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THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

Volume XIX, Number 2

November 1979

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

unveils

The Mathematics Curriculum of the 1980s

FIFTY-EIGHTH ANNUAL MEETING
April 16-19, 1980

NCTM 1980

SEATTLE

delta-k

THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

Volume XIX, Number 2 **November 1979**

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EDPRESS

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A — B — C — U in SEATTLE

(An activity sponsored by The NCTM-1980 Seattle Meeting)

TOPIC: Addition and Problem-Solving

MATERIALS: Paper and pencil

FORMAT: Large group working individually

DIRECTIONS:

1. The teacher writes the following chart on the chalkboard or an overhead transparency.

A = 1¢	E = 5¢	I = 9¢	M = 13¢	Q = 17¢	U = 21¢	Y = 25¢
B = 2¢	F = 6¢	J = 10¢	N = 14¢	R = 18¢	V = 22¢	Z = 26¢
C = 3¢	G = 7¢	K = 11¢	O = 15¢	S = 19¢	W = 23¢	
D = 4¢	H = 8¢	L = 12¢	P = 16¢	T = 20¢	X = 24¢	

2. The value of a word is the sum of the values of the letters.

   Examples: CAB is worth 6¢ — C A B: 3¢ + 1¢ + 2¢ = 6¢

   MATH is worth 42¢ — M A T H: 13¢ + 1¢ + 20¢ + 8¢ = 42¢

   NCTM is worth 50¢ — N C T M: 14¢ + 3¢ + 20¢ + 13¢ = 50¢

3. How much is "Seattle" worth?
4. What is the value of your first name; your last name?
5. Find the value of these words: FUN, CENTER, APRIL, YOU.
6. Find a three-letter word worth 10¢ (BAG), a four-letter word worth 20¢ (FEED).
7. Find a word worth 25¢, 75¢, $1, $1.25, $1.50.
8. If the value of each letter is doubled, will the word value double?

+ = +

The National Council of Teachers of Mathematics (NCTM) unveils "The Curriculum of the 1980s" at the 58th Annual Meeting - April 16-19, 1980, The Seattle Center, Seattle, Washington.

+ = +

Guest Editorial

Popular and professional literature more and more frequently uses terms like "basic skills," and "minimal competencies." Twenty-four states or more have established minimal competency requirements for advancement in school or for graduation from high school. Several others have statewide assessment programs that try to determine the degree of acquisition of basic skills in the three Rs. The experience of some states has led several organizations from the NCTM and the National Council of Supervisors of Mathematics to the Missouri, Michigan, and Montana Councils of Teachers of Mathematics to issue position papers on "basic skills."

Inevitably concern over basic skills, whether at the state or local level, gets translated into administering some written test to determine whether such skills have been acquired. This, in turn, more often than not is reduced to using a standardized test of some kind and an interpretation of the normalized scores that are furnished to render decisions about individuals.

Once such scores are available, the temptation is great to assume that teacher effectiveness can be measured by changes in these scores from grade to grade. Terms like "accountability" then join "basic skills" and "minimal competencies."

Three very important questions need thorough debate before this process goes much farther and, through a need for self-preservation, teachers begin to teach to the tests.

What are basic skills?

Memorized number facts or formulas? paper and pencil computations with whole numbers, decimals, and fractions? or are they thinking? imagining? estimating? predicting? inferring? sorting relevant from irrelevant? weighing alternatives and making choices? generating and testing hypotheses? identifying patterns? creating problems, arrangements? and other problem-solving behaviors?

Mathematics teachers, not text and test writers alone, or administrators alone, or legislators alone, should help determine what "basic skills" are, if money will be spent on testing for them, remediating the lack of them, et cetera.

What are minimal competencies?

Are they skills in doing computation better done by calculators or computers? or mathematical literacy? or ability to use graphs and tables? or? Do minimally acceptable performances become the goals of instruction?

Teachers must help determine what minimal performances are appropriate and achieveable for entry into and exit from educational programs.

What do standardized tests test?

What objectives of instruction are measured by commonly used standardized tests?

What learnings are sampled by these test items?

Are single-answer responses the major goal of mathematics teaching?

Are errors indicators of lack of skill or knowledge, or indicators of the state of concept development?

Do standardized tests measure teacher effectiveness?

Is the teacher the major factor, let alone the sole factor, in determining what children learn?

These are not new questions, but they do need continual discussion by teachers, especially now in the days of increased concern by the taxpayer for what his dollar is delivering in the way of outcomes in education. Some helpful readings are:

The Mathematics Teacher, February, 1978.

The Mathematics Teacher, March, 1978.

"The Problems of Minimalcy," Barry Mitzman, Math Learning Center Report, Salem, Oregon.

Phi Delta Kappa, May, 1978.

Phi Delta Kappa, March, 1977.

Phi Delta Kappa, November, 1977.

Phi Delta Kappa, December, 1976.

"Forward to Basics," Thomas O'Brien, incidental paper of the Teacher Center, Southern Illinois U., Edwardsville, Ill. 62026.

Let's add the voices of mathematics teachers to those of administrators, legislators, editors and other newspaper writers, textbook publishers and others in the discussion of these issues.

Dean Hendrickson

Editor
Minnesota Council of Teachers of Mathematics Newsletter

BULLETIN BOARD LAYOUTS WANTED

Because of the wide popularity of its *Bulletin Board Ideas for Elementary and Middle School Mathematics*, NCTM is preparing a similar publication aimed at the secondary school level. Your help is requested. Photographs or sketches of successful bulletin board layouts that teachers have designed and used in their secondary school mathematics classrooms are needed. Here is a chance to help the NCTM create a useful publication by sharing your successful ideas. Possible topics include general mathematics, algebra, geometry, trigonometry, mathematical analysis, probability, statistics, and consumer mathematics. Credit will be given for all pictures or layouts used. Materials should be sent to Seaton E. Smith, Jr., Route 2, Box 153, Gulf Breeze, FL 32561, U.S.A.

Holt Math Bindings

The following letter sent to Mr. Bruce Stonell of the Department of Education will be of interest to mathematics teachers.

Mr. Bruce Stonell
Education Consultant
Red Deer Regional Office
Department of Education
4th Floor, Royal Trust Building
4814 Ross Street, P.O. Box 5002
Red Deer T4N 5Y5

Dear Bruce,

David Durbin and I have discussed the problem of the Holt Math bindings with Holt's production people. We have also had the bindings tested by a firm in Montreal that specializes in these problems. To date, the findings have been inconclusive. There does not appear to be a universal problem with the bindings; some of the books are holding up well.

However, this does not solve your problems. Clearly some of the books sent to Alberta have not stood up under normal use. To solve your problem, we will do the following:

1. All stock of Holt Math 1, 2, 3 and 4, at the advice of the Montreal firm, will be strengthened by placing reinforcement tape front and back. This will include the inventory held at the Alberta School Book Branch.

2. Defective books can be returned to Holt or to the Alberta Book Branch. They will be replaced. Defects in the binding are not widespread outside of Alberta, so we hope your problems are a result of a bad batch of glue, or some such production problem.

I apologize, on behalf of Holt, Rinehart and Winston, for all inconveniences this may cause. To expedite the replacing of any defective copies, please address any queries to my attention at 55 Horner Avenue, Toronto M8Z 4X6.

Yours very truly,

Doug Panasis
Manager
School Division
HOLT, RINEHART AND WINSTON OF CANADA, LIMITED

National Council of
Teachers of Mathematics

A Position Statement on

Calculators in the Classroom

The National Council of Teachers of Mathematics encourages the use of calculators in the classroom as instructional aids and computational tools. Calculators give mathematics educators new opportunities to help their students learn mathematics and solve contemporary problems. The use of calculators, however, will not replace the necessity for learning computational skills.

As instructional aids, calculators can support the development and discovery of mathematical concepts. As computational tools, they reduce the time needed to solve problems, thereby allowing the consideration of a wider variety of applications. Furthermore, the use of calculators requires students to focus on the analysis of problems and the selection of appropriate operations. The effective use of calculators can improve student attitudes toward, and increase interest in, mathematics.

Other electronic devices, programmed to generate questions and activities, that provide immediate feedback to students, are not to be confused with calculators or computers. These devices can be used to reinforce computational skills through drill.

The NCTM Instructional Affairs Committee has identified the following justifications for using the calculator in the schools.

- The calculator can be used to encourage students to be inquisitive and creative as they experiment with mathematical ideas.
- The calculator can be used to assist the individual to become a wiser consumer.
- The calculator can be used to reinforce the learning of the basic number facts and properties in addition, subtraction, multiplication and division.
- The calculator can be used to develop the understanding of computational algorithms by repeated operations.
- The calculator can be used to serve as a flexible "answer key" to verify the results of computation.
- The calculator can be used as a resource tool that promotes student independence in problem solving.
- The calculator can be used to solve problems that previously have been too time-consuming or impractical to be done with paper and pencil.
- The calculator can be used to formulate generalizations from patterns of numbers that are displayed.
- The calculator can be used to decrease the time needed to solve difficult computations.

The Fixed Element Match Problem:
An Illustration with Cards

by *Bonnie H. Litwiller* and *David R. Duncan*
Professors of Mathematics
University of Northern Iowa
Cedar Falls, Iowa

In probability and statistics units, the following two problems are often considered. Suppose that each of n-persons is given a standard deck of 52 playing cards. At a given time, each person draws at random a card from his/her deck.

1. What is the probability that at least two people draw exactly the same card (same suit, same face value)?

2. If a specific card is designated in advance, what is the probability that at least one person will draw it?

Many students have difficulty in distinguishing between these two problems; let us solve both.

1. Let a group of n-persons be arranged in some order. If each person consecutively draws a card, there can result $\underbrace{52 \cdot 52 \cdot 52 \cdot \cdots \cdot 52}_{\text{n factors}}$ or 52^n distinct n-tiples of cards. For each person, each of the cards is equally likely to be drawn. Since the n-persons are drawing their cards independently, each of the 52^n n-tiples is equally likely to occur.

How many of these n-tiples contain *no* matches? Again, visualize the n-persons selecting their cards consecutively. The first person can select any of 52 cards; to avoid a match, the second person can select from 51 cards, the third from 50 cards, and so on. Finally, the n^{th} person can select from any of $(52 - n + 1)$ cards. The number of ways that these n cards can be selected with no matches is thus $52 \cdot 51 \cdot 50 \cdot \cdots \cdot (52 - n + 1)$. The probability of no matches is thus $\frac{52 \cdot 51 \cdot 50 \cdot \cdots \cdot (52 - n + 1)}{52^n}$; the probability of at least one match is therefore $1 - [\frac{52 \cdot 51 \cdot 50 \cdot \cdots \cdot (52 - n + 1)}{52^n}]$.

2. For each draw, the probability of selecting a given card is 1/52. The probability of not selecting a given card is 51/52. Since the n-draws are independent, the probability that none of the n-persons will select the given card is $\underbrace{\frac{51}{52} \cdot \frac{51}{52} \cdot \frac{51}{52} \cdot \cdot \cdot \cdot \cdot \frac{51}{52}}_{n \text{ factors}}$ or $(\frac{51}{52})^n$. Consequently, the probability that at least one person selects the given card is $1 - (\frac{51}{52})^n$.

The students who think the two problems are substantially the same are very surprised that distinctly different probabilities arise from them. Table I reports these probabilities for varying sizes of n.

TABLE I

n (number of decks)	*Probability of at least one match (Problem 1)*	*Probability of matching a given card (Problem 2)*
1	0	.0192
2	.0192	.0381
3	.0570	.0566
4	.1114	.0747
5	.1793	.0925
6	.2586	.1100
7	.3441	.1271
8	.4324	.1439
9	.5197	.1603
10	.6029	.1765
11	.6792	.1923
12	.7471	.2079
13	.8055	.2231
14	.8541	.2380
15	.8934	.2527
16	.9241	.2671
17	.9475	.2812
18	.9646	.2950
19	.9769	.3085
20	.9853	.3218
21	.9910	.3349
22	.9946	.3477
23	.9969	.3602
24	.9983	.3725
25	.9991	.3846
26	.9995	.3964
27	.9998	.4080
28	.9999	.4194
29	.9999	.4306

n (number of decks)	*Probability of at least one match (Problem 1)*	*Probability of matching a given card (Problem 2)*
30	1.0000 (rounded to 4 decimal places)	.4415
31		.4523
32		.4628
33		.4731
34		.4833
35		.4932
36		.5029

Observe from the table that for Problem 1 only 9 decks are needed before the probability of at least one match exceeds 1/2, while to match a fixed card (Problem 2) 36 decks are needed before the probability of such a match exceeds 1/2.

Logarithms can also be used to achieve the "break-even" value of n in Problem 2. Find the number of decks needed in order that the probability of at least one match with a fixed card exceeds 1/2. Algebraically, solve the following inequality for n: $1 - (\frac{51}{52})^n > \frac{1}{2}$.

Equivalently: $(\frac{51}{52})^n < \frac{1}{2}$

$$n \log_e (\frac{51}{52}) < \log_e (\frac{1}{2})$$

$$n > \frac{\log_e (\frac{1}{2})}{\log_e (\frac{51}{52})} \quad \text{(Since } \frac{51}{52} < 1\text{, the log is negative.)}$$

$$n > \frac{-.6931}{-.0194}$$

$$n > 35.70$$

Consequently, at least 36 decks are needed.

For Problem 1 with 30 decks, the probability of at least one match exceeded .99995 and thus rounded to 1.0000 (4 decimal places). In Problem 2, how many decks are needed so that the probability of matching the fixed card exceeds .99995 and rounds to 1.0000?

$$1 - (\frac{51}{52})^n > .99995$$

$$(\frac{51}{52})^n < .00005$$

$$n \log_e \left(\frac{51}{52}\right) < \log_e .00005$$

$$n > \frac{\log_e 0.00005}{\log_e \left(\frac{51}{52}\right)}$$

$$n > \frac{-9.9034876}{-.01941809}$$

$$n > 510.01$$

Therefore, 511 decks are needed.

Challenge for the reader: A well-known prabability situation involves comparing birthday anniversaries for each of n-randomly-selected persons. State problems analogous to Problems 1 and 2 in the birthday situations and construct an appropriate table for comparison purposes.

Providing for Individual Differences and The Mathematics Curriculum

by *Dr. Marlow Ediger*
Professor of Education
Northeast Missouri State University
Kirksville, Missouri

Educators have long experimented with numerous means to provide for individual differences in learners. Among other methods, the following have been utilized: contracts, sequential specific objectives, learning centers, programmed materials, packaged materials, computer-assisted instruction, individualized reading and the project method.

Teachers, principals, and supervisors also need to study, analyze, and utilize recommended principles and theories from the psychology of learning. Thus, pupils may be guided to achieve optimal development in the mathematics curriculum, as well as in other curricular areas.

A Hierarchy of Objectives

Robert Gagné, educational psychologist from Florida State University, emphasizes the importance of *educators* in the school-class setting determining what pupils are to learn. The initial question that teachers then need to raise and answer in any unit of study is, "What do I want pupils to learn?" The answer to the stated question becomes a vital goal for pupils to achieve. In order to attain this objective, the teacher must analyze and state specific objectives learners need to achieve sequentially. For example, a teacher may have selected the following goal, among others, for learner achievement:

> Given the number of times a batter hits safely compared to the number of times at bat, the pupil will compute correctly batting averages of five baseball players.

To attain the measurably stated end, the teacher needs to analyze which sequential specific ends pupils need to achieve. These sequential objectives may involve understanding a $\frac{\text{numerator}}{\text{denominator}}$ ratio concept such as $\frac{\text{the number of times hit safely}}{\text{the number of times at bat}}$.

Pupils may also need to achieve computation skills in dividing the numerator by the denominator. Perhaps learners need to achieve skills pertaining to checking their computations. Thus, the teacher needs to determine present achievement levels of pupils and arrange objectives sequentially to guide learners to compute accurately batting averages of selected baseball players. Achieving specific objectives sequentially in a *logical curriculum* may assist each pupil to develop optimally in the mathematics arena.

Jean Piaget and Stages of Pupil Development

Jean Piaget, clinical psychologist from Geneva, Switzerland, in studying pupil development for over 50 years, emphasizes that pupils generally progress through specific stages as a result of maturation.

Piaget has identified the following general stages of individual development:

1. Sensory-motor intelligence (this stage includes, approximately, the first 18 months of an infant's life).
2. Preoperational stage (from age 18 months to seven years, approximately). Pupils in this stage of development lack the ability to think logically. They perceive one variable only, such as the length of an object, or the width of an item.
3. Stage of concrete operations (age seven through age 11, approximately). Learners can think logically; the continued use of concrete materials in teaching and learning is important.
4. Stage of formal operations (age 12 and continuing). Pupils need less of concrete experiences and benefit increasingly more from abstract experiences.

Readiness for moving sequentially through each of the above named stages of development basically cannot be hastened. Thus, biological maturation on the part of each learner is necessary in progressing from the sensory-motor stage of development to the preoperational stage, and ultimately achieving the stage of formal operations.

According to Piaget's school of thought in the psychology of learning, the teacher needs to follow criteria, such as the following, in ongoing units of study:

1. Learning activities must be chosen based on the individual pupil's present level of maturation.
2. Rather heavy emphasis must be placed upon the utilization of concrete materials in teaching-learning situations through the stage of concrete operations.
3. Objectives emphasizing the abstract may be emphasized increasingly so, beginning with the stage of formal operations.
4. The learner must actively operate mentally in ongoing activities and experiences, otherwise learning generally does not take place.

The Structure of Knowledge

Jerome Bruner, psychologist from Harvard University, emphasizes the importance of utilizing enactive (manipulative materials), iconic (semi-concrete experiences including pupils obtaining mental images of concepts), and symbolic (abstract) materials in teaching-learning situations. Thus, within each learning activity, enactive, iconic, and abstract content would be stressed in ongoing units of study. No specific levels of maturation that pupils progress through are emphasized by Jerome Bruner. Thus, readiness for learning may be hastened within each pupil if proper materials and methods are utilized in teaching-learning situations. Jerome Bruner, in the well known book *The Process of Education*, states the following hypothesis: We begin with the hypothesis that any subject can be taught effectively in some intellectually honest form to any child at any stage of development (page 33).

Thus, pupils may achieve complex structural concepts such as the commutative property of addition and multiplication, as well as the associative property of multiplication and addition at a relatively young age in the mathematics curriculum. Pupils may achieve these structural ideas on each achievement level but at increasing levels of complexity. Jerome Bruner places heavy emphasis upon pupils learning content inductively. Thus, the stage needs to be set for pupils to achieve structural ideas through induction. Excitement in learning occurs if pupils find out on their own rather than hearing explanations to acquire facts, concepts, and generalizations.

To provide for individual differences, then, Jerome Bruner recommends utilizing enactive, iconic, and symbol learnings within the framework of each learning activity for pupils to achieve key structural ideas inductively.

Humanism and the Curriculum

To provide for individual differences, humanism, as a psychology of learning, emphasizes pupils sequentially choosing objectives and learning activities in a stimulating environment. Within a flexible framework, pupils may then select mathematics experiences from among the following in ongoing units of study:

1. Programmed learning and management systems of instruction.
2. Reputable single or multiple series of textbooks.
3. Contract systems and project methods.
4. Mathematics laboratory approaches.
5. Films, filmstrips, slides and transparencies.
6. Learning stations and open spaces.

To implement a humane learning environment, pupils need to have ample opportunities to choose and select that which is purposeful and self-fulfilling.

A.H. Maslow, a leading humanist, emphasizes a hierarchy of needs that learners have which desire fulfillment. These needs in ascending order of complexity are:

1. *Physiological needs*, including adequate nutrition, rest, clothing and shelter.
2. *Safety needs*, including feelings of security.
3. *Love and belonging needs*, including being accepted positively by others.
4. *Esteem needs*, including feelings of being prized for abilities possessed.
5. *Self-actualization needs*, including the learner becoming what he/she desires to become.

The simplest needs - physiological - must be met first, generally, before increasingly more complex needs are to be met, such as safety, love and belonging, esteem, and self-actualization needs. The mathematics curriculum may then assist in need fulfillment on the part of each pupil. Each of the above named needs, except physiological, can be met in part in stimulating objectives, learning activities and evaluation techniques within the framework of mathematics units of study. Developing feelings of security, belonging, importance, and realizing one's optimal self may well become vital objectives for pupil achievement in the mathematics curriculum.

Selected References

Bruner, Jerome S. *The Process of Education*. New York: Vintage Books, 1960.

Ediger, Marlow. *The Elementary Curriculum, A Handbook*. Kirksville, Missouri: Simpson Publishing Company, 1977.

Eliot, John (ed.) *Human Development and Cognitive Processes*. New York: Holt, Rinehart and Winston, Inc., 1971.

Goodwin, William L., and Herbert J. Klausmeier. *Facilitating Student Learning*. New York: Harper and Row, Publishers, 1975.

$x = \frac{9}{x} - 8$,

A Programmable Calculator Activity

by *Dr. Stephen L. Snover* and *Dr. Mark A. Spikell*
University of Hartford
West Hartford, Connecticut

Lesley College
Cambridge, Massachusetts

The authors are presenting variations of this activity in several journals in order to invite reader reaction and correspondence about nonstandard problems that can be solved with programmable calculators.

Programmable calculators, as well as computers, are ideal problem-solving tools for secondary students and ideal teaching aids for secondary teachers. In fact, programmable calculators are often better devised for teaching and learning purposes than computers for several reasons, including the following:

1. they now cost so little (a TI 57 is less than $50, and a HP 33 E less than $90) that individuals and schools are increasingly able to afford them;
2. they are small, hand-held, and can be operated from rechargeable battery packs for complete portability;
3. they provide instant access - there is no waiting for that one, always busy, terminal; and
4. they solve many of the same kinds of problems handled by computers.

The purpose of this article is to share one problem (activity) which you and your students can explore with a calculator. The activity has two key features. First, it is non-standard; that is, it is an activity that would not be easily explored without the use of a programmable calculator. Second, the activity can be used to motivate a discussion of some interesting mathematics, as will be noted following the statement of the activity.

The Activity

(a) Start with any number;

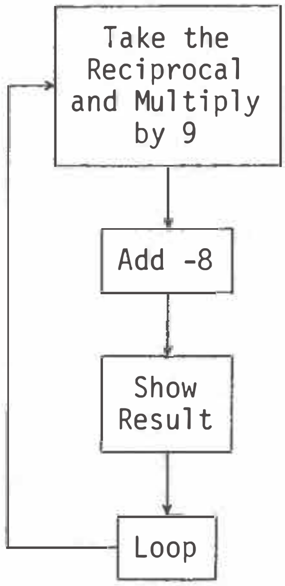
(b) Take the reciprocal and multiply by 9;

(c) Add -8; and

(d) Repeat from step (b).

Figures 1 and 2 present flow charts and programs for the TI 57 and HP 33 E programmable calculators, respectively. These machines are chosen as they represent the least expensive models of the two major domestic manufacturers and will likely, therefore, be more widely available than other models.

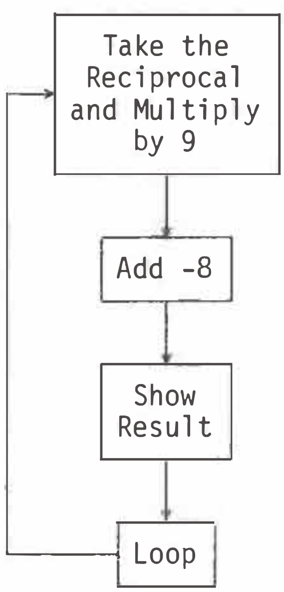
Figure 1




```
00   1/x
01   x
02   9
03   =

04   +    06   +/-
05   8    07   =

08   2nd Pause

09   RST
```

initialize:
RST, enter any number, R/S

Figure 2




```
00   (R/S)

01   g 1/x
02   9
03   x

04   8
05   CHS
06   +

07   f PAUSE

09   GTO 01
```

initialize:
f PRGM, enter any number, R/S

Some relevant questions which might be asked about this particular activity include:

1. For a given starting number, say 19, what happens (to the value in the display) as the number of times through the loop increases?
2. For different positive integer starting numbers, what happens (to the value in the display)?
3. For a starting number of 0, what happens (to the value in the display)? Are there other integers which give the same result?
4. For different negative integer starting numbers, what happens (to the value in the display)?
5. For different non-integer starting numbers, what happens (to the value in the display)?
6. For what starting number(s) does the value in the display remain constant?

Analysis

If you explore the activity, you will find that for virtually any starting number the value in the display tends to have the value -9 as a limit. We like this type of activity because it motivates the need for a technique to convince us that the limit is really what it appears to be, namely, -9.

Interestingly, it is easy to show that the limit of the sequence of numbers in the display is really -9. Think of the limiting value as x; then x has the property that nine times its reciprocal plus negative eight is itself. That is,

$$(1) \quad x = \frac{9}{x} - 8$$

This equation leads to the quadratic $x^2 + 8x - 9 = 0$, whose roots are $\frac{-8 \pm 10}{2}$. The root $\frac{-8 - 10}{2}$ gives the value -9, which is the limit.

Related Activities

How does one create activities of this type? Since the activity leads to equation (1), which can be rearranged as a quadratic equation, start with a quadratic equation and reverse the process. For example, start with the quadratic $(x - 5)(x + 3) = 0$ and rewrite it as $x^2 = 2x + 15$, and then either take the square root obtaining equation

$$(2) \quad x = \sqrt{2x + 15}$$

or divide by x obtaining equation

$$(3) \quad x = 2 + \frac{15}{x} .$$

Each of these forms suggests a related activity which you and your students can explore.

Equation (2) indicates the following activity:

(a) Start with any number;

(b) Multiply by 2 and add 15;

(c) Take the square root; and

(d) Repeat from step (b).

Similarly, equation (3) indicates the following additional activity:

(a) Start with any number;

(b) Divide it into 15;

(c) Add 2; and

(d) repeat from step (b).

What happens when you explore these activities? ... similar activities derived from other quadratic equations? ... activities derived from other equations such as cubics?

Conclusion

Hand-held programmable calculators offer an inexpensive alternative to computers as a tool for solving a variety of nonstandard problems. Much study, research, and classroom experimentation is needed to find suitable problems (activities) and ways to use these powerful calculators to enhance the teaching and learning of mathematics at the pre-college level. The authors would enjoy corresponding with persons who are using or know of nonstandard problems which can be solved at the secondary level with programmable calculators, particularly the TI 57 and HP 33 E (or their predecessors, the TI 56 and HP 25). Please direct any correspondence to Professor Spikell at 20 Pinebrook Road, Wayland, MA 01778.

References

Snover, Stephen L., and Mark A. Spikell. *How To Program Your Programmable Calculator.* Englewood Cliffs, NJ: Prentice Hall, Inc., 1979.

______. "Because of Programmable Calculators, Why Avoid These Problems Any Longer?" (submitted January, 1979).

______. "Generally, How Do You Solve Equations?" *Mathematics Teacher* (to appear).

______. "The Role of Programmable Calculators and Computers in Mathematical Proofs," *Mathematics Teacher*, December, 1978, pp.745-50.

activities

Calculator Graphing

by *William A. Miller* and *Donald W. Hazekamp*
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Mount Pleasant, Michigan

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Teacher's Guide

Grade Level: 7–12

Materials: One set of activity sheets and a hand-held calculator for each student. If you do not have an adequate supply of calculators, students may work in pairs.

Objectives: To provide experience in using calculators to compute squares, reciprocals, and square roots. Experience also is gained in rounding numbers to the nearest hundredth, in plotting points, and in drawing graphs.

Directions: Distribute the activity sheets, one at a time, to each student. Have the students complete sheet 1 before beginning sheet 2.

Sheet 1. This initial activity familiarizes students with using a calculator to compute squares of numbers from zero to one and in rounding off these numbers to the nearest hundredth. A nice extension is to graph the curve $y = x^3$ in the same interval. Some students may be bothered by the fact that the cubes of 0.00, 0.05, 0.10, and 0.15 are each 0.00 when rounded off to the nearest hundredth.

Sheet 2. This activity provides more practice in using the calculator, in rounding off, and in graphing. If your calculators do not have the reciprocal function, these values can be computed by using the division function ($y = 1 \div x$).

Sheet 3. This activity requires calculators with a square-root key. It may be helpful to have your students graph the curve $y = \sqrt{x}$ in the interval $0 \leq x \leq 1$ before working on this sheet. In completing this activity, have your students round off x^2 to the nearest hundredth. The corresponding values of $\sqrt{1 - x^2}$ are sufficiently accurate when computed from that data. With students who have the appropriate background, you may wish to develop the relationship between the equations $x^2 + y^2 = 1$ and $y = \sqrt{1 - x^2}$. For further study, students may be interested in comparing the graphs of $y = \sqrt{4 - x^2}$ or $y = \sqrt{9 - x^2}$ on the corresponding intervals $0 \leq x \leq 2$ or $0 \leq x \leq 3$. For these graphs, change the horizontal scale so that ten to twenty points are generated.

Bibliography

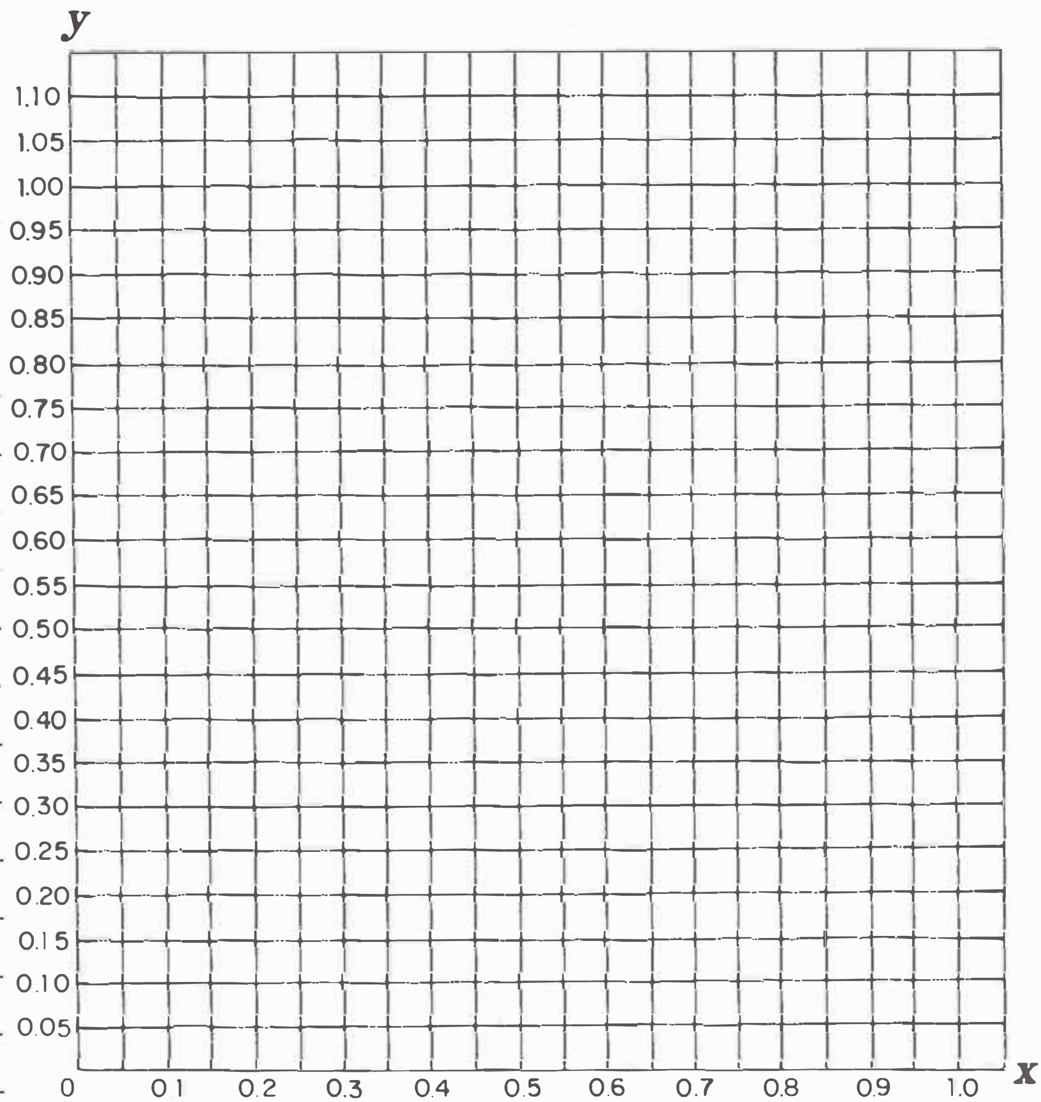
Miller, William A. *Laboratory Activities in Algebra.* Teacher ed. Portland, Maine: J. Weston Walch, Publisher, 1974, pp. T28–T30 and 52–57.

SHEET 1

Graphing $y = x^2$

1. Use your calculator to complete the following table. Round off your answers to the nearest hundredth.

x	$y = x^2$
.00	
.05	
.10	
.15	
.20	
.25	
.30	
.35	
.40	
.45	
.50	
.55	
.60	
.65	
.70	
.75	
.80	
.85	
.90	
.95	
1.00	



2. After you have completed the table, graph the ordered pairs (x , y).

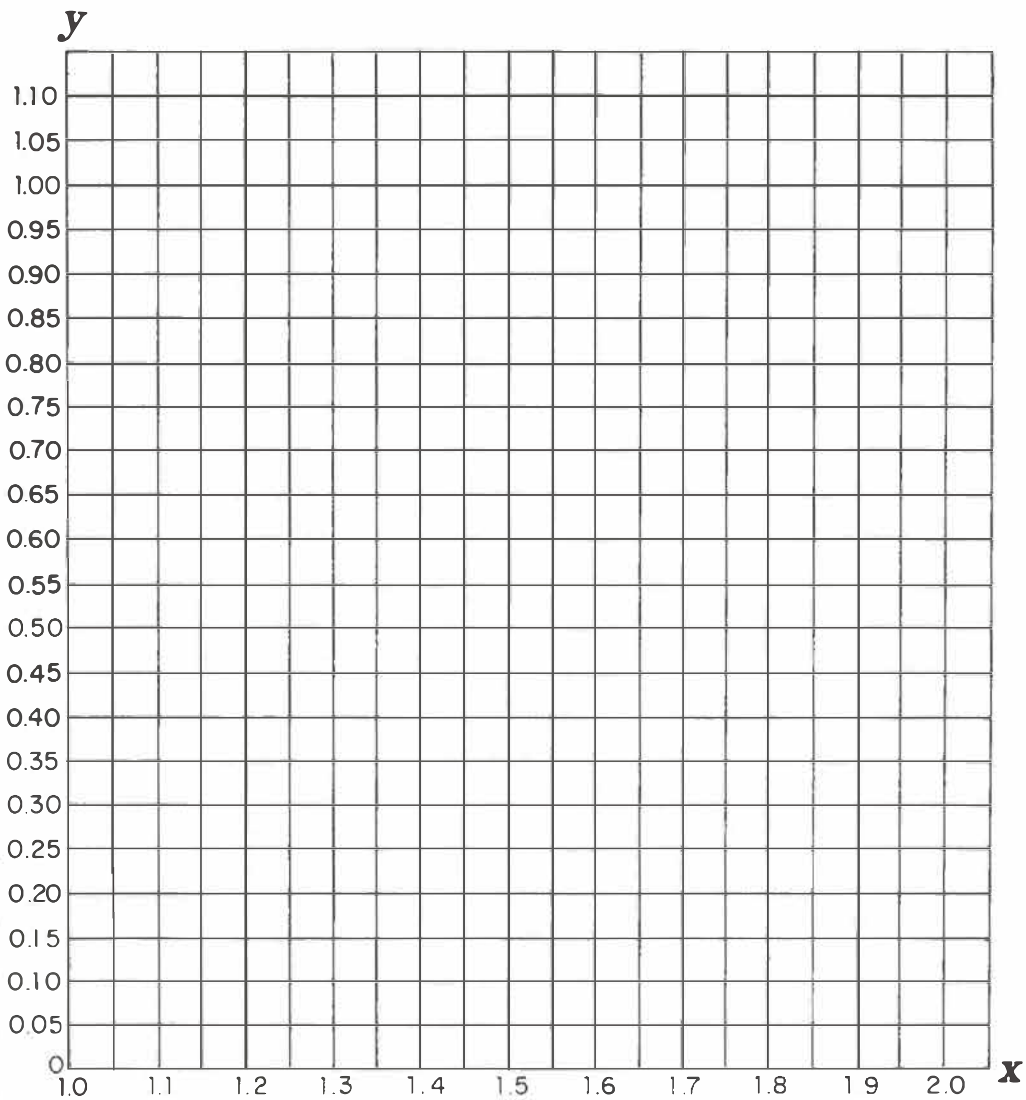
3. Connect the points with a smooth curve to complete the graph.

SHEET 2

Graphing $y = \frac{1}{x}$ from 1 to 2

1. Use your calculator to complete the following table. Round off your answers to the nearest hundredth.

x	$y = \frac{1}{x}$
1.00	
1.05	
1.10	
1.15	
1.20	
1.25	
1.30	
1.35	
1.40	
1.45	
1.50	
1.55	
1.60	
1.65	
1.70	
1.75	
1.80	
1.85	
1.90	
1.95	
2.00	



2. After you have completed the table, graph the ordered pairs (x , y).

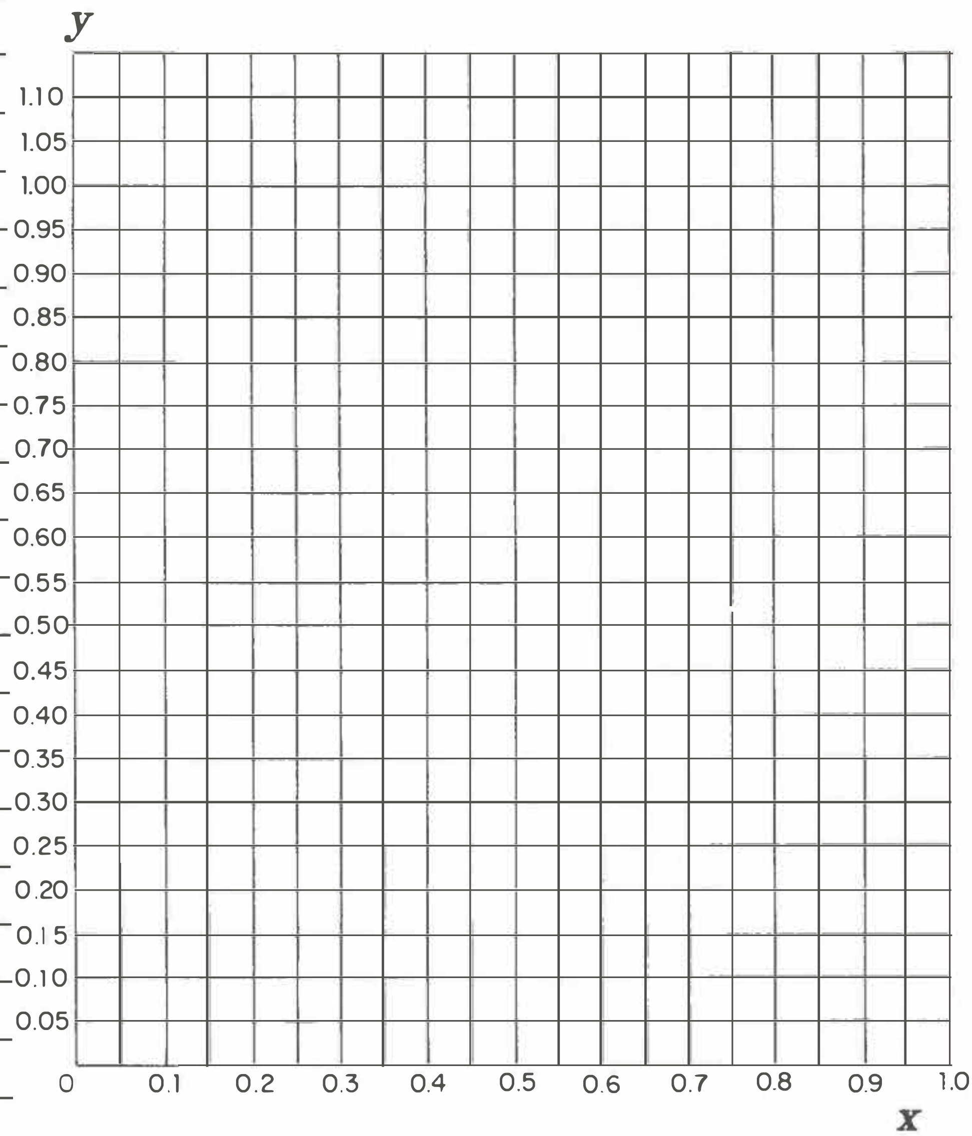
3. Connect the points with a smooth curve to complete the graph.

SHEET 3

Graphing $x^2 + y^2 = 1$

1. Use your calculator to complete the following table. Round off your answers in each column to the nearest hundredth.

x	x^2	$1-x^2$	$y=\sqrt{1-x^2}$
.00			
.05			
.10			
.15			
.20			
.25			
.30			
.35			
.40			
.45			
.50			
.55			
.60			
.65			
.70			
.75			
.80			
.85			
.90			
.95			
1.00			



2. After you have completed the table, graph the ordered pairs (x , y).

3. Connect the points with a smooth curve to complete the graph.

Ideas

Prepared by *George W. Bright*
Northern Illinois University
DeKalb, Illinois

Ideas for this month provides activities to build the concept of area measure. Considerable experience with the idea of comparing and measuring areas seems to be needed for students to understand area measure fully. The introduction of units of area measure, nonstandard or standard, should be delayed until students are able to compare regions directly and to put such regions together to cover a larger region. Only then will the repeated use of a single unit be meaningful. Finally, a standard unit and formulas for area can be used.

IDEAS for Teachers: Levels K, 1, 2

Objective: To practice comparing and ordering areas.

Directions for teachers:

1. Give each student a copy of the worksheet and scissors.
2. Read the directions to the students.
3. You may want to have students paste the figures on sheets of construction paper.
4. When everyone is finished, conduct a discussion with the class. Ask students to explain why they ordered the figures as they did. Help the students use the words *bigger (larger)* and *smaller.*

IDEAS for Teachers: Levels 2, 3, 4

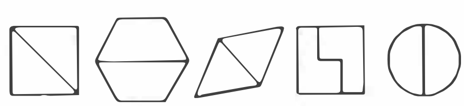
Objective: To practice covering regions to illustrate additivity of area.

Directions for teachers:

1. Give each student a copy of the worksheet and scissors.
2. You may want the students to paste the two shaded regions on each black region.
3. In exercises 3 and 5, the shaded regions have to be rotated before they will fit.

Answers:

1. 2. 3. 4. 5.



Going further:

Ask students to make up puzzles like these.

IDEAS for Teachers: Levels 3, 4, 5

Objective: To measure areas by covering regions with a nonstandard unit.

Directions for teachers:

1. Give each student a copy of the worksheet and scissors.
2. Be sure students understand that some of the diamonds may need to be cut into pieces in order to cover the figures completely.
3. Some students may prefer to work in small groups.

Answers:

1. 4	2. 2	3. 6
4. 1	5. 12	6. $4\frac{1}{2}$

Going further:

Ask students to make a figure that could be covered with five diamonds, 10 diamonds, or 20 diamonds.

IDEAS for Teachers: Levels 6, 7, 8

Objective: To practice measuring in centimetres, computing areas, and computing ratios.

Directions for teachers:

1. Give each student a copy of the worksheet and a ruler marked in centimetres. (Measuring in customary units will produce numbers that are too complex for students to handle.)
2. Be sure students know the formula for the area of a triangle, $A = \frac{1}{2}bh$.
3. The pattern students should see is that the ratio of areas is the square of the ratio of length (or width or base or height).

Answers:

	ratio of lengths (base)	*ratio of areas*
1.	4:1 or $\frac{4}{1}$	16:1 or $\frac{16}{1}$
2.	2:1 or $\frac{2}{1}$	4:1 or $\frac{4}{1}$
3.	2:1 or $\frac{2}{1}$	4:1 or $\frac{4}{1}$
4.	3:1 or $\frac{3}{1}$	9:1 or $\frac{9}{1}$
5.	3:2 or $\frac{3}{2}$	9:4 or $\frac{9}{4}$

Going further:

Ask students to check their generalizations by drawing their own pairs of similar figures and measuring areas.

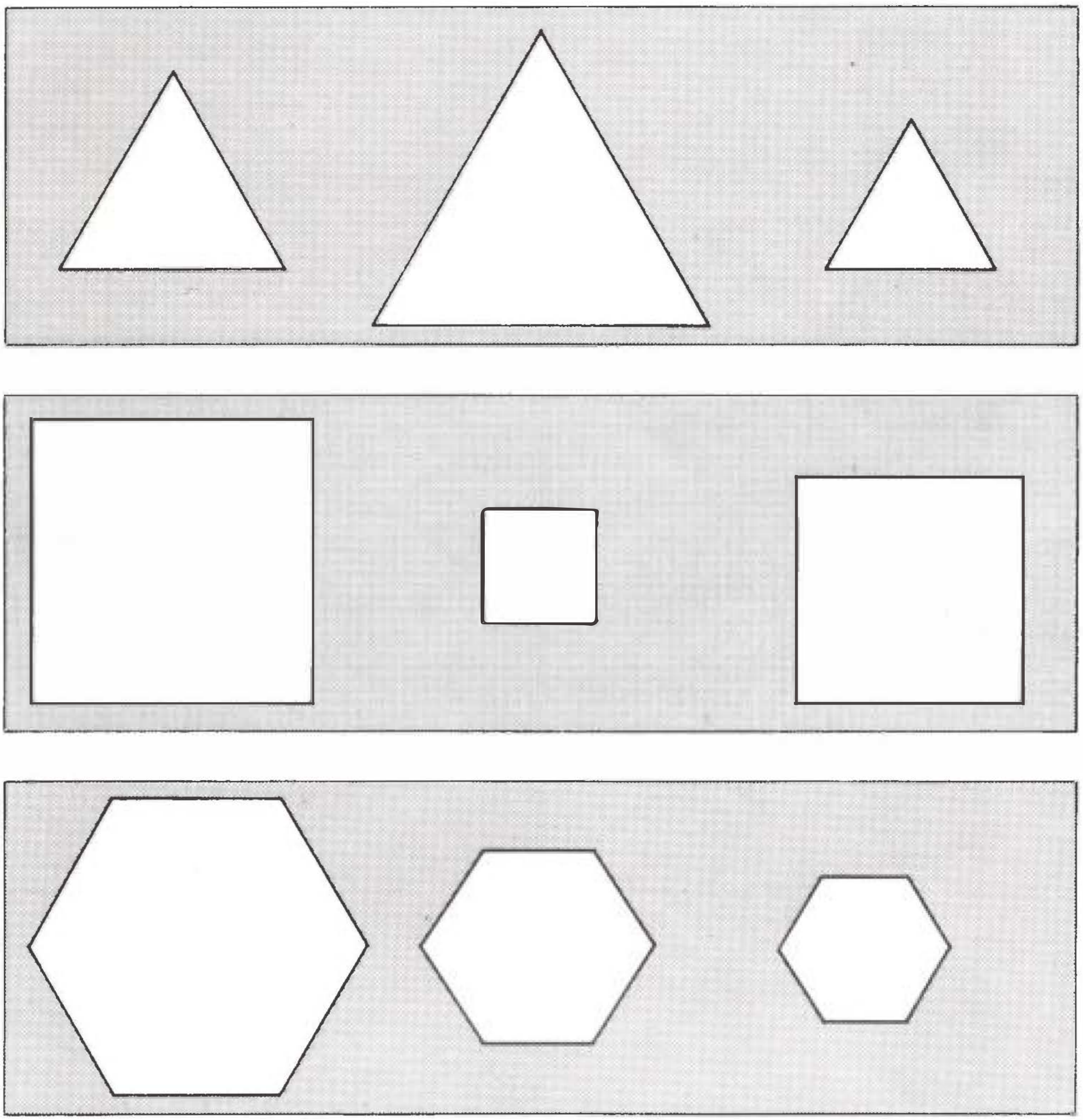
ID[redacted]AS

Levels K, I, 2

Name ____________________

Cut out the figures in each box.

Put them in order from smallest to largest.



From the *Arithmetic Teacher*

IDEAS

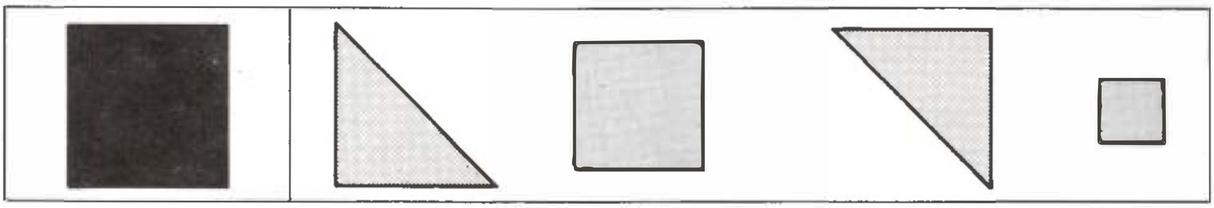
Name ______________________

Levels 2, 3, 4

In each row, choose two shaded regions that exactly cover the black region when you put them together.

Check your answers by cutting out the two shaded regions and laying them on top of the black region.

1.



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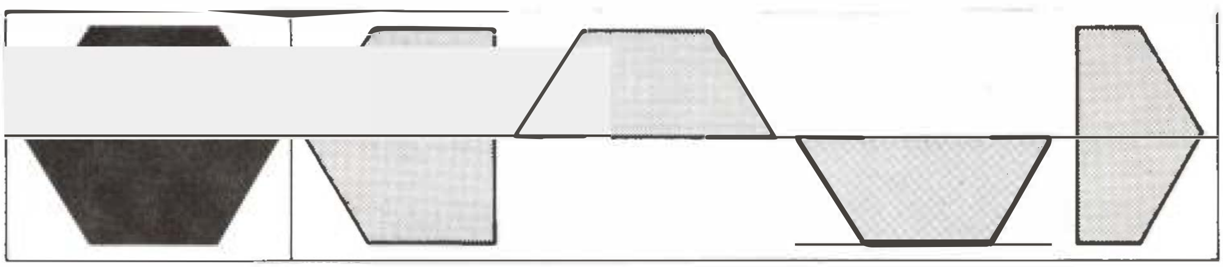

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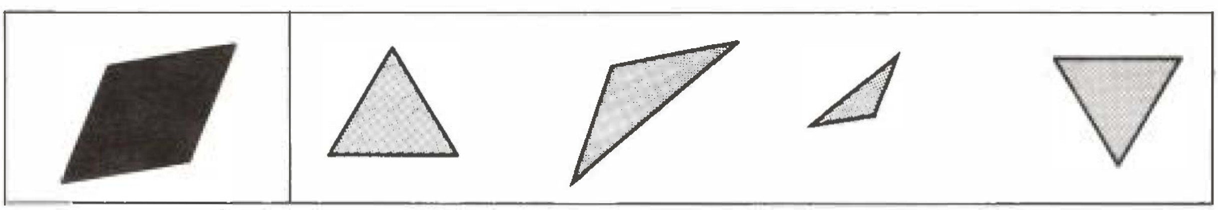

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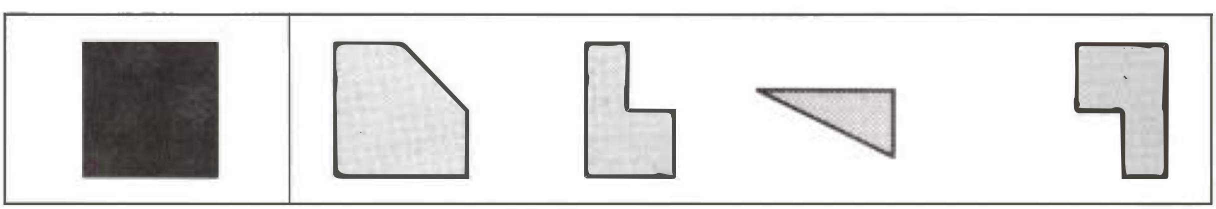

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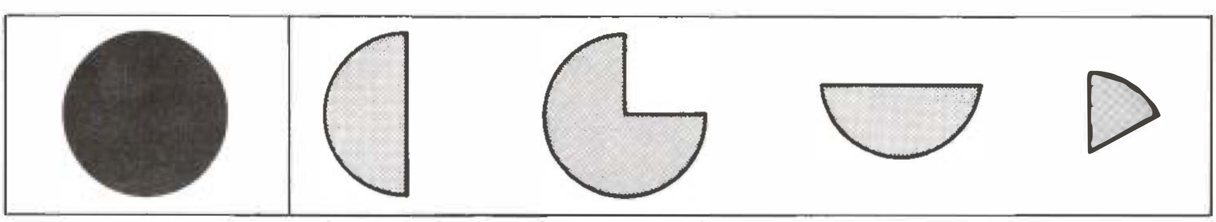
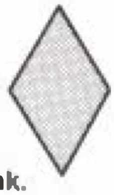


From the *Arithmetic Teacher*

IDEAS

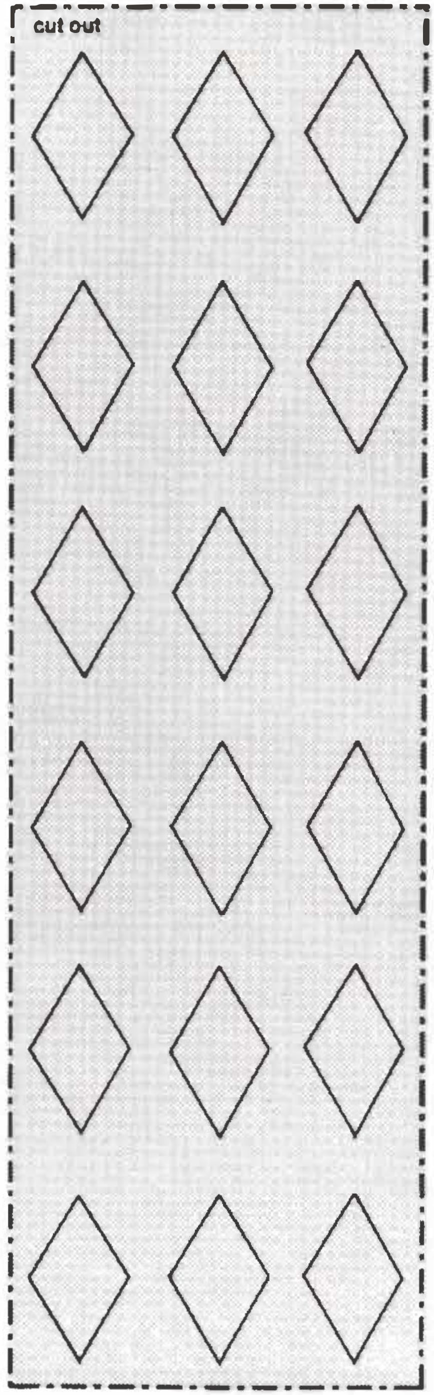
Name ____________________

Levels 3, 4, 5

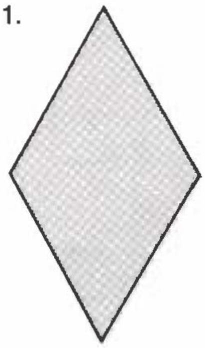
How many of these  will cover each figure?

Write the number in the blank.

You may want to cut some of the diamonds into pieces.



1.



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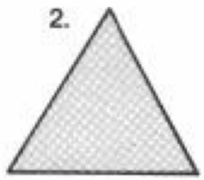

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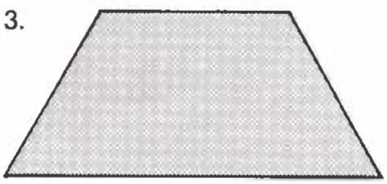

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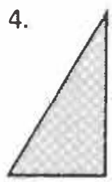

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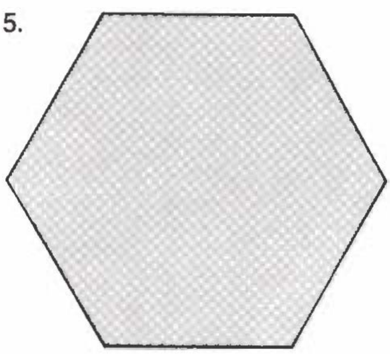

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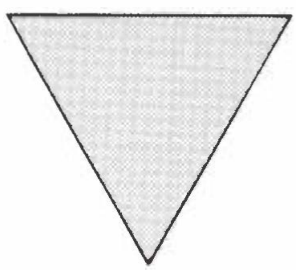


From the *Arithmetic Teacher*

IDEAS

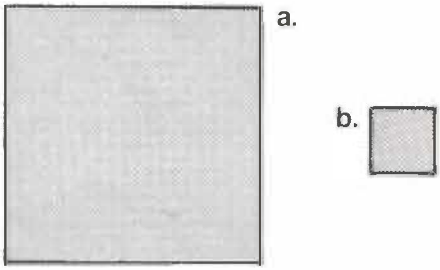
Name ____________________

Levels 6, 7, 8

Complete each table. Use a centimetre as the unit of measure.

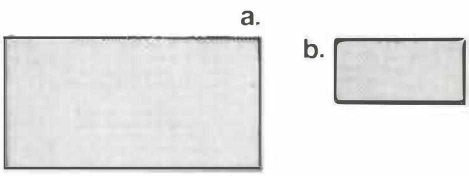
What pattern do you see? ____________________

1.



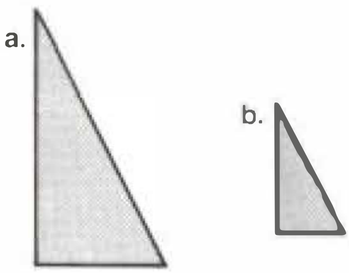
	length	ratio of length	width	ratio of width	area	ratio of area
a						
b						

2.



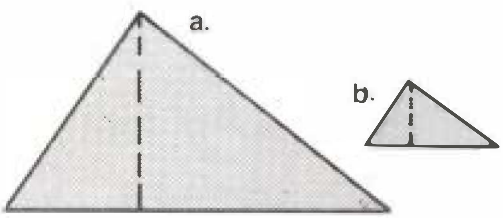
	length	ratio of length	width	ratio of width	area	ratio of area
a						
b						

3.



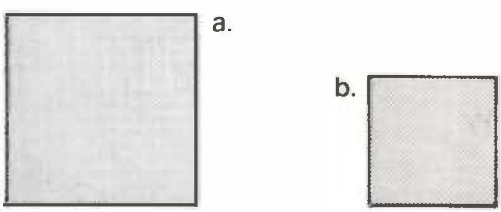
	base	ratio of base	height	ratio of height	area	ratio of area
a						
b						

4.



	base	ratio of base	height	ratio of height	area	ratio of area
a						
b						

5.



	length	ratio of length	width	ratio of width	area	ratio of area
a						
b						

From the *Arithmetic Teacher*

Instructive Prime Number Reduction Algorithms

by *William J. Bruce*

The purpose of this article is to provide teachers of elementary through high school with a collection of algorithms that are suitable as source material. These can be used either for practice with the simple arithmetical operations alone or can be used as an introduction to graphical representation, flow charts, and computer programming.

Certain types of algorithms that utilize only prime numbers as elements for reduction are possible, but not always easy to design, let alone prove. In each one, the purpose is to design an algorithm on the set of integers N, such that N ≥ 2, in which the first chosen operation is always division by one or more prime numbers. Also, these algorithms must reduce every integer, N ≥ 2, to unity. Furthermore, in this reduction, they must succeed only with the inclusion of the first named operation and fail, otherwise.

The following set of such algorithms has been designed by the author, except where otherwise indicated:

1. N/2
   Odd 3N + 1
   Even N/2

2. (a) N/3
   Odd N - 1
   Even 2N + 1

   (b) N/3
   Odd (N + 1)/2
   Even 3N - 1

3. (a) N/5
   Odd 3N - 1
   Even N/2

   (b) N/5
   Odd (N + 1)/2
   Even 3N - 1

   (c) N/5
   Odd (N + 1)/2
   Even 2N + 1

   (d) N/5 N/3
   Odd N + 1
   Even 2N + 1

   If N/5 is deleted, failure occurs for N = 6.

4. (a) N/7 N/5
   Odd N - 1
   Even 2N + 1

   If N/7 is deleted, failure occurs for N = 34.

   (b) N/7 N/5
   Odd 3N + 1
   Even N/2

   If N/7 is deleted, failure occurs for N = 27.

5. N/11 N/5
   Odd 3N + 1
   Even N/2

   If N/11 is deleted, failure occurs for N = 27.

6. N/13 N/5
   Odd N - 1
   Even 2N + 1

   If N/13 is deleted, failure occurs for N = 34.

7. N/17 N/5 — If N/17 is deleted, failure occurs for N = 34.
   Odd N - 1
   Even 2N + 1

8. N/19 N/5 — If N/19 is deleted, failure occurs for N = 4.
   Odd (N - 1)/2
   Even 3N + 1

9. N/23 N/5 — If N/23 is deleted, failure occurs for N = 34.
   Odd N - 1
   Even 2N + 1

10. N/29 N/11 N/5 N/3 — If N/29 is deleted, failure occurs for N = 19.
    Odd 2N - 1 — If N/11 is deleted, failure occurs for N = 31.
    Even N + 1 — If N/5 is deleted, failure occurs for N = 67.
    If N/3 is deleted, failure occurs for N = 21.

In each step, the operations given in the first row are to be performed first, in any order. If these operations are not possible, the separate instructions for odd and even integers are to be used, followed by another attempt to apply the operations of the first row, and so on.

For each algorithm for which deletion of the first named operation causes a failure, a sample value of N for which this failure occurs is given. In order that the algorithm be valid for the first named operation, it must fail for the other first-row operation(s) taken separately.

Algorithms 1 and 2(a) have been considered separately in two previous papers by the author. After some attempts by P. Erdös (Member of the Hungarian Academy of Science), algorithm 1 has not been proven yet. Proofs by A. Meir (Professor of Mathematics, University of Alberta) and by the author have been found for 2(a). So far, algorithms 3(c) and 3(d) are the only other ones listed for which proofs have been completed. This proof by Meir is by induction involving arithmetic mod 15 and proceeds as follows:

We want to prove that the algorithm, namely, N/5 and/or N/3, if possible,
otherwise, if $N = \text{odd} \rightarrow N + 1$
if $N = \text{even} \rightarrow 2N + 1$,
reduces any given integer, $n \geq 2$, to 1.

Suppose that we have proved that this theorem is true for $1 < n < m$. We shall show, then, that it is true also for $n = m$. Since 15 is the lowest common multiple of 5 and 3, we shall use m mod 15. Then, either m is divisible by 5 and/or 3 or it is off by some integer 1 to 14, inclusive. Therefore, we distinguish 15 cases according to the remainder of m mod 15 and apply the algorithm to each of these cases as follows:

1. $m = 15k + 1$ <
   $k = \text{odd} \rightarrow 30k + 3 \rightarrow 10k + 1 < m.$
   $k = \text{even} \rightarrow 15k + 2 \rightarrow 30k + 5 \rightarrow 6k + 1 < m.$

2. $m = 15k + 2$ <
   $k = \text{odd} \rightarrow 15k + 3 \rightarrow 5k + 1 < m.$
   $k = \text{even} \rightarrow 30k + 5 \rightarrow 6k + 1 < m.$

3. $m = 15k + 3 \to 5k + 1 < m.$

4. $m = 15k + 4 \left< \begin{array}{l} k = \text{odd} \to 15k + 5 \to 3k + 1 < m. \\ k = \text{even} \to 30k + 9 \to 10k + 3 < m. \end{array} \right.$

5. $m = 15k + 5 \to 3k + 1 < m.$

6. $m = 15k + 6 \to 5k + 2 < m.$

7. $m = 15k + 7 \left< \begin{array}{l} k = \text{odd} \to 30k + 15 \to 6k + 3 < m. \\ k = \text{even} \to 15k + 8 \to 30k + 17 \to 30k + 18 \to 10k + 6 < m. \end{array} \right.$

8. $m = 15k + 8 \left< \begin{array}{l} k = \text{odd} \to 15k + 9 \to 5k + 3 < m. \\ k = \text{even} \to 30k + 17 \to 30k + 18 \to 10k + 6 < m. \end{array} \right.$

9. $m = 15k + 9 \to 5k + 3 < m.$

10. $m = 15k + 10 \to 3k + 2 < m.$

11. $m = 15k + 11 \left< \begin{array}{l} k = \text{odd} \to 30k + 23 \to 30k + 24 \to 10k + 8 < m. \\ k = \text{even} \to 15k + 12 \to 5k + 4 < m. \end{array} \right.$

12. $m = 15k + 12 \to 5k + 4 < m.$

13. $m = 15k + 13 \left< \begin{array}{l} k = \text{odd} \to 30k + 27 \to 10k + 9 < m. \\ k = \text{even} \to 15k + 14 \to 30k + 29 \to 30k + 30 \to 6k + 6 < m. \end{array} \right.$

14. $m = 15k + 14 \left< \begin{array}{l} k = \text{odd} \to 15k + 15 \to 3k + 3 < m. \\ k = \text{even} \to 30k + 29 \to 30k + 30 \to 6k + 6 < m. \end{array} \right.$

15. $m = 15k + 15 \to 3k + 3 < m.$

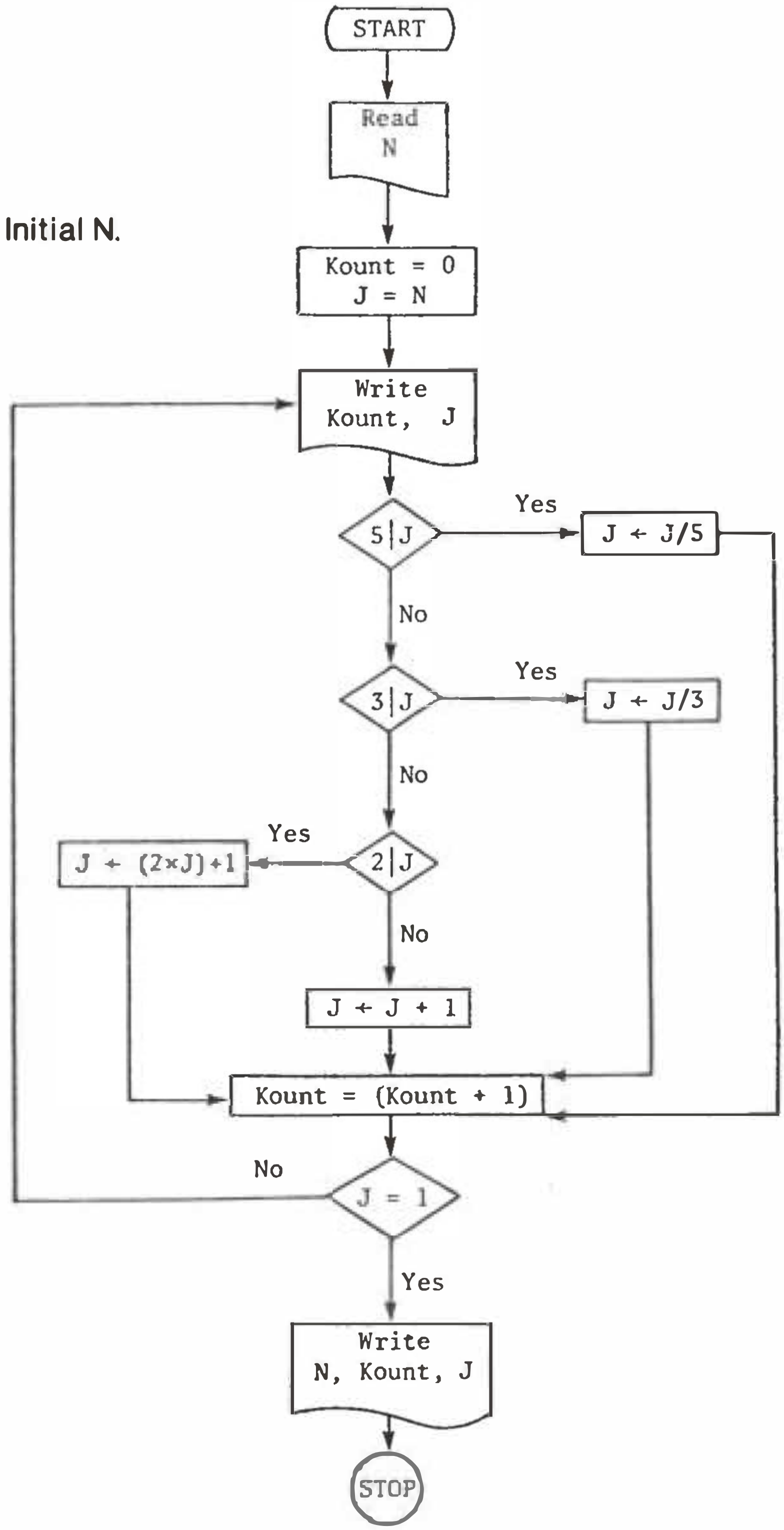
After a few steps in each case, the number obtained on the right-hand side is *smaller than the initial m*. Thus, if the theorem is true for $1 < n < m$, it is true also for $n = m$. But simple checking establishes that it is true for $n = 2$. Hence, by the principle of induction, the theorem is true for all integers $n \geq 2$.

For $N = 55$, the tabulation for this algorithm becomes:

Step Number	*Procedure*	*n*
0		55
1	n/5	11
2	n + 1	12
3	n/3	4
4	2n + 1	9
5	n/3	3
6	n/3	1

A simple form of a flow chart for algorithm 3(d) for any initial N, and based on the language of the Fortran program, might be as shown in Figure 1. With minor changes, the flow charts for the other algorithms are easily constructed. The Fortran programs are similar to those previously used by the author and require very little experience to write. All of the listed algorithms have been computer checked for $2 \leq N \leq 500$.

FIGURE 1
Flow Chart for Chain with Initial N.



For those who want to start with a very simple prime number reduction algorithm, E. Phibbs (Professor Emeritus of Mathematics, University of Alberta) suggests:

N/5
Odd N + 1
Even 2N + ½d, where d is the last digit of the previous number obtained.

It is easy to see why this one always works. Try it.

The author has not attempted to design reduction algorithms for prime numbers greater than 29.

References

Bruce, William J. "Crazy Roller Coasters." *The Mathematics Teacher*, Vol. 71, No. 1, January, 1978, pp.45-49.

________________. "An Instructive Algorithm Involving a Number Theoretic Problem." *Ontario Mathematics Gazette*, Vol. 17, No. 2, December, 1978, pp.44-50. (This article will be published in the February 1980 issue of *delta-k*.

Algebra Holiday Quiz

Here is a quiz for the Christmas holiday season. U R 2 solve for the value of (x) in each of the equations below. Place your answer in the space provided at the left. When you have solved all the equations, unscramble the letters to get the message.

______	$x + M = 2M$	______	$bx + be = 2be$
______	$3x - s = 2s$	______	$2x - 2s = 0$
______	$\frac{vt^2}{g} = \frac{vxt}{g}$	______	$5i + x = 6i$
______	$swx = swh$	______	$x - b = -b + C$
______	$pr = xp$	______	$5x - 2r = r + 2x$
______	$\frac{x}{a} = 1$	______	$m^2x = m^3$
______	$7 + ax = ay + 7$	______	$8x + mv = 8r + mv$

PLUS+++

The following material is reprinted from Issue No. 2 of Plus + + +, *a short magazine informing mathematics educators across Canada about important events, research, curriculum development and items of national interest.*

Canadian Mathematical Society Winter Meeting

(Ottawa, December 14-16, 1979)

Part of the program of the C.M.S. Winter Meeting will be devoted to matters of interest to secondary school teachers as well as university mathematicians. A keynote speaker will be Professor Peter Lancaster of the University of Calgary, who will discuss the role of C.M.S. in promoting the teaching of mathematics and problem-solving in the applied sense in secondary schools. Professor Ed Williams of Memorial University will chair a session of six 15-minute talks on contests and other resources. Professor Ed Barbeau of the University of Toronto will chair a panel, including a teacher and a professor, on the mathematician's role in assisting schools. All those events are planned for Saturday, December 15, 1979, in Ottawa. For further details, write to Professor D.A. Dawson, C.M.S. Winter Meeting, Department of Mathematics, Carleton University, Ottawa, Ontario K1S 5B6.

Samuel Beatty Mathematics Essay Contest

The Beatty Essay Contest is open to Canadian elementary and secondary school students. Sponsored by the Samuel Beatty Fund, established by University of Toronto graduates of mathematics and physics, the first contest was held in 1978-79 with a first prize of $200 going to Doug Bonn, a Grade XIII student of Northern C.I. and V.S. in Sarnia, Ontario. Topics and rules for the second contest are obtainable from E.J. Barbeau, Secretary, Samuel Beatty Fund, University College, B201, University of Toronto, Toronto, Ontario M5S 1A1. Deadline for submissions: March 31, 1979.

Waterloo University Contests

Entry deadlines: Gauss contest - May 1, 1980; Euclid contest - March 9, 1980. For reprints of previous Gauss, Euclid, and Junior contests, write to the secretary of the contest at the Faculty of Mathematics, University of Waterloo, Waterloo, Ontario N2L 3G1. Cost: Gauss - $1.25 for bundle of 10; Euclid, Junior - $1 for bundle of 10. Contests are administered through provincial chairman.

Mathematical Association of America Mathematics Examination

The sponsors of this contest are the M.A.A., Mu Alpha Theta, Society of Actuaries, N.C.T.M. and the Casualty Actuarial Society. For information about local arrangements, write to the executive director at the Mathematics Department, 917 Oldfather Hall, University of Nebraska, Lincoln, Nebraska 68588, U.S.A.

Fourth International Conference on Mathematical Education

(Berkeley, August, 1980)

ICME will be held at Berkeley, California, from August 10 to 16, 1980. The scientific program examines problems in education at all levels and for every type of learner. Topics of emphasis: universal primary education, research, technology, applications, profession of teaching, relationship of languages and mathematics. For further information, write to ICME IV, Mathematics Department, University of California, Berkeley, California 94720, U.S.A. Specify your language (English, Español, Française) and level of interest (elementary, secondary, college, teacher training, supervision, university).

Canadian Mathematics Education Study Group

In June, 1979, the C.M.E.S.G. met for the third time at Kingston, Ontario - to discuss preservice and inservice teacher education, mathematization, and research in hand. The group has become more formally organized with an advisory committee and a secretary, Professor Joel Hillel, Department of Mathematics, Concordia University, S.G.W., Montreal, Quebec H3G 1M8, to whom requests for a report on the proceedings should be sent.

Power Play! A Hockey Math Book

Power Play! A Hockey Math Book
by Don Fraser. Toronto: Collier Macmillan Canada Ltd.,
1125B Leslie Street, Don Mills, Ontario M3C 2K2. 122 pp.
Paperback 02991180X $2.75

There is a hoary joke about a real estate agent who tells a southern belle, "This is a house without a flaw," only to have the puzzled lady respond, "Mah, what do yewall walk on?" The story comes to mind because it would appear that Professor Don Fraser of the Faculty of Education at the University of Toronto has written a book "without a flaw."

Included in the book are 60 fascinating units all built around some aspect of professional hockey. The first 15 units (First Period) deal with the addition, subtraction, and multiplication of whole numbers. In the Second Period, the mathematical ideas considered in the 23 units include division, fractions, decimals, percent, averages and simple probability. The Third Period treats some elementary aspects of data presentation in 12 units, and in Overtime, 10 units of review are given. The 15-page Appendix includes Game Summaries (1978 01 05-18), and Team Rosters. Answers to exercises and an index constitute the last four pages of the book.

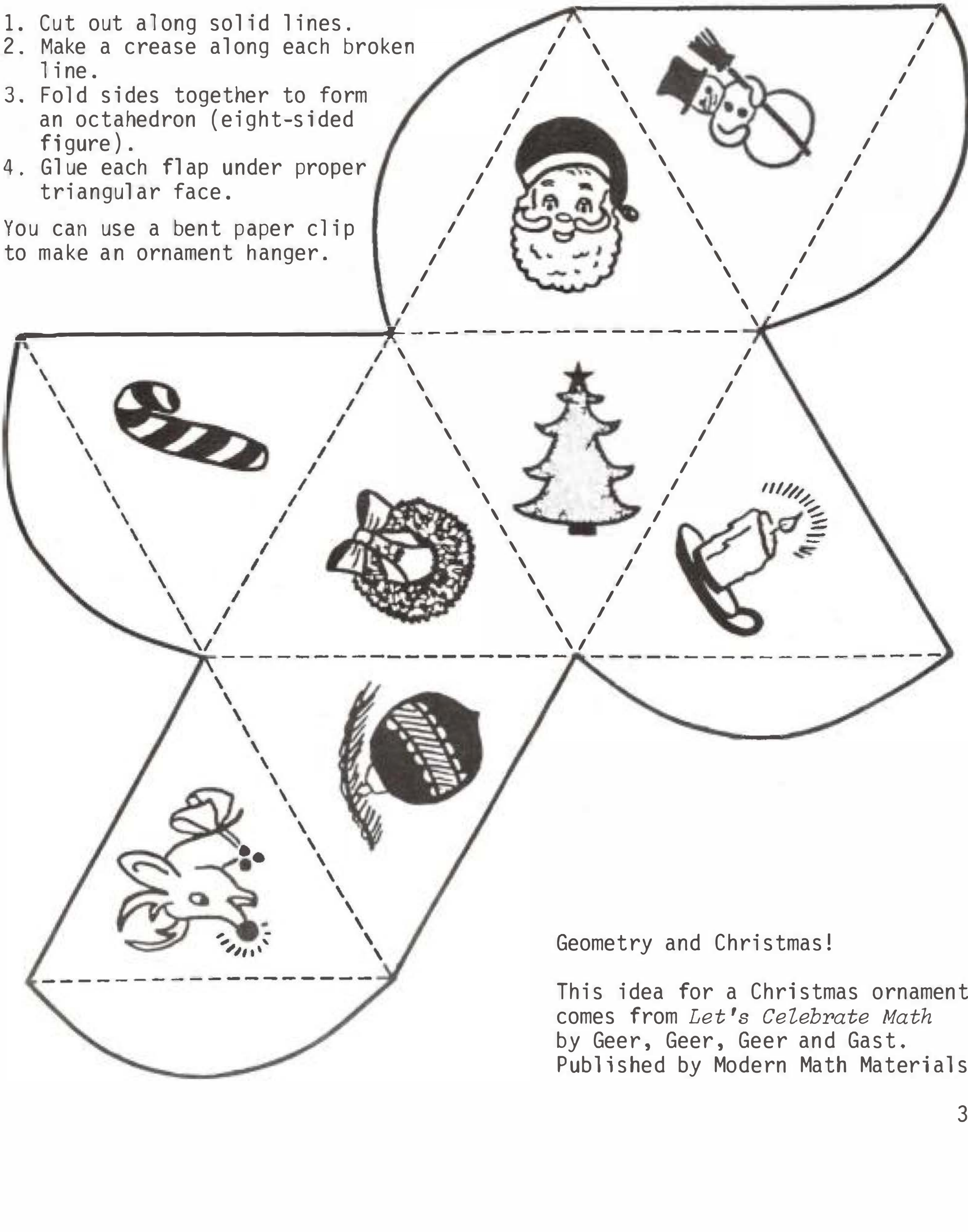
Reprinted from Ontario Mathematics Gazette, *Vol. 17, No. 2, December 1978.*

A Christmas Ornament

REPRODUCE THIS ORNAMENT ON STIFF PAPER.

1. Cut out along solid lines.
2. Make a crease along each broken line.
3. Fold sides together to form an octahedron (eight-sided figure).
4. Glue each flap under proper triangular face.

You can use a bent paper clip to make an ornament hanger.

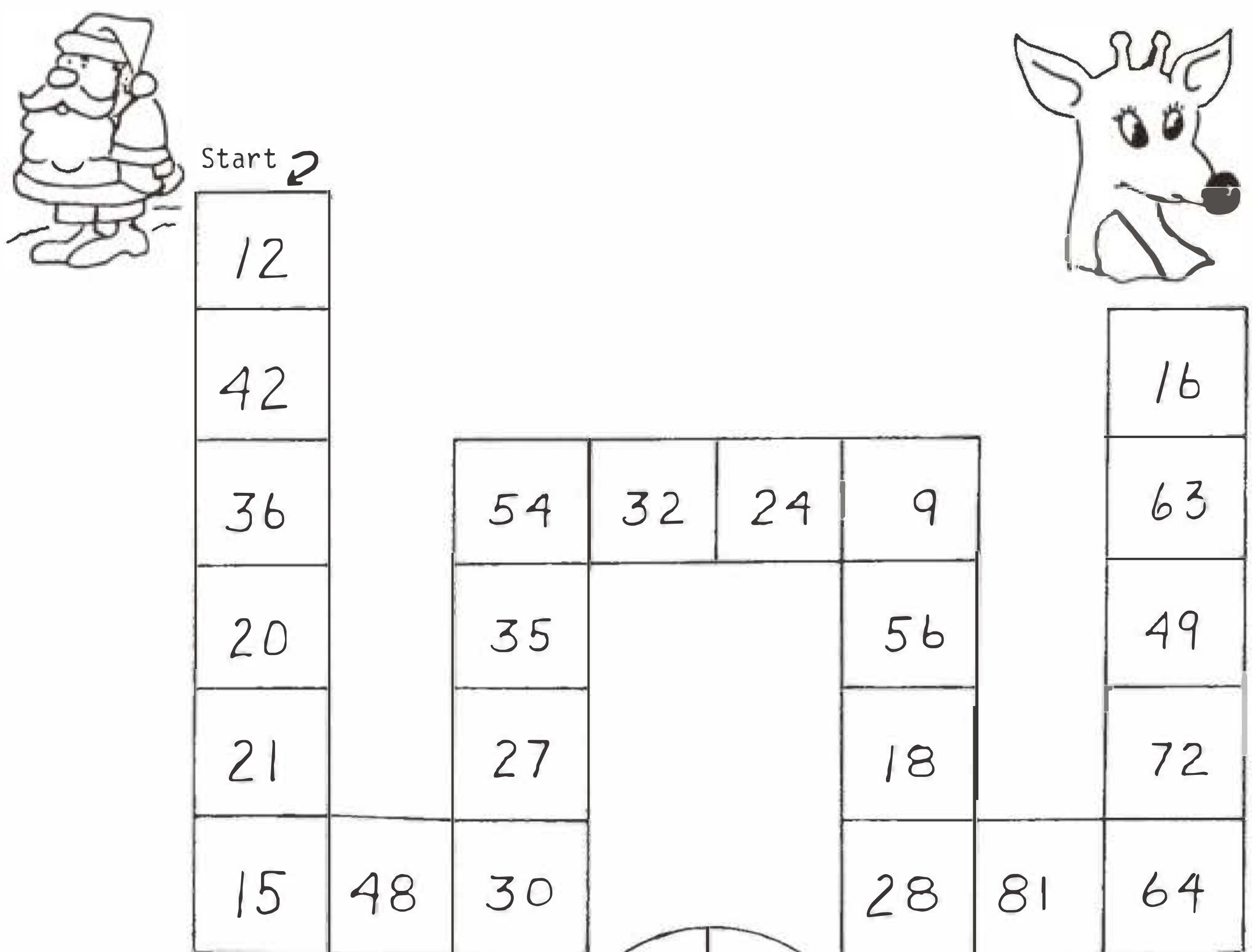


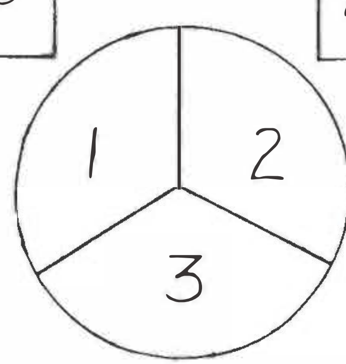
Geometry and Christmas!

This idea for a Christmas ornament comes from *Let's Celebrate Math* by Geer, Geer, Geer and Gast. Published by Modern Math Materials.

Help Santa Get to Rudolph

Using the point of a pencil and a paper clip to complete the spinner, thump the paper clip to see which player gets the larger number to go first. Each player in turn thumps the paper clip and moves the number of spaces shown on the spinner. Player then gives the factors of the number landed on. (One is not used as a factor.) If player misses, he must go back to his last stop. The first player to get to Rudolph is the winner.





The book *Holiday Math* by Latellya Smith, published by Educational Teaching Aids, is the source of this Christmas idea.

SEATTLE SPIN

(An activity sponsored by The NCTM-1980 Seattle Meeting)

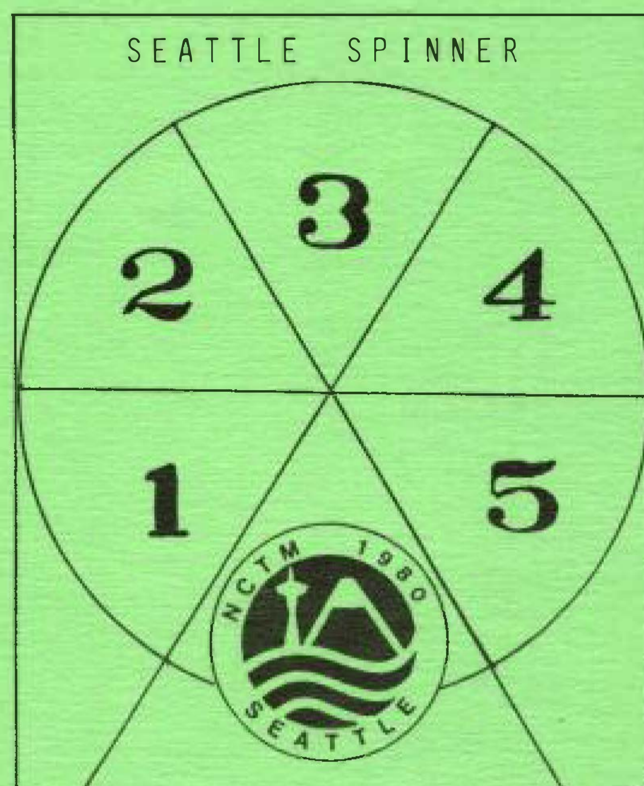
TOPIC: Simple addition and subtraction facts

MATERIALS:
1 Seattle Spinner
1 gameboard for each player
10 markers for each player

NUMBER OF PLAYERS: 2 to 5

HOW TO PLAY:
1. A player spins the Seattle Spinner twice and places a marker on the sum or difference of the 2 numbers.
2. If the sum and difference are already covered, the player loses that turn.
3. When "SEATTLE" is spun, the player may choose it to be any number 1 to 5. In other words: "Seattle is wild!!"

WHO WINS: The first player to cover all of his numbers is the winner.



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The National Council of Teachers of Mathematics (NCTM) unveils "The Curriculum of the 1980s" at the 58th Annual Meeting - April 16-19, 1980, The Seattle Center, Seattle, Washington.

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EPIE Analysts Needed

Alberta Education has need of individuals with valid Alberta teaching certificates who are interested in analyzing educational materials in the areas of science and mathematics at a Grade I-XII level. The individuals must undertake a three-day workshop in Edmonton to be presented at a date yet to be determined. This workshop will provide these individuals with the background knowledge necessary for an examination leading to their certification as analysts for EPIE (Educational Products Information Exchange).

Those educators who become certified will be eligible for contracted assignments at prevailing fees to analyze materials for Alberta Education. If you are interested in becoming a certified analyst, please register now, using the form below. Registration is limited to 30 participants.

EPIE REGISTRATION FORM

(Please print)

Name ______________________________ Home phone __________

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____

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Name of Superintendent ______________________________

District Office Address ______________________________ Phone no. __________

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Grade level presently teaching ______________________________

Subject area assignment ______________________________

Other grades recently taught ______________________________

Other subjects recently taught ______________________________

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