → 4 1 5 2 6 3 7 4
Row 5 → 5 1 6 2 7 3 8 4 9 5
_ _ 7 _ _ _ _ _ _ _ _ _ _
_ _ _ _ _ _ _ _ _ _ _ _ 13 _

1. Study the mixed-up number triangle shown. Then extend it by two more rows.
2. Find the number of terms for each of the first seven rows. Then predict the number of terms in row 75.

Row number	1	2	3	4	5	6	7	...	75
Number of terms								...	

3. The sum of the numbers in row 2 is 2 + 1 + 3 + 2 = 8 = 2 x 4. Find the sums of the numbers in the other rows and complete the table below.
Next represent each sum as 2 times a number.
Then predict the sum for row 100.

Row number	1	2	3	4	5	6	7	...	100
Sum		8						...	
Sum, as 2 x □		2x4	2x_					...	

4. Alternately subtracting and adding terms in row 3 gives
3 - 1 + 4 - 2 + 5 - 3 = 6 = 2 x 3.
Similarly, for row 4: 4 - 1 + 5 - 2 + 6 - 3 + 7 - 4 = 12 = 3 x 4.
Do the same for the rest of the first seven rows and predict the result for row 100.

Row number	1	2	3	4	5	6	7	...	100
Alternately subtract and add terms			6	12				...	
Result, as □ x △			2x3	3x4				...	

5. A five-numeral diamond has been outlined in the triangle. The sums of both pairs of opposite corner numbers are 13: 5 + 8 = 13, and 6 + 7 = 13. The products of these opposite pairs of numbers are 40 and 42: 5 x 8 = 40, and 6 x 7 = 42. Find these sums and products for several more five-numeral diamonds. Then state two relationships that appear always to exist for such diamonds.

Sheet 3

1. In row 100, what are the first, second, third, fourth, and sixtieth terms?
2. For any four consecutive terms in a row, compare the sum of the first and fourth terms with the sum of the second and third terms.

Many of the problems on the three activity sheets can be generalized for row n. The generalized activities for row n could be used as supplementary problems for upper-level students.

Answers:

Sheet 1: (2) 0, 2, 30, 42; 9900; (3) 0, 3, 24, 35, 48; 9999; (4) 0, 6, 120, 210, 336; 999 900. *Sheet 2:* (3) 1, 1; 2, 4; 4, 16; 5, 25; 6, 36; 7, 49; 100; 10 000; (4) 8, 28, 30, 5, 18, 35; 7100; 60; (5) the sums differ by 1; the products are equal. *Sheet 3:* (2) 2, 4, 6, 8, 10, 12, 14; 150; (3) 2, 18, 32, 50, 72, 98; 20 000; (4) 0, 2, 20, 30, 42; 9900; (5) the sums are equal; the products either are equal or differ by 2.

REFERENCE

Ouellette, H. "Number Triangles—a Discovery Lesson." *Mathematics Teacher* 68 (December 1975): 671–74.

A publication featuring math games, tricks, and puzzles for holiday occasions is available from Modern Math Materials. *Let's Celebrate Math* by Geer, Geer, Geer and Gast is a collection of holiday happenings. The following puzzle is found in the book.

HALLOWEEN

George, Geraldine, Gertrude and Gilbert were dressed as a goblin, ghost, gorilla and ghoul. From the clues below, can you find out which costume each person wore?

1. No one is wearing the costume they wore last year.
2. One of the boys is the ghost.
3. Geraldine and the goblin are sisters.
4. The gorilla has no brothers or sisters.
5. George and the goblin are sixth graders.
6. Only the ghoul and Gilbert like bubblegum.
7. Last year Geraldine was the best student in the fifth grade.
8. George isn't the ghost or ghoul.
9. Last Halloween Gertrude was a ghost and Gilbert a gorilla.

Answer: George (gorilla), Gilbert (ghost), Geraldine (ghoul), Gertrude (goblin).

Reprinted from *The Math Post.*

Ideas

Prepared by
Earl Ockenga and *Joan Duea*
Malcolm Price Laboratory School
University of Northern Iowa
Cedar Falls, Iowa

Each *Ideas* issue presents activities that are appropriate for use with students at various levels in the elementary school. The activity sheets are so arranged that they can be easily removed and reproduced for classroom use. Permission to reproduce them for such use is not necessary.

IDEAS For Teachers

1. *Objective:* Experience in solving addition problems.

   Levels: 1,2

2. *Objective:* Experience in solving addition and subtraction problems.

   Levels: 3,4

3. *Objective:* Experience in solving two-step addition, subtraction, and multiplication problems.

   Levels: 5,6

4. *Objective:* Experience in solving two-step addition, subtraction, multiplication and division problems.

   Levels: 7,8

Directions for teachers:

1. Remove the master and reproduce one copy for each student.

2. Students fill in their tic-tac-toe by randomly writing the numbers in the nine empty spaces.

3. Students choose any problem at the bottom of the page, solve it, and mark (X) the answer on their tic-tac-toe.

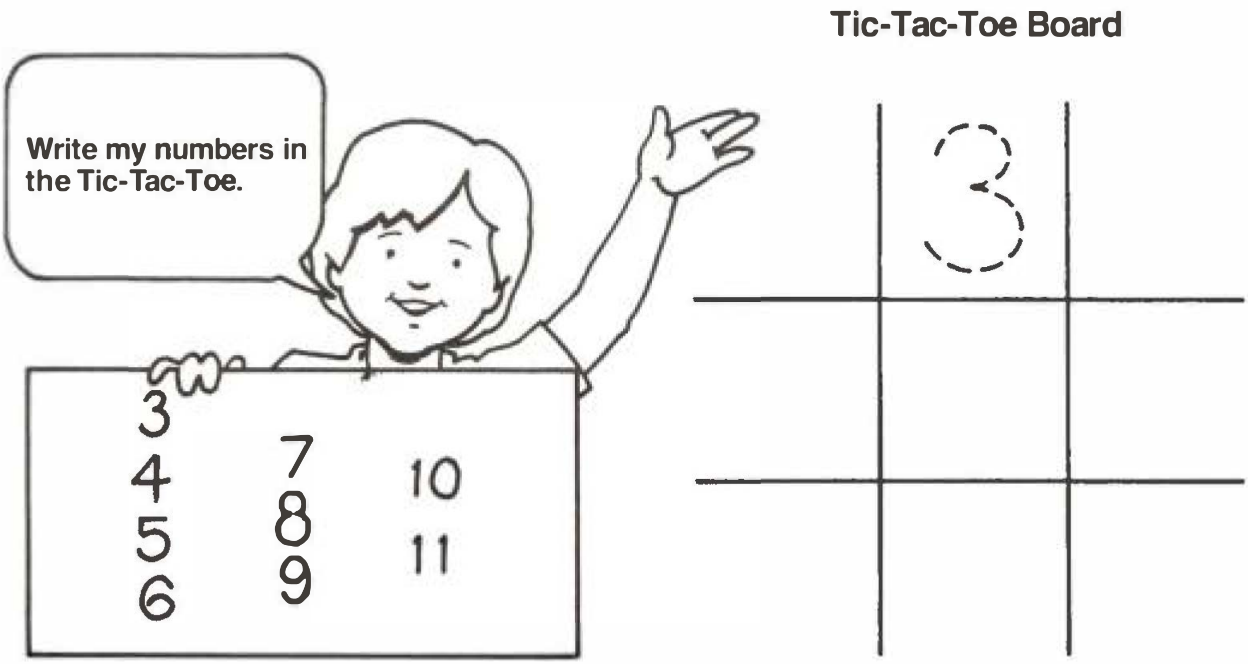
4. At the lower levels, students win with a simple tic-tac-toe (three Xs in a line vertically, horizontally, or diagonally). With older students, *two* tic-tac-toes (three Xs in a line twice) are needed to win.

Depending on your class, you may want to read the problems to your students. If the problems are read aloud, have the students select the order in which the problems are read.

The game may be played in partnerships. Players take turns selecting a problem, then mark their answer on the tic-tac-toe with either an X or O.

IDEAS Levels: 1,2

Problem Solving Tic-Tac-Toe



Pick any problem below.
Find the answer to the problem.
Put an *X* on the answer in the tic-tac-toe.
Three *X*s in a line wins.

Problems

You had 3 toy cars. Sue gave you 4 more. How many cars do you have?

You had 5 cents. You got 5 cents more. How many cents do you have?

You have 1 yellow pencil, 1 red pencil, and 1 blue pencil. How many pencils do you have?

There are 6 fish in a bowl.
There are 3 fish in another bowl.
How many fish altogether?

The park has 4 swings. It has 2 slides. How many things are there to play on?

Bill counted 5 boys playing kick ball. Ann counted 6 girls playing kick ball. How many are playing kick ball?

We had 2 bats. Coach has 6 new bats. How many bats do we have?

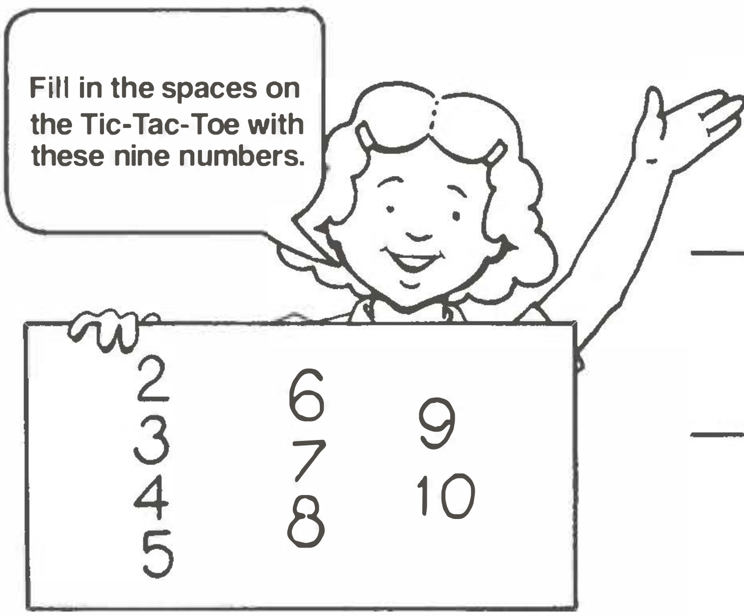
Your mom gave you 2 cookies. Your brother gave 1 more. Your sister gave you 1 more. How many cookies did you get?

You ate 2 hamburgers. Your dad ate 3 hamburgers. How many hamburgers were eaten?

From the *Arithmetic Teacher*

IDEAS Levels: 3,4

Problem Solving Tic-Tac-Toe



Tic-Tac-Toe Board

Now choose any problem below. Solve it.
Put an *X* on the answer on the tic-tac-toe board.
Three *X*s in a line wins.

Problems

Add 7 to my mystery number and you get 15. What's my mystery number?

Subtract 6 from my mystery number and you get 3. What's my mystery number?

Subtract 5 from my mystery number. Then add 8 and you get 10. What's my mystery number?

Add my mystery number to itself. Then subtract 4 and you get 0. What's my mystery number?

Subtract 7 from my mystery number and you get 3. What's my mystery number?

Add 17 to my mystery number and you get 20. What's my mystery number?

Subtract 3 from my mystery number. Then add 9 and you get 10. What's my mystery number?

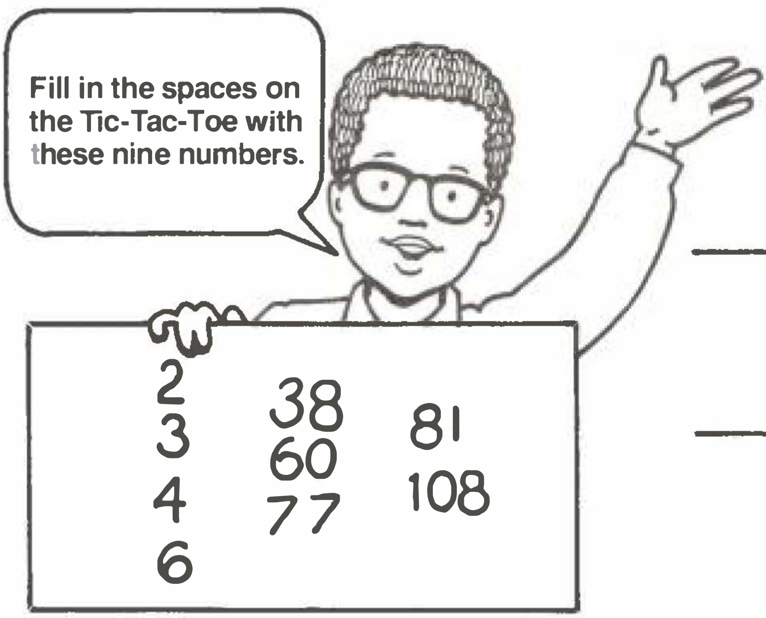
Add 8 to my mystery number. Then subtract 8 and you get 5. What's my mystery number?

Add my mystery number to itself and you get 12. What's my mystery number?

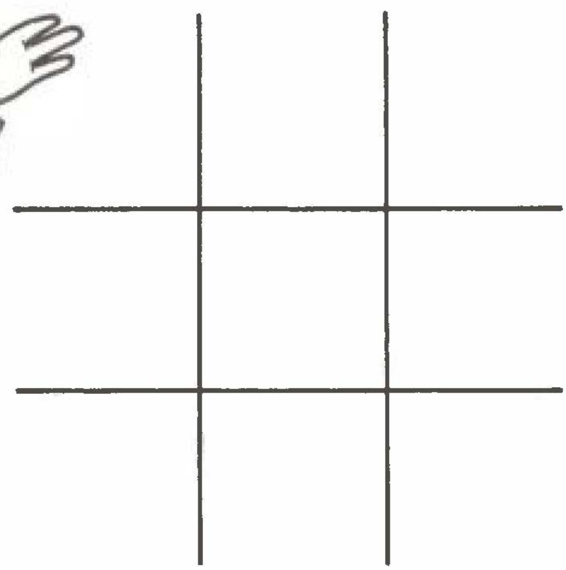
From the *Arithmetic Teacher*

IDEAS **Levels: 5,6**

Problem Solving Tic-Tac-Toe



Tic-Tac-Toe Board



Now choose any problem below. Solve it. Put an *X* on the answer on the tic-tac-toe board. You win when you get two tic-tac-toes.

Problems

I had 45 comic books. I gave my friend 24 of them. My friend gave me 17 of her comic books. How many comic books do I have now?

How many legs on 12 dogs and 15 cats?

How much money is 31 dimes and 18 nickels?

I bought 4 records at $4.25 each. I gave the clerk $20. How much change did I receive?

How many hours in 3 days and 5 hours?

Roller coaster rides cost 25¢ each. Ferris wheel rides cost 50¢ each. What's the cost for four roller coaster rides and 2 ferris wheel rides?

I had $20. I spent $2.50, $4.25, and $7.25. How much is left?

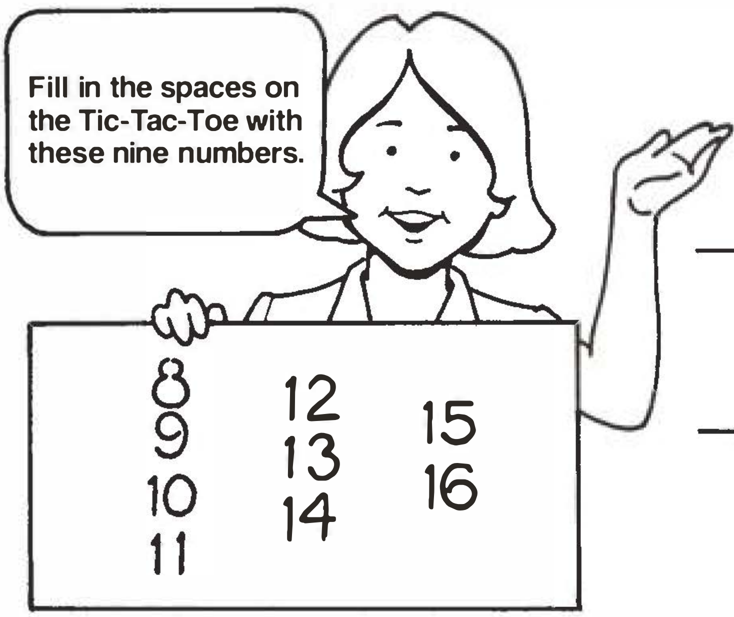
How many legs on 9 cows and 12 chickens?

There are 4 teams of fifth graders and 5 teams of sixth graders. Each team has 9 players. How many players are there?

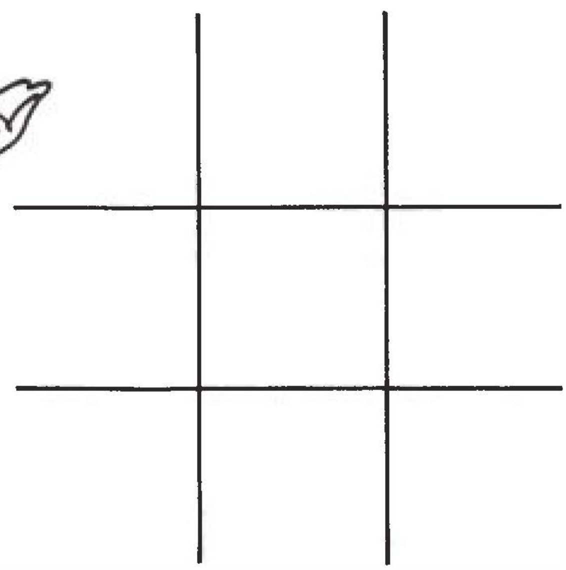
From the *Arithmetic Teacher*

IDEAS **Levels: 7,8**

Problem Solving Tic-Tac-Toe



Tic-Tac-Toe Board



Now choose any problem below. Solve it. Put an *X* on the answer on the tic-tac-toe board. You win when you get two tic-tac-toes.

Problems

If you add 58 to my mystery number and then divide by 7, you get 10. What's my mystery number?

If you multiply my mystery number by 5 and then add 10, you get 75. What's my mystery number?

If you divide my mystery number by 7 and then add 18, you get 20. How old am I?

If you multiply my mystery number by 6 and then divide by 12, you get 4. What's my mystery number?

If you multiply my mystery number by 9 and then subtract 49, you get 50. What's my mystery number?

If you multiply my mystery number times itself and then add 19, you get 100. What's my mystery number?

Seven times my mystery number is 20 more than 5 times my mystery number. What's my mystery number?

My mystery number is between 10 and 20. If you divide it by 11, you get a remainder of 4. What's my mystery number?

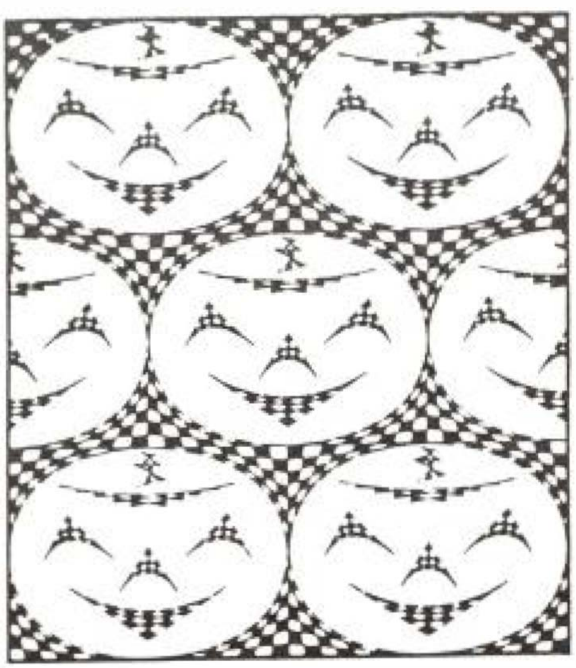
If you divide my mystery number by 4 and then multiply by 10, you get 40. What's my mystery number?

From the *Arithmetic Teacher*

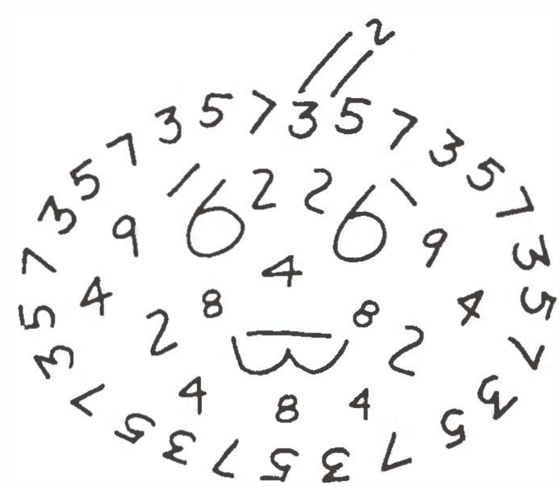
Halloween

The following activities were reprinted from The Math Post.

This Halloween design is drawn entirely with straight lines. Other similar designs are found in *Line Designs* by Seymour, Silvey, and Snider. The book is available from SETSCO Educational Limited.



The Great Pumpkin is formed using the digits 1 through 9. Find the mass of the monster by finding the sum of the digits.



Ans. 227

A black cat before two black cats.
A black cat between two black cats.
A black cat behind two black cats.

How many black cats?

This activity and others similar can be found in the Midwest Publications Company book entitled *Math Amusements in Developing Skills* by Clack and Leitch.

Graphing Greats! First Quadrant Quickies by McClees, Borne, and Helmbrecht is the source of this Halloween idea. The book is published by The Math Group. A few changes from the original have been made.

Connect each group of points in order.

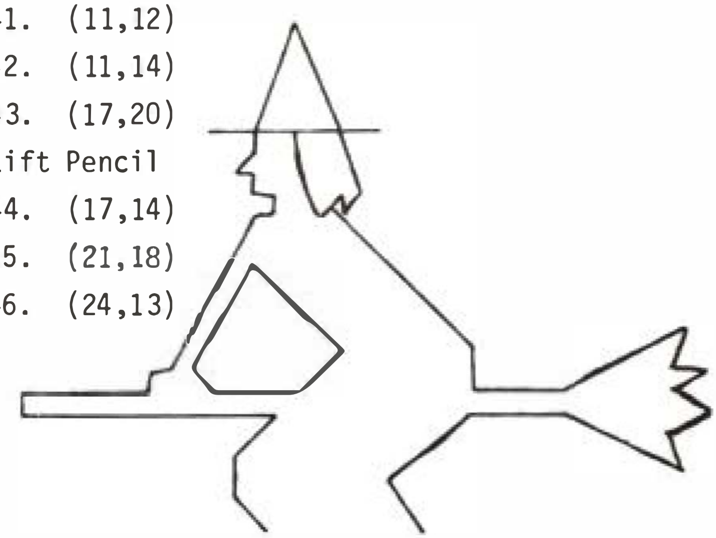
1. (19,24)
2. (19,22)
3. (18,20)
4. (17,21)
5. (17,20)
6. (16,21)
7. (17,24)
8. (19,29)
9. (21,24)
10. (21,23)
11. (22,22)
12. (21,22)
13. (21,21)
14. (20,21)
15. (20,20)
16. (21,20)
17. (25,13)
18. (26,13)
19. (26,12)
20. (32,12)
21. (32,11)
22. (20,11)
23. (22,9)
24. (22,7)
25. (16,0)
26. (16,2)
27. (14,1)
28. (15,4)
29. (12,3)
30. (15,8)
31. (11,11)
32. (7,11)
33. (1,8)
34. (2,10)
35. (0,11)
36. (2,12)
37. (0,13)
38. (2,13)
39. (1,15)
40. (7,12)
41. (11,12)
42. (11,14)
43. (17,20)

Lift Pencil

44. (17,14)
45. (21,18)
46. (24,13)
47. (23,12)
48. (19,12)
49. (17,14)

Lift Pencil

50. (23,24)
51. (15,24)



APPLE GRAPH

Connect the dots in order.

1. (5,11)
2. (7,11)
3. (7,13)
4. (5,14)
5. (5,11)
6. (3,11)
7. (1,10)
8. (0,7)
9. (0,5)
10. (1,3)
11. (2,1)
12. (4,0)
13. (6,1)
14. (8,0)
15. (10,1)
16. (11,3)
17. (12,5)
18. (12,7)
19. (11,10)
20. (9,11)
21. (7,11)

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