$$x = \frac{9}{x} - 8,$$

A Programmable Calculator Activity

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The authors are presenting variations of this activity in several journals in order to invite reader reaction and correspondence about nonstandard problems that can be solved with programmable calculators.

Programmable calculators, as well as computers, are ideal problem-solving tools for secondary students and ideal teaching aids for secondary teachers. In fact, programmable calculators are often better devised for teaching and learning purposes than computers for several reasons, including the following:

1. they now cost so little (a TI 57 is less than $50, and a HP 33 E less than $90) that individuals and schools are increasingly able to afford them;

2. they are small, hand-held, and can be operated from rechargeable battery packs for complete portability;

3. they provide instant access - there is no waiting for that one, always busy, terminal; and

4. they solve many of the same kinds of problems handled by computers.

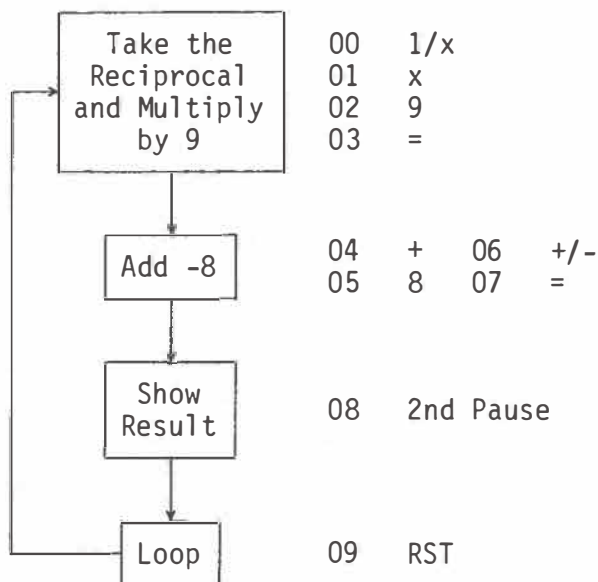The purpose of this article is to share one problem (activity) which you and your students can explore with a calculator. The activity has two key features. First, it is non-standard; that is, it is an activity that would not be easily explored without the use of a programmable calculator. Second, the activity can be used to motivate a discussion of some interesting mathematics, as will be noted following the statement of the activity.

The Activity

(a) Start with any number;

(b) Take the reciprocal and multiply by 9;
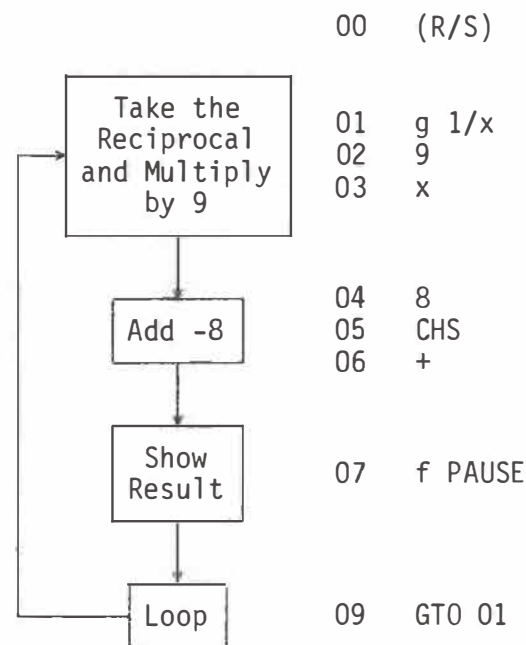
(c) Add -8; and

(d) Repeat from step (b).

Figures 1 and 2 present flow charts and programs for the TI 57 and HP 33 E programmable calculators, respectively. These machines are chosen as they represent the least expensive models of the two major domestic manufacturers and will likely, therefore, be more widely available than other models.

Figure 1

Box	Step	Code	Step	Code
Take the Reciprocal and Multiply by 9	00	1/x		
	01	x		
	02	9		
	03	=		
Add -8	04	+	06	+/-
	05	8	07	=
Show Result	08	2nd Pause		
Loop	09	RST		

initialize:
RST, enter any number, R/S

Figure 2

Box	Step	Code
	00	(R/S)
Take the Reciprocal and Multiply by 9	01	g 1/x
	02	9
	03	x
Add -8	04	8
	05	CHS
	06	+
Show Result	07	f PAUSE
Loop	09	GTO 01

initialize:
f PRGM, enter any number, R/S

Some relevant questions which might be asked about this particular activity include:

1. For a given starting number, say 19, what happens (to the value in the display) as the number of times through the loop increases?

2. For different positive integer starting numbers, what happens (to the value in the display)?

3. For a starting number of 0, what happens (to the value in the display)? Are there other integers which give the same result?

4. For different negative integer starting numbers, what happens (to the value in the display)?

5. For different non-integer starting numbers, what happens (to the value in the display)?

6. For what starting number(s) does the value in the display remain constant?

Analysis

If you explore the activity, you will find that for virtually any starting number the value in the display tends to have the value -9 as a limit. We like this type of activity because it motivates the need for a technique to convince us that the limit is really what it appears to be, namely, -9.

Interestingly, it is easy to show that the limit of the sequence of numbers in the display is really -9. Think of the limiting value as x; then x has the property that nine times its reciprocal plus negative eight is itself. That is,

$$(1) \quad x = \frac{9}{x} - 8$$

This equation leads to the quadratic $x^2 + 8x - 9 = 0$, whose roots are $\frac{-8 \pm 10}{2}$. The root $\frac{-8 - 10}{2}$ gives the value -9, which is the limit.

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Related Activities

How does one create activities of this type? Since the activity leads to equation (1), which can be rearranged as a quadratic equation, start with a quadratic equation and reverse the process. For example, start with the quadratic $(x - 5)(x + 3) = 0$ and rewrite it as $x^2 = 2x + 15$, and then either take the square root obtaining equation

$$(2) \quad x = \sqrt{2x + 15}$$

or divide by x obtaining equation

$$(3) \quad x = 2 + \frac{15}{x} .$$

Each of these forms suggests a related activity which you and your students can explore.

Equation (2) indicates the following activity:

(a) Start with any number;

(b) Multiply by 2 and add 15;

(c) Take the square root; and

(d) Repeat from step (b).

Similarly, equation (3) indicates the following additional activity:

(a) Start with any number;

(b) Divide it into 15;

(c) Add 2; and

(d) repeat from step (b).

What happens when you explore these activities? ... similar activities derived from other quadratic equations? ... activities derived from other equations such as cubics?

Conclusion

Hand-held programmable calculators offer an inexpensive alternative to computers as a tool for solving a variety of nonstandard problems. Much study, research, and classroom experimentation is needed to find suitable problems (activities) and ways to use these powerful calculators to enhance the teaching and learning of mathematics at the pre-college level. The authors would enjoy corresponding with persons who are using or know of nonstandard problems which can be solved at the secondary level with programmable calculators, particularly the TI 57 and HP 33 E (or their predecessors, the TI 56 and HP 25). Please direct any correspondence to Professor Spikell at 20 Pinebrook Road, Wayland, MA 01778.

References

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