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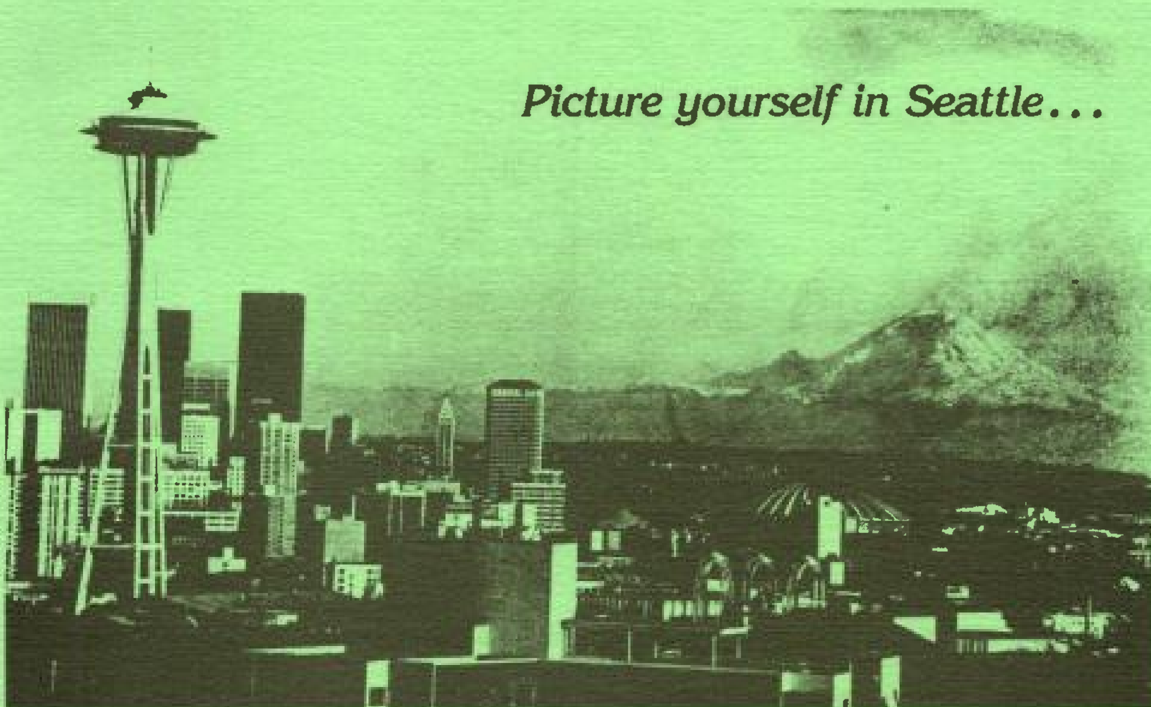
delta-k

THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

Volume XIX, Number 3

March 1980



The National Council of
Teachers of Mathematics
Unveils

The Mathematics Curriculum of the 1980's

CONTENT • METHODS • TECHNOLOGY

NCTM 1980 SEATTLE

58th ANNUAL MEETING
APRIL 16-19, 1980
The Seattle Center
Seattle, Washington

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Contents

EDPRESS

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Mathematics Council Executive 1979-80

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THE SEATTLE DIVIDE

(An activity sponsored by The NCTM-1980 Seattle Meeting)

TOPIC: Division.

MATERIALS: List of the Seattle Divide Numbers. (The leader may write these on the chalkboard or overhead projector.) Paper and pencil.

FORMAT: Leader, 4-40 players, teams of 6-8.

DIRECTIONS: The leader calls any number between 2 and 50 (inclusive). The first team member to find a Seattle Divide number that is divisible by the leader's number wins a point.

THE SEATTLE DIVIDE NUMBERS

564 1,548 874
296
1,056 837
12,600 820
1,326 1,421

EXAMPLES:

The leader calls 15.	(15 evenly divides 12,600)
The leader calls 23.	(23 evenly divides 874)
The leader calls 34.	(34 evenly divides 1,326)
The leader calls 7.	(7 evenly divides 12,600 and 1,421)

WHO WINS: The first team to score 5 points wins.

The National Council of Teachers of Mathematics (NCTM) unveils "The Curriculum of the 1980s" at the 58th Annual Meeting - April 16-19, 1980, The Seattle Center, Seattle, Washington.

From the Editor's Desk

Among the materials for classroom use included in the past three issues of *delta-k*, you will have found sections entitled "Activities" for secondary schools and "Ideas" for elementary students. These were used for two reasons: one, to assist you; and two, to point out one of the services available if you join the NCTM and subscribe to the appropriate magazine(s) for your area(s) of interest. For further information, contact Klaus Puhlmann, MCATA/NCTM representative, or any other executive member of MCATA.

You may have noticed we are emphasizing the NCTM Annual Meeting this year. Remember the success we had in Calgary? It was possible through our support of NCTM and their interest in us as a member group. To maintin this we must have interested individuals as members and supporters of NCTM. This year the Annual Meeting is close to home - in Seattle - which gives us an opportunity to show our support and learn what NCTM really means to us and how we can all benefit. Now can you see why "NCTM Seattle" is being promoted?

With this issue we are introducing a new section called the "Problem Corner." Its pages are open to all to express themselves, and its size will depend upon the number of articles contributed. We would challenge people in the elementary area to contribute items suitable for elementary teachers and their students. We encourage university personnel to contribute articles as has been done in this issue. Original ideas from the classroom would be welcome.

Is it worthwhile including guest editorials or position statements such as the articles on pages 3, 6, and 11 of the November 1979 issue of *delta-k*? I have had no feedback concerning these ideas. Does this mean agreement, indifference, or disagreement? How about expressing your support or opposition? In a "Letters to the Editor" column your editor will present your bias, no matter how diametrically opposed to his own. You have received material in the past which definitely was not in agreement with the opinions of either MCATA or the ATA, and we will continue to leave the expression of controversial ideas open. We will even withhold the name of the author where desired. However, this is your *delta-k* only insofar as you make it - and we have need for more interest or it is soon going to be only your editor's publication.

Ed Carriger

An Instructive Algorithm Involving a Number Theoretic Problem

by *William J. Bruce*
University of Alberta
Edmonton, Alberta

The following article is reprinted with permission from the Ontario Mathematics Gazette, *Volume 17, Number 2, December 1978.*

This activity-oriented problem is rich in mathematical experiences that are adaptable to students from elementary through high school. The emphasis is on simple arithmetical operations, flow charts, and graphical representation. Another such algorithm has been considered previously by the author.[1]

Consider any natural number $N > 1$. If N is divisible by 3, divide it by 3 and repeat the division by 3 as often as possible. When N is not divisible by 3 and is an odd number, subtract 1 from N and try division by 3 again. Repeat this division if possible. Should N be even and not divisible by 3, multiply N by 2 and add 1. Again try division by 3 and repeat if possible. Division by 3 is attempted at every step. Whenever N is even and division by 3 fails, multiply N by 2 and add 1, but if N is odd and division by 3 fails, subtract 1. The procedure continues until N is reduced to 1.

For $N = 14$ we tabulate as follows:

Step Number	Procedure	n
0		14
1	M2 + 1	29
2	S1	28
3	M2 + 1	57
4	D3	19
5	S1	18
6	D3	6
7	D3	2
8	M2 + 1	5
9	S1	4
10	M2 + 1	9
11	D3	3
12	D3	1

---

[1]William J. Bruce, "Crazy Roller Coasters." *The Mathematics Teacher,* Vol. 71, No. 1, January 1978, pp.45-49.

In this table, M2+1 means multiply by 2 and add 1, whereas, S1 means subtract 1, while D3 means divide by 3. The natural number n is the number obtained in each step when one of these operations has been performed.

For n = 57 the tabulation becomes:

Step Number	Procedure	n
0		57
1	D3	19
2	S1	18
3	D3	6
4	D3	2
5	M2 + 1	5
6	S1	4
7	M2 + 1	9
8	D3	3
9	D3	1

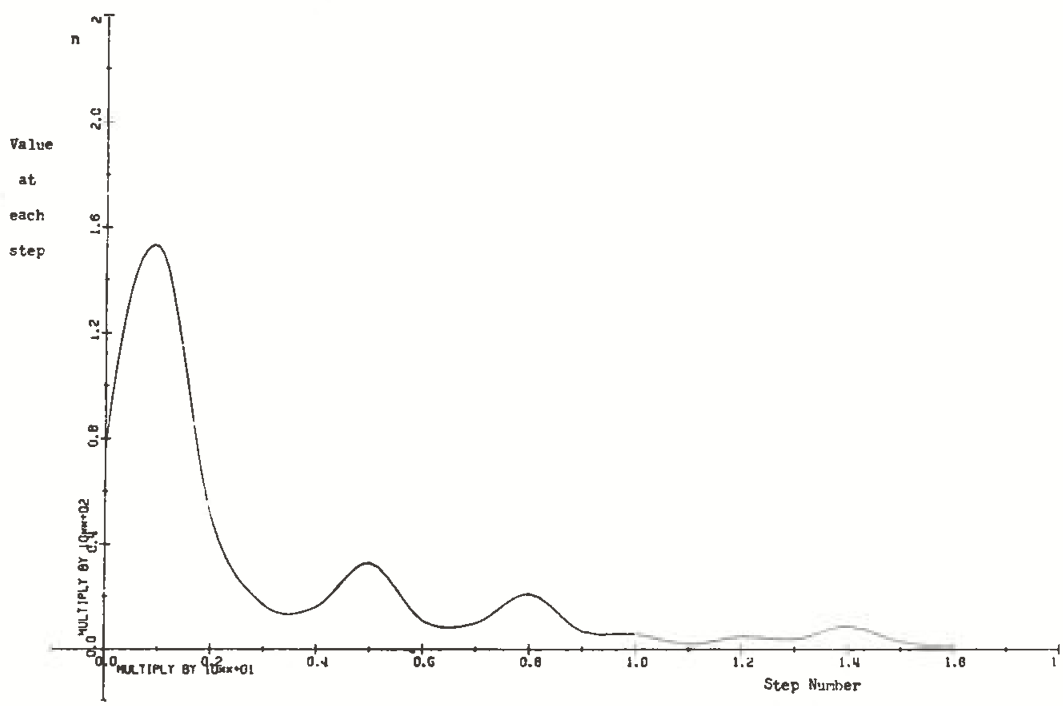
It will be shown later in this paper that for all natural numbers $N > 1$ the final number in the chain is always 1. The following table gives the number of steps required to complete the chains for natural numbers $2 \leq N \leq 101$. Clearly the number of steps do not tend to increase very rapidly. In this table, the largest number of steps is 24 and this occurs for $N = 80$ and $N = 86$. Even for numbers up to 500 the greatest number of steps is 31 and this occurs for $N = 494$. Incidentally, $N = 194$ requires 30 steps.

N	Number of Steps	N	Number of Steps	N	Number of Steps	N	Number of Steps
2	5	27	3	52	13	77	17
3	1	28	10	53	14	78	16
4	3	29	11	54	8	79	17
5	4	30	10	55	9	80	24
6	6	31	11	56	21	81	4
7	7	32	18	57	9	82	11
8	14	33	11	58	8	83	12
9	2	34	10	59	9	84	11
10	9	35	11	60	8	85	12
11	10	36	5	61	9	86	24
12	4	37	6	62	16	87	12
13	5	38	18	63	9	88	11
14	12	39	6	64	16	89	12
15	5	40	5	65	17	90	11
16	12	41	6	66	8	91	12
17	13	42	13	67	9	92	11
18	7	43	14	68	16	93	12
19	8	44	13	69	9	94	11
20	7	45	6	70	16	95	12
21	8	46	13	71	17	96	19
22	7	47	14	72	16	97	20
23	8	48	13	73	17	98	19
24	15	49	14	74	16	99	12
25	16	50	13	75	17	100	11
26	15	51	14	76	16	101	12

New chains can be generated by reference to previously determined chains without doing every step because, eventually, a number less than N is reached. An interesting phenomenon frequently occurs for certain groups of adjacent numbers. For example, for $87 \leq N \leq 95$ the numbers of steps are, respectively, 12, 11, 12, 11, 12, 11, 12, 11, 12. For $334 \leq N \leq 350$ the numbers of steps are 20, 21, 20, ..., 21, 20. Also, by arguing backward, it is easily shown that, whenever three or more steps are needed, the last three values of n are always 9, 3, 1.

Graphical experience can be provided along with each chain completed by having the values at each step plotted against the step numbers to obtain a set of points. If desired, these points can be connected by line segments or by a smooth curve. The result, in the latter case, tends to remind one of a roller coaster run. Some of these are quite interesting and can, again, be associated with the term crazy roller coasters.[2] Since every graph will be different, pupils can produce their personalized roller coaster run. Figure 1 illustrates the plot for the chain for which N = 76. A cubic curve has been fitted to the set of points to produce a smooth path. This chain requires only 16 steps so is very quickly completed. Some attention to scale units will be necessary in that smaller scale units on the vertical scales than on the horizontal scales often will be advisable. In this example, the largest value of n is 153 and this occurs in step 1.

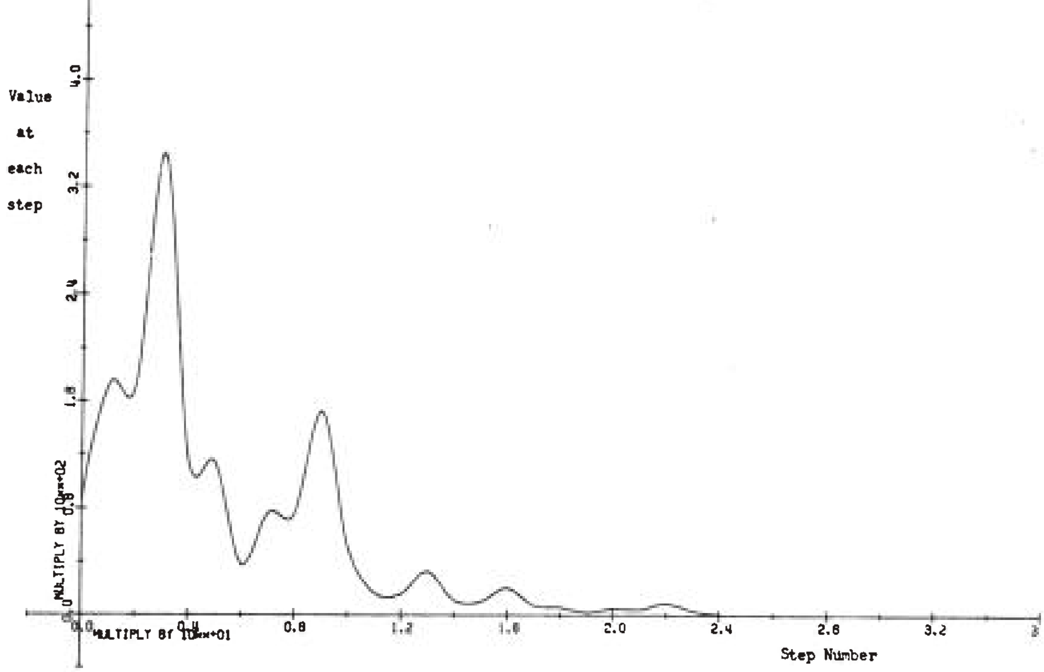
Fig. 1. Plot for N = 76



Consider now the chain associated with N = 86. Only 24 steps are needed to find the values of n. The plot, in this case, with a smoothly fitted cubic curve is shown in Figure 2. The graph reaches its highest point quickly in step 3 and attains a value n = 345 at this time, after which it drops quickly for awhile, but in step 9 it peaks again at n = 153 before dropping quickly again, and then slowly the rest of the way.

[2]Ibid.

Fig. 2. Plot for N = 86



The chain for N = 494, which has only 31 steps, builds up very quickly until it peaks in step 3 at a value of n = 1977. Its next greatest value is n = 1317 at step 6. Certainly this run should provide quite a thrilling ride. Power assist will be necessary only to step 3 and from there on free descent will be rapid and exciting. Note, in Figure 3, that the vertical scale probably should be given so as to require 100 as a multiplier. Such an interesting example could be used as a pin-up model.

Fig. 3. Plot for N = 494

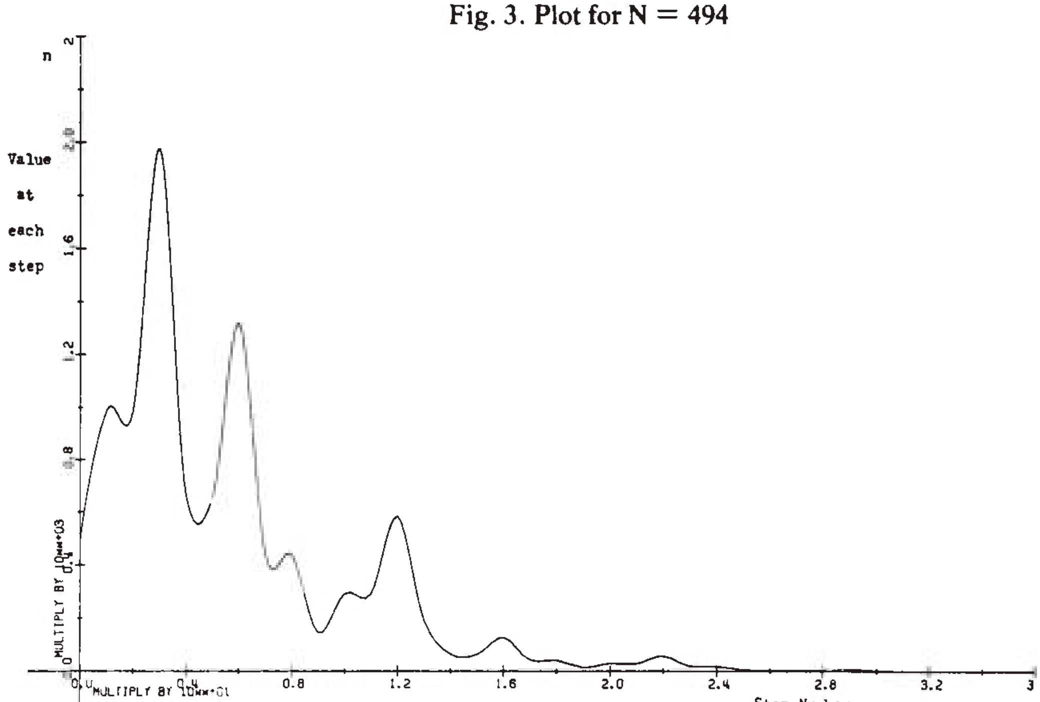


Fig. 3. Plot for N = 494

Since the algorithm used to generate the chains is very simple, the concept of a flow chart can be introduced again, as recommended previously.[3] In this case, the flow chart is only slightly more difficult to construct than before. Certainly, one based entirely on the computational procedure, as outlined in the first paragraph, should be attempted at first. Then one based on the language of the Fortran program could be tried. Figure 4 is one suggested format for the latter.

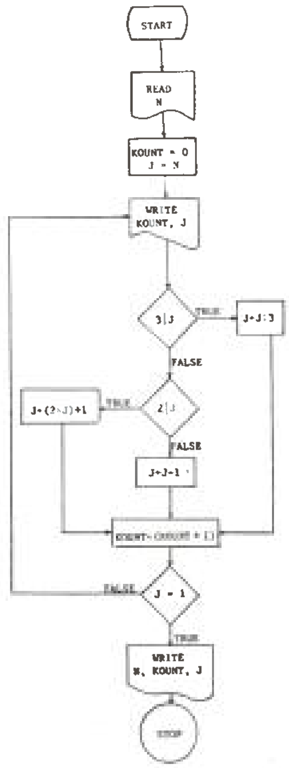


Fig. 4. Flow chart for a chain with initial N

```
*
*
*list c
>        100   FORMAT(I4)
>              READ(5,100)N
>       C
>              J=N
>       C      THE NUMBER TO BE TESTED IS N.
>       C      IT WILL TAKE ON VALUES NAMED J
>       C      AS IT IS REDUCED TO 1.
>       C
>              KOUNT=0
>       C      WE WILL COUNT THE NUMBER OF STEPS
>       C      IT TAKES TO REDUCE J TO 1.
>       C
>         66   WRITE(6,200)KOUNT,J
>        200   FORMAT(1X,2I5)
>       C
>              IF(J.EQ.3*(J/3))GO TO 3
>       C      WE TEST TO SEE IF J IS
>       C      EVENLY DIVISIBLE BY 3.
>       C
>              IF(J.EQ.2*(J/2))GO TO 4
>       C      WE TEST TO SEE IF J IS EVEN.
>       C
>              J=J-1
>              GO TO 5
>       C
>          3   J=J/3
>              GO TO 5
>       C
>          4   J=1+2*J
>       C
>          5   KOUNT=KOUNT+1
>       C      WE INCREMENT THE COUNTER
>       C      EVERY TIME AN OPERATION
>       C      IS PERFORMED ON J.
>       C
>              IF(J.EQ.1) GO TO 16
>              GO TO 66
>       C
>       16     WRITE(6,200) KOUNT,J
>              WRITE(6,300) N,KOUNT
>        300   FORMAT(1X,/,'THE NUMBER',I4,
>             *' WAS REDUCED TO 1 IN',
>             *I4,' STEPS.',//)
>       C
>              STOP
>              END
*END OF FILE
*
```

Fig. 5. Annotated Fortran program.

For those who want to generate chains for selected large values of N, Figure 5 provides an annotated Fortran program to facilitate application of the algorithm.

That the algorithm is always true for any natural number $N > 1$ can be shown easily by mathematical induction.[4] It is clearly true for $N = 2$. We assume the algorithm true for $N \leq m - 1$ and show that it is then true for $N = m$. It suffices to consider m even, since if m were odd, and whether m is divisible by 3 or not, the algorithm will reduce it to a number less than m in one step.

Since m is either divisible by 3 or off by 1 or 2, three cases need by considered, namely, (i) $m = 3p$, (ii) $m = 3p + 1$, and (iii) $m = 3p + 2$. In case (iii) there are three sub-cases since one must consider whether p itself is divisible by 3 or is off by 1 or 2. If we let $p = 3k$, $p = 3k + 1$, and $p = 3k + 2$, we get the three sub-cases (a) $m = 9k + 2$, (b) $m = 9k + 5$, and (c) $m = 9k + 8$. Apply the algorithm to all of the cases as follows:

(i) Procedure	n	(ii) Procedure	n
	$3p$		$3p + 1$
D3	$p < m$.	M2 + 1	$6p + 3$
		D3	$2p + 1 < m$.

(iii)

[3]Ibid.

[4]In consultation with A. Meir, Professor of Mathematics, University of Alberta.

(a)		(b)		(c)	
Procedure	n	Procedure	n	Procedure	n
	9k + 2		9k + 5		9k + 8
M2 + 1	18k + 5	M2 + 1	18k + 11	M2 + 1	18k + 17
S1	18k + 4	S1	18k + 10	S1	18k + 16
M2 + 1	36k + 9	M2 + 1	36k + 21	M2 + 1	36k + 33
D3	12k + 3	D3	12k + 7	D3	12k + 11
D3	4k + 1 < m.	S1	12k + 6	S1	12k + 10
		D3	4k + 2 < m.	M2 + 1	24k + 21
				D3	8k + 7 < m.

The requirements of induction have been satisfied. Hence the algorithm is true for any natural number $N > 1$.

Another proof that requires little more than a knowledge of elementary arithmetic is possible. This proof considers all of the natural numbers as being written in base 3 or in the equivalent expanded forms. For example,

91 (base 10) = $3^4 + 3^2 + 1$ = 10101 (base 3)

19 (base 10) = $2(3^2) + 1$ = 201 (base 3)

16 (base 10) = $3^2 + 2(3) + 1$ = 121 (base 3).

In base 3 the last two digits of any natural number $N > 1$ can be none other than one of the following:

00	10	20
01	11	21
02	12	22

We call the starting number N and the number at each step is called n. For numbers that end in 00, 10, and 20 division by 3 is immediate and we get $n < N$. The other endings need to be examined further. Since every odd number can be converted to an even number by subtraction of 1, as provided for in the algorithm, we need consider only even numbers. Thus the remaining six endings are taken as even at the outset and these give rise to six cases. The algorithm is applied in the following table to these six cases. Note that cases 5 and 6 in step 4 lead to three different routes in each case. Eventually, in all of these cases, the starting number N is reduced so that $n < N$. Thus we have shown that reduction occurs in all possible cases and since all possible endings have been included, the algorithm eventually will reduce all natural numbers $N > 1$ to unity.

(1) Procedure	Endings for n base 3		(2) Procedure	Endings for n base 3	
	01	Even		11	Even
M2 + 1	10	Odd	M2 + 1	00	Odd
D3	01, 11, 21	n < N	D9	00	n < N

(3)

Procedure	Endings for n base 3	
	21	Even
M2 + 1	20	Odd
D3	02, 12, 22	$n < N$

(4)

Procedure	Endings for n base 3	
	02	Even
M2 + 1	12	Odd
S1	11	Even
M2 + 1	00	Odd
D9	00	$n < N$

(5)

	12	Even
M2 + 1	02	Odd
S1	01	Even
M2 + 1	10	Odd
D3	(a) 01, (b) 11, (c) 21	

(a)

	01	Odd
S1	00	Even
D9	00	$n < N$

(b)

	11	Odd
S1	10	Even
D3	01, 11, 21	$n < N$

(c)

	21	Odd
S1	20	Even
D3	02, 12, 22	$n < N$

(6)

	22	Even
M2 + 1	22	Odd
S1	21	Even
M2 + 1	20	Odd
D3	(a) 02, (b) 12, (c) 22	

(a)

	02	Odd
S1	01	Even
M2 + 1	10	Odd
D3	01, 11, 21	$n < N$

(b)

	12	Odd
S1	11	Even
M2 + 1	00	Odd
D9	00	$n < N$

(c)

	22	Odd
S1	21	Even
M2 + 1	20	Odd
D3	02, 12, 22	$n < N$

The Four What's of Mathematics

by *V. P. Madan*
Red Deer College, Red Deer, Alberta

1. What is mathematics?
2. What do mathematicians do?
3. What kind of individuals become mathematicians?
4. What is the nature of mathematics?

What is mathematics? This is indeed hard to describe, due to the complex nature of the various disciplines and the role different professionals currently play. It has become a kind of international activity.

Therefore, we first discuss: What do mathematicians do?

Looking at the *American Mathematical Reviews*, a publication of the American Mathematical Society which publishes review articles on numerous papers and books handled by world mathematicians, one notes that there are at least 100 subclassifications of mathematics. To name a few, we have:

> history, logic, set theory, combinatorics, graph theory, number theory, algebra, geometry, topology, calculus, probability theory, statistics, numerical analysis, computer science, elasticity, plasticity, fluid mechanics, acoustics, optics, electromagnetic theory, thermodynamics, quantum mechanics, relativity, astronomy, astrophysics, geophysics, operations research, systems, control, information and communication, automata.

The list is almost endless, as this is a vast field which seems to grow and grow!

Mathematical reviews seem to be very useful for gleaning mathematical ideas. They are presently being published in English, French, German and Italian. Reviews are published for papers or books in almost any language in which mathematical ideas are transmitted, and since they contain only a special limited vocabulary, some familiarity with the subject enables mathematicians to read most papers or at least make a good guess at their contents.

International Mathematics Conferences have been very successful due to the fact that mathematicians from all parts of the world can understand each other. These conferences, held every four years, provide willing mathematicians with ample opportunities to make personal contacts and exchange viewpoints. The attendance at such conferences is quite high - usually well over 3,000. Two prizes of great value are awarded at each conference for the best work done by a mathematician under age 40, the award recipient being chosen by a jury of internationally famous mathematicians. The human aspect of such a large gathering of mathematical specialists is one of the great attractions.[1]

What kind of individuals become mathematicians? It is said that people turn to mathematics when they are unable to cope with the problems of the world! This is not always the case. Mathematicians are all sorts of people. There are mathematicians who are genial giants, mathematicians who are rotund Rotarians, mathematicians who adore Bach, mathematicians who play jazz and so on. There are also mathematicians who would never be suspected of being mathematicians!

[1]The next International Mathematics Conference happens to be in North America. It will be held in Berkeley, California, on August 10-16, 1980.

The one faculty common to all mathematicians is a capacity for the *abstract* thought.

Therefore, to answer the question "What is mathematics?", perhaps one can ask, "What is it not?" As for the nature of mathematics, Professor Alfred J. Ayer, in *Language, Truth and Logic*,[2] contends that an infinitely intelligent person would find mathematics dull, since he would be able to see at a glance all the possible consequences of any set of axioms! This is a meaningless statement, since we cannot devise a test to see whether any person is infinitely intelligent. But If we give meaning to the statement, we assume that mathematical development is exclusively occupied with the logical deductions from the given axioms or theorems.

Poincaré writes in his *Science and Method*[3]:

> ... That the mathematical discovery does not consist in making new combinations with entities that are already known. This indeed can be done by anyone in infinitely many ways. Discovery consists not in constructing useless combinations, but constructing those that are useful - which are an infinitely small minority.
>
> ... Mathematical facts worthy of being studied are those which, by their analogy with other facts, are capable of leading us to the knowledge of a mathematical law, in the same way as the experimental facts lead us to the knowledge of a physical law.

Mathematics may be summed up as a vast adventure in ideas. It is a collection of thoughts of several generations. It has been influenced by agriculture, commerce, engineering, philosophy and many other disciplines. The mathematics of the present century has so many aspects that it is impossible to do justice with even main trends. We suggest the following books on the history of mathematics to serve as a supplement for further exploration of the subject:

Bell, E.T. *Men of Mathematics*, Second Ed., Pelican Books. New York: Simon & Schuster, 1953.

Jacobs, Harold R. *Mathematics - A Human Endeavor (A Book for Those Who Think They Don't Like the Subject)*. San Francisco: Freeman and Co., 1970.

Kline, M. *Mathematics in Western Culture*. New York: Oxford University Press, 1953.

Kline, Morris. *Mathematics: An Introduction to Its Spirit and Use.* San Francisco: Freeman and Company, 1978.

Miller, G.A. "A First Lesson in the History of Mathematics," "A Second Lesson...," et cetera - a series of 10 articles in *National Mathematics Magazine*, Vols. 13-19 (1935-45).

Sarton, G. *The Study of the History of Mathematics*. London: Cambridge Dover, (reprint) 1957.

Sedgwick, W.T. and H.W. Tyler. *A Short History of Science*, Third Ed. New York: Macmillan Co., 1948.

Smith, D.E. *A Source Book in Mathematics*. London: Cambridge University Press, 1929, and New York: Dover Publications Inc. (reprint), 2 volumes, 1959.

Struick, Dirk J. *A Concise History of Mathematics*, Third Ed. New York: Dover Publications, Inc., 1967.

Acknowledgement:
The author would like to thank the Dover Publications, Inc., for granting permission to use the reference material of *The Gentle Art of Mathematics*, by Dan Pedöe.

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[2]Alfred J. Ayer, *Language, Truth and Logic* (New York: Dover Publications, Inc., 1947), p.85.

[3]Poincaré, *Science and Method* (New York: Science Press).

Pentominoes

by *Emil Dukovac*
Head of Mathematics
Kapuskasing District High School
Kapuskasing Board of Education
Kapuskasing, Ontario

If five squares are joined in all possible ways, the 12 shapes that result are called pentominoes.

Materials:

1. floor tiles
2. gummed squares
3. cubes
4. construction paper
5. scissors
6. squared paper
7. overhead
8. geoboard
9. mira

Exploratory Period:

- Use items 1, 2, or 3 and arrange them to make the 12 pentomino shapes.
- Once the pupils have discovered all the shapes, have them build these shapes on graph paper.
- You can then hand out construction paper with the images printed on them. Have the students cut these out.
- Now let's get familiar with the 12 shapes. Can you make some of these pieces fit these geometric shapes?

Game: Pent Up[1]

"Pent Up" is a strategy game for two or three players. It uses the 12 pentominoes and the 8-by-8 playing field.

Rules for Pent Up:

1. Place the set of 12 pentominoes in a pile near the players.
2. Players take turns. In his turn, a player chooses a pentomino from the draw pile and places it so that it covers five squares on the playing field.
3. Any pentomino may be placed either side up.
4. The last player who can play a pentomino is the winner.

Pentominoes and Perimeter

Let's make a *conjecture*.

Since the area of the 12 shapes is the same, it follows that their respective perimeter is the same. Discovery actively follows to prove or disprove the conjecture.

Other Activities:

- Place two pieces together; trace the figure. Do this for a number of examples (six). Compare the perimeters.
- Place three pieces together; trace the figure. Do this for six examples. Compare the perimeters.
- What conclusion can you draw?

[1]Dr. John L. Ginther, *Math Experiments with Pentominoes* (Midwest Publications Company, Incorporated), p.LR-9. Available through Western Educational Activities Ltd., 10234 - 103 Street, Edmonton, Alberta.

Do Pentominoes Tesselate?

Discovery Activity:

Assign each student to work with a certain pentomino shape. Attempt to use that shape to tesselate the plane. See if you can make it tesselate in more than one way.

Follow Up:

- Color them; decorate the classroom with them.
- Work with different geometric shapes to tesselate the plane. Are there certain shapes that do not tesselate?
- Devise other activities.

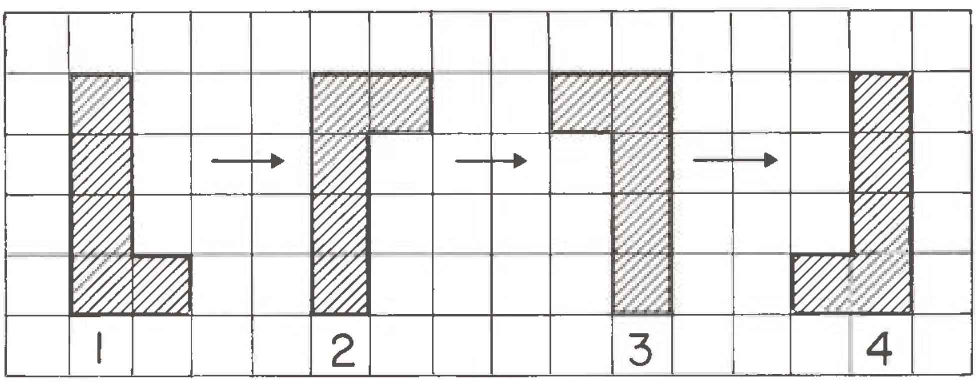
Build an Open Box

- Have the students tape the edges of the five pieces.
- Will the figure fold to form an open box?
- Rearrange the five tiles in the other 11 ways and repeat.
- Will all the 12 pentomino shapes form an open box?
- Devise other activities.

Flips, Slides, and Turns with Pentominoes

Describe the motion as a flip, slide, or turn.

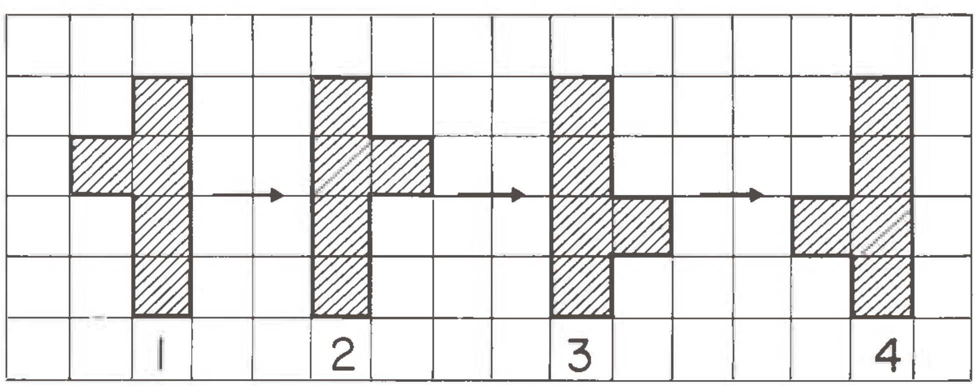
1.



$1 \rightarrow 2$ $\quad$ $1 \rightarrow 4$

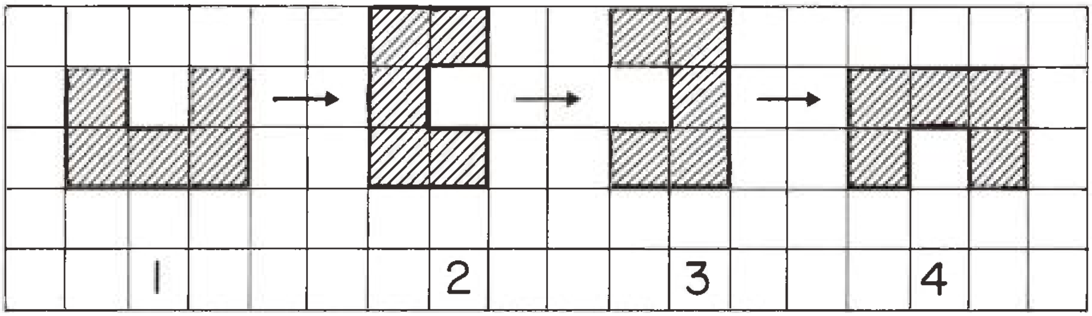
$1 \xrightarrow{flip} 2$ $\quad$ $2 \rightarrow 3$ $\quad$ $\equiv$ $\quad$ $1 \xrightarrow{R_{180}} 3$

2.



From the above four figures, show 4 flips and 2 rotations.
On square paper using this shape, demonstrate a slide, and two flips = R_{180}.

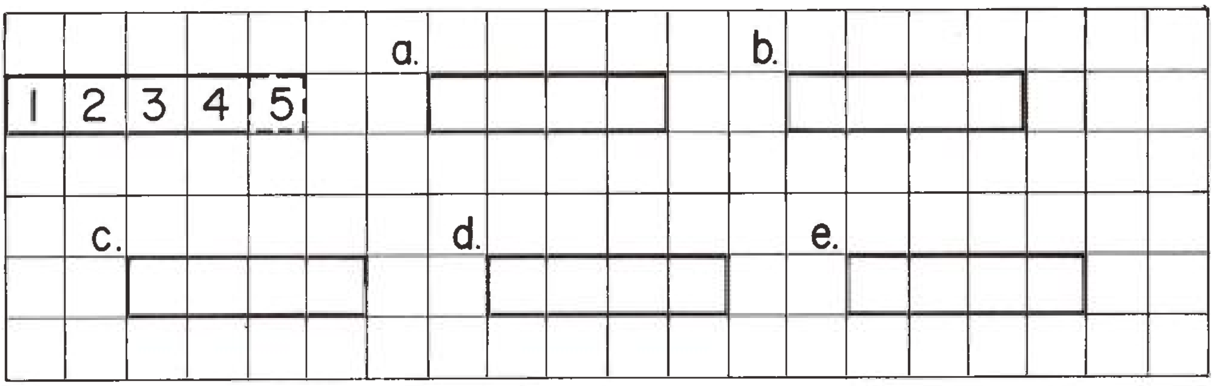
3.



From the above diagrams, describe the following motions:

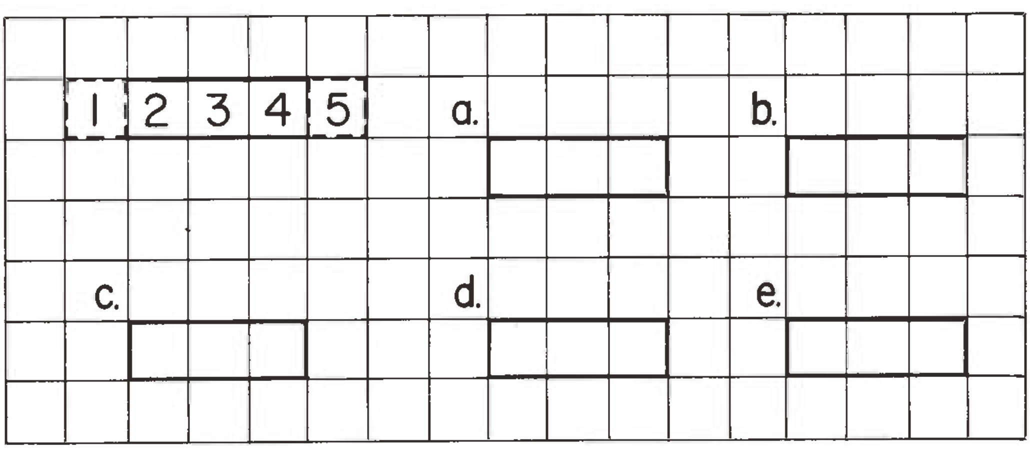
1 → 2   1 → 3   1 → 4   2 → 3   2 → 4   3 → 4

4.



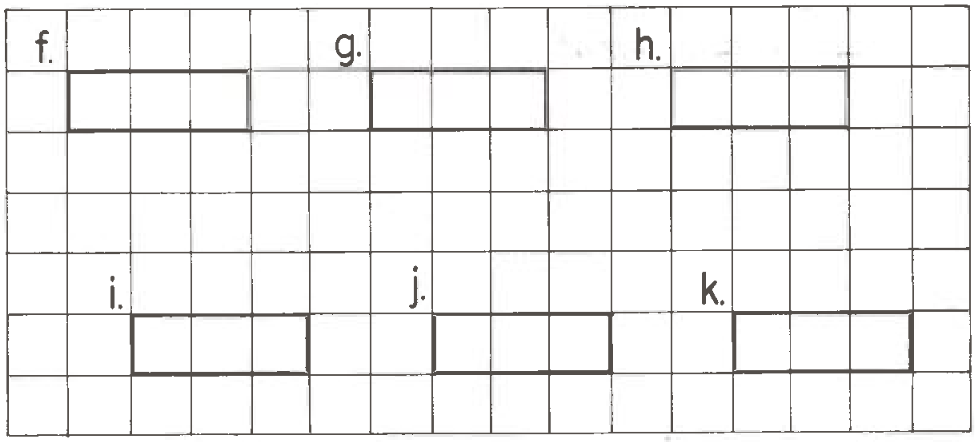
Place square #5 in a different position in figures a, b, c, d, e.
Compare these figures.

5.



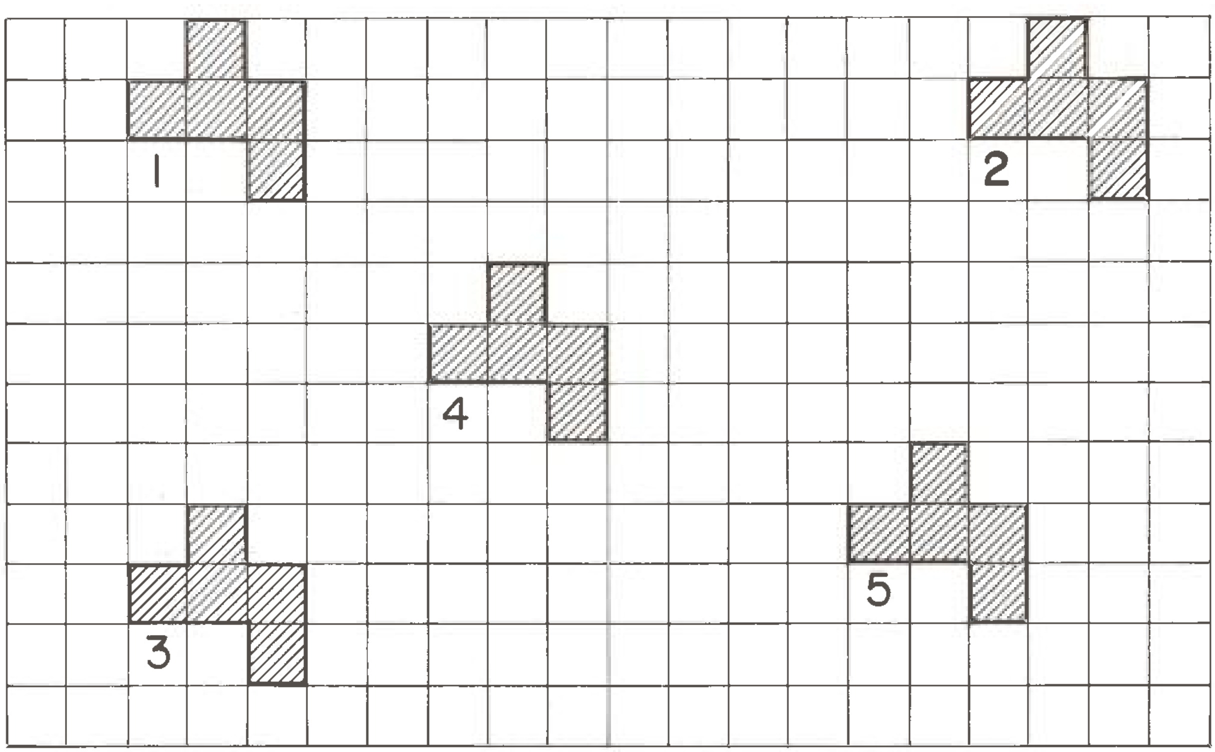
How many shapes can you build by sliding #1 and #5?

5.



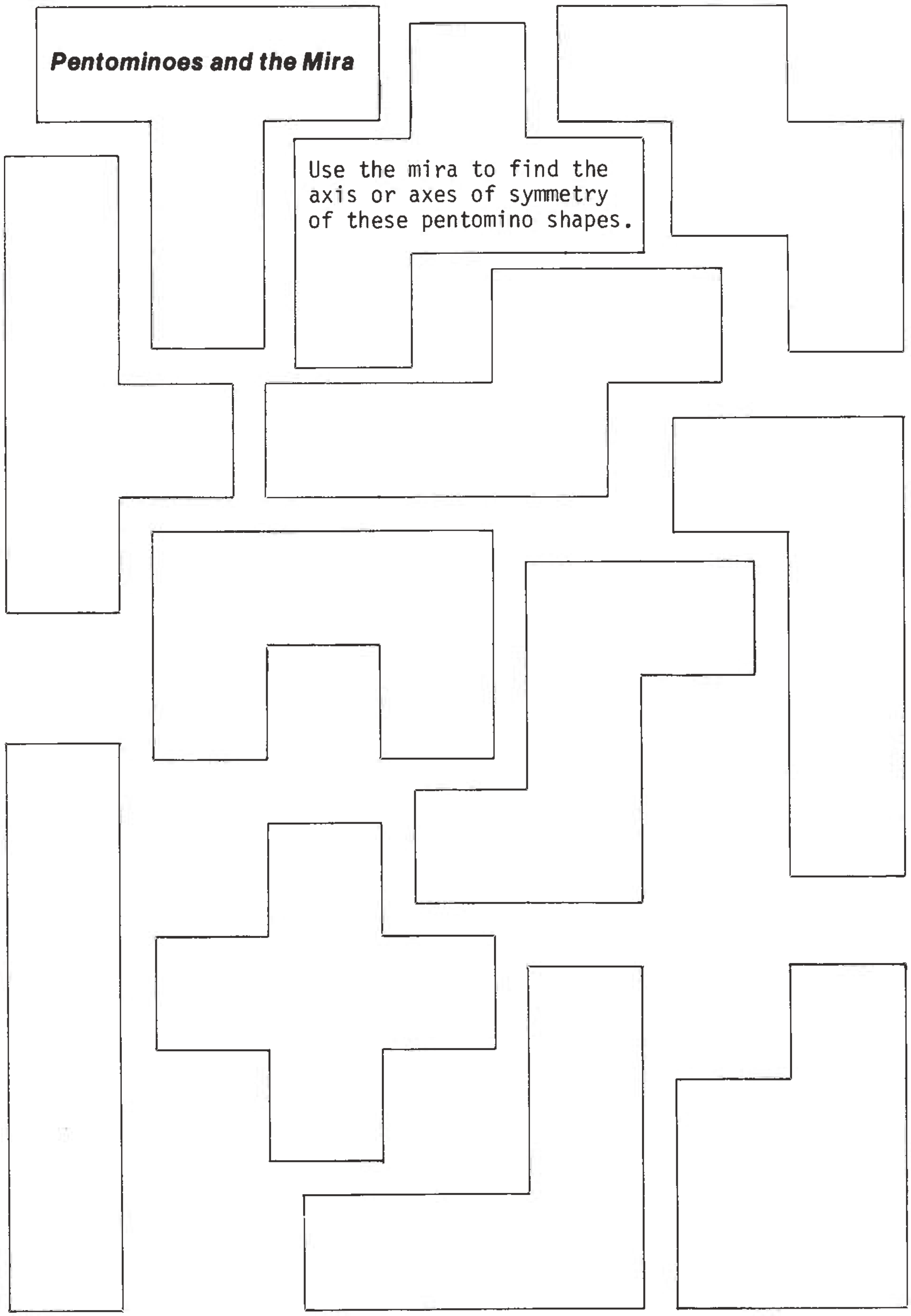
6. Describe the following slides: $1 \xrightarrow{\overset{(14)}{0}} 2$ 1 → 3 1 → 5

4 → 2 4 → 3 4 → 5 3 → 2 5 → 3



Pentominoes and the Mira

Use the mira to find the axis or axes of symmetry of these pentomino shapes.



Arithmetic and Fingers

from *Minnesota Council of Teachers of Mathematics Newsletter*

Reprinted with permission from Colorado Mathematics Teacher, *Vol. 12, No. 2, December 1979.*

A task force consisting of Ev Schendel, C. Eileen Oslund, Marge Lucarelli, Casey Humphreys and Lous Cohen has prepared a report on CHISANBOP/FINGERMATH for the MCTM Board of Directors. The complete report will be available to members at the fall meeting. The essence is included in the conclusions and recommendation below.

1. One of the Ten Basic Skill Areas identified by the National Council of Supervisors of Mathematics is Appropriate Computational Skills. Two essentials listed are knowledge of single digit number facts and mental arithmetic. There is no apparent attempt to teach either when Chisanbop is used.

2. Chisanbop is not really a new process - finger reckoning has been around for a long time. Serious attempts to use some form of finger reckoning were made by such men as Trachtenburg, Fibonacci, and others. Each had some specific base to work from as does Chisanbop that uses the adaptations of the Korean Abacus.

3. Success in speed and accuracy is based on psycho-motor and eye-hand dexterity. These are different in all children.

4. There seems to be no apparent correlation between the learning of Chisanbop as a young student and using it in mathematics of the real world in later life.

5. There has been unwarranted publicity - the type that appeals to parents saying in essence, "When all else fails, try this." Many forms of media have delivered this message.

6. The teaching of Chisanbop does not make association with the concrete - a one-to-one correspondence. It does, however, make association with counting to a certain extent.

7. Chisanbop teaches the four operations in different order than the mathematics programs generally do. the proponents of Chisanbop say the best order to use is addition, multiplication, subtraction and division and that is the way their material is presented.

8. It has a fun appeal to all ages and is relatively easy to learn by the teacher as well as the student - almost anyone can use it.

9. Cost for equipment is definitely minimal - the greatest expense seems to be the 25-hour workshop sponsored by Chisanbop Inc. The cost is $130 per teacher if 20 teachers register. There are simple workbooks and teacher's manuals for fingermath available at an average cost from the publisher. The publisher will conduct free workshops for teachers.

10. Society is looking at this finger reckoning method and so should we as mathematics educators.

11. Attitude from Chisanbop promoters from even a year ago has changed. On one hand, some feel that a company-based workshop is necessary for teachers to be qualified to teach. They also feel it is a complete basic program in itself. Other factions are saying it is a supplementary approach to computation in mathematics and can be learned with minimal workshop direction plus practice.

12. If used as a complete program, there is no correlation or provision for computer literacy or calculator competencies that appear to be high priorities in society.

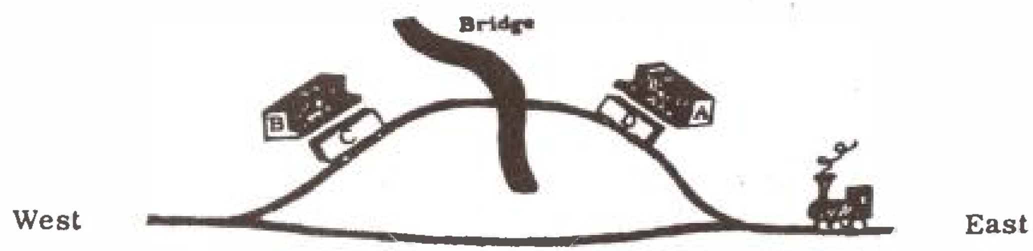
13. There is apparent enthusiasm by teachers and students but no evidence was found that this enthusiasm lasts.

Recommendation

We as a task force recommend Chisanbop/Fingermath as another supplementary approach to the teaching/learning of computational skills with whole numbers. Teachers should not be discouraged from trying out this "new" approach but should be made aware that Chisanbop is not a complete basic program in mathematics. Children have many different learning styles and teachers might find success for some using this method to supplement the basic program. It is easy and fun to learn for those in the complete spectrum of student abilities from remediation to gifted.

The following logic problem is from The Math Wizard *by Louis Grant Brandes, published by J. Weston Walch and distributed by Western Educational Ltd.*

Boxcar Problem.



A mistake was made in placement of boxcars in front of warehouses A and B, as illustrated in the diagram above. A bridge passes over the spur track which permits boxcars to pass under it, but not the engine. The engine will not fit between the bridge and the warehouse unloading points. How can the engineer switch boxcar D to warehouse B and boxcar C to warehouse A?

SOLUTION: Use the engine to push D to bridge and leave there. Take engine west, then back on spur to couple C and D; leave C and D coupled under bridge. Move engine to west end of spur and pick them up. Move C and D to a point past the west spur intersection. Leave C, then move D back past A and leave under bridge. Now pick up C and move it to warehouse A. Move the engine back to the west spur and pull D to warehouse B.

??? PROBLEM CORNER ???

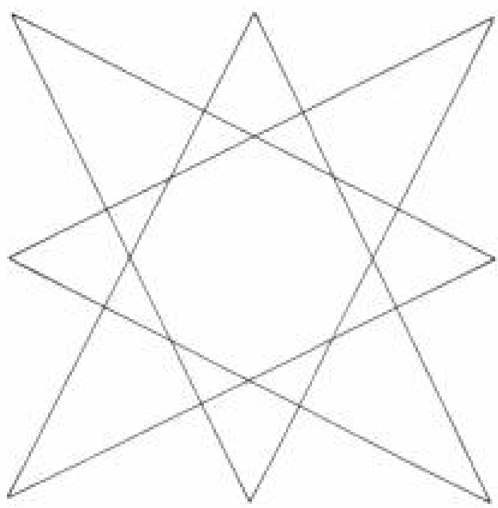
edited by *William J. Bruce* and *Roy Sinclair*
University of Alberta
Edmonton, Alberta

Problems suggested here are aimed at students of both the junior and senior high schools of Alberta. Solutions are solicited and a selection will be made for publication in the next issue of *delta-k*. Names of participants will be included. All solutions must be received (preferably in typewritten form) within 30 days of publication of the problem in *delta-k*. *Mail solutions to:*

Dr. Roy Sinclair
Department of Mathematics
University of Alberta
Edmonton, Alberta T6G 2G1

PROBLEM 1:

An 8-point star is formed in a square region of side S units by drawing two lines from each mid-point of a side of the square to opposite corners of the square. Note that these lines also form two identical smaller squares as well as one octagon.



(a) If the star is cut out of the square region, what fraction of the square is wasted?

(b) In terms of S, what is the area of one of the smaller square regions?

(c) In terms of S, what is the area of the octagonal region?

(submitted by William J. Bruce, University of Alberta)

News Items from ...

NATIONAL COUNCIL OF

Teachers of Mathematics

NCTM

1906 Association Drive, Reston, Virginia 22091 ■ (703) 620-9840

Mathematics & Careers for the 1980s

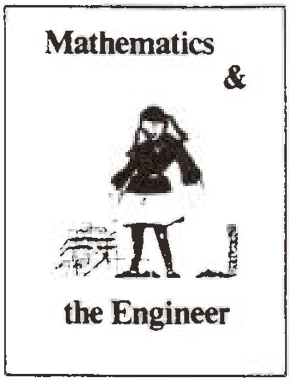
Educators interested in stressing the importance of the mathematics we all use can now call on the new Mathematics & Careers resources just released by the National Council of Teachers of Mathematics.

Mathematics & Careers is a series of posters and brochures developed by the NCTM Career Education Committee. These full-color 43 x 56 cm posters, suitable for framing, present a new, more dynamic image of mathematics. Each career packet includes a poster and 10 copies of a brochure describing that career. Interviews with members of the featured occupation are included in each brochure.

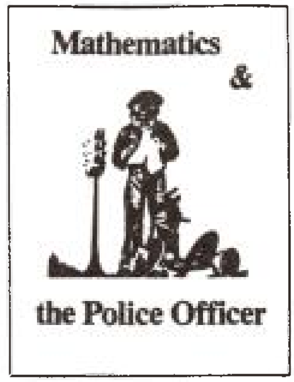
The Engineer and the Police Officer are the first two career packets in this series. These and future packets, such as the Actuary, the Banker, the Carpenter, the Electrician, the Metal Worker, the Nurse, the Pilot and the Realtor, are certain to help popularize mathematics and each of the occupations.

Priced at $4.30 per career packet, with individual member and quantity discounts available, Mathematics & Careers can be obtained from the National Council of Teachers of Mathematics, 1906 Association Drive, Reston VA 22091; phone (703) 620-9840. An order form is provided on the following page. (Review copies of Mathematics & Careers are available on request.)

NEW CAREER EDUCATION MATERIALS



Now Available



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Engineer brochures (10 per package)	#0503		1.35	
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Bulletin Board Resources

BULLETIN BOARD IDEAS FOR ELEMENTARY AND MIDDLE SCHOOL MATHEMATICS, by Seaton E. Smith, Jr. #82. $4.00. Gives guidelines for effective titles, borders, color, background, arrangement, et cetera; 82 photos, 31 in full-color. 1977. 56pp.

MATHEMATICS AND HUMOR, edited by Aggie Vinik, Linda Silvey, and Barnabas Hughes. #266. $4.00. A collection of jokes, riddles, and cartoons to add levity to bulletin boards and test papers and pique the interest of students from junior high school up. 1978. 58pp.

MATHEMATICS IN USE, AS SEEN ON POSTAGE STAMPS, by William L. Schaaf. #18, $2.25. Set of eight 37 x 28 cm full-color posters, each portraying international stamps representing a particular use of mathematics. Based on illustrations from Schaaf's article in the January 1974 *Mathematics Teacher*. 1976.

POSTERS: COUNTING THE PETALS OF A FLOWER. #264. $4.00. Set of four 41 x 53 cm full-color posters based on photographs from the popular book *I Can Count the Petals of a Flower*, by John and Stacey Wahl. 1978.

POSTERS FROM THE ARITHMETIC TEACHER. #253. $2.25. Set of nine imaginative, stimulating posters for the elementary classroom, each 28.5 x 36.5 cm; bright and colorful. 1978.

MATHEMATICS & CAREERS, a series of posters and brochures being developed by the NCTM Career Education Committee. These full-color 43 x 56 cm posters, suitable for framing, will attract students to your room, display, or career education program, while stressing the importance of mathematics. Each career poster comes with 10 copies of a brochure describing that career. Interviews with members of the featured occupation are included in each brochure. Start a standing order plan and be the first to receive each Mathematics & Careers packet in the series as soon as it is available.

Special NCTM Bulletin Board Resources Order Form

Title	Stock #	Quantity	Unit Price	Total Price
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MATHEMATICS IN USE, AS SEEN ON POSTAGE STAMPS	18		$ 2.25	
POSTERS: COUNTING THE PETALS OF A FLOWER	264		$ 4.00	
POSTERS FROM THE ARITHMETIC TEACHER	253		$ 2.25	

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PLUS +++

The following material is reprinted from Issue No. 3 of Plus + + +, *a short magazine informing mathematics educators across Canada about important events, research, curriculum development and items of national interest.*

Associate Editors Needed

Plus +++ is now appearing in several Canadian journals. To make it more effective at gathering and disseminating information, there should be associate editors who can comb their own regions for items of national interest. Volunteers and nominations are requested. It would be desirable to set up the editorship on a rotating basis.

Newsletter on Problem Solving

In June 1979, the Franklin Institute Press began to produce a monthly newsletter entitled *Problem Solving*. According to Managing Editor Julia S. Hough, it will keep the reader "up-to-date on material presented at conferences, as well as books and papers available on the subject." It will include reports on programs to develop methods of teaching problem solving and on research conducted in university and industry. Write to The Franklin Institute Press, 20th and Race Streets (Box 226), Philadelphia, PA 19103, U.S.A.

Second International Mathematics Study

In 1964, the International Association for the Evaluation of Educational Achievement (IEA) sponsored a study of the mathematics, achievement, interests and attitudes of students aged 13, 16, and 18 in each of 12 countries. A second study is now under way. There are three components: curriculum analysis (intended objectives and methodologies for mathematics teaching and learning), classroom process (instructional practices analyzed and compared), student processes (attitudes and achievement in the light of curricular emphases and classroom practices). Two sessions at ICME IV in Berkeley, California, in August 1980 will have progress reports: (1) curriculum analysis (A.I. Weinzweig, H. Steiner), (2) classroom process, with the results of a pilot study of teacher questionnaires in seven countries (K.J. Travers). International reports are scheduled to appear in December 1982.

Several countries have national committees associated with the study. Canada is not one of these, although the following have been active in pilot testing and other developmental activities: David Bale (Regina U.), John Del Grande (North York B.E.), Lars Jansson (Manitoba U.), Thomas Kieren (Alberta U.), Ronald Ragsdale (O.I.S.E.), David Robitaille (U.B.C.), Howard Russell (O.I.S.E.).

Literature available:

1. IEA ACTIVITIES:
   description of IEA and projects, participating national institutions; from Dr. T. Neville Postlethwaite, Department of Comparative Education, University of Hamburg, Sedanstrasse 19, 2000 Hamburg 13, Federal Republic of Germany.
2. THE SECOND INTERNATIONAL MATHEMATICS STUDY:
   purposes and plans for study; from Roy W. Phillipps, Chairman,

Mathematics Project Council, Private Bag, Department of Education, Wellington, New Zealand.

3. SECOND STUDY OF MATHEMATICS, BULLETIN NO. 3: purposes and designs, sampling plan, analyses, IEA papers relevant to study, timetable, personnel; from Dr. Kenneth J. Travers, Chairman, International Mathematics Committee, 341 Armory, University of Illinois, Urbana, Illinois 61801, U.S.A.

Master of Science in Teaching at Toronto

The Mathematics Department and Faculty of Education at the University of Toronto have embarked on a part-time joint graduate program for experienced teachers. It will include four courses, two jointly given by the two divisions (problem solving, mathematical modelling), one given in the Faculty of Education and one given in the Department of Mathematics. For further information, write Professor D. Alexander, Mathematics Department, Room 373, Faculty of Education, University of Toronto, Toronto M5S 1A1.

French IREM Experiment

From Professor Jean Dhombres, Scientific Counsellor of the Embassy of France to Canada, comes an account of an experiment in continuous teacher training, designed to meet the mathematical needs of a modern society. France is divided into 24 academies, each in the charge of a rector responsible for education from nursery to university levels. In 1968, the Ministry of National Education decided to create in each zone an Institute for Research in Mathematics Education (IREM). These are involved in inservice teacher training, pedagogical research and evaluation, and coordinating the sequence of instruction on various topics in a given grade. IREM trainees from schools are given time off to meet in groups, led by a high school teacher and assisted by a research amateur from a university, and study a predetermined theme. Further details are available from Professor Dhombres or from Professor E. Barbeau, the editor. During the fall of 1979, Queen's University was visited by Professor Bouvier who is associated with IREM.

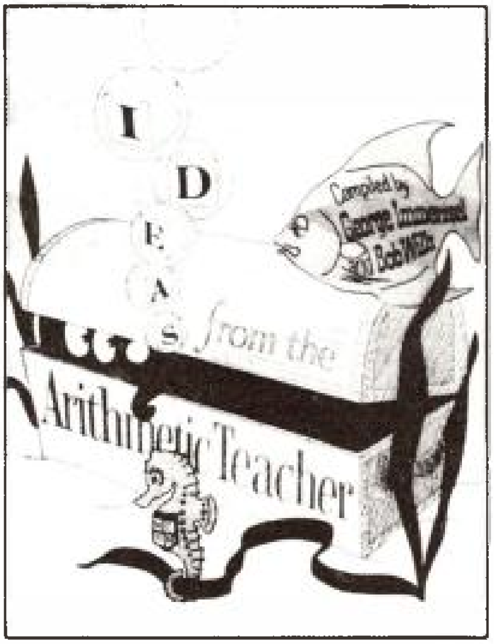
Concourse des Jeunes Mathematiciens

(du niveau secondaire)

Pour des renseignements, ecriere à Départment de Mathématiques, Université d'Ottawa, Ottawa, Ontario K1N 9B4.

IDEAS from the

Arithmetic Teacher



- IDEAS for Fractions
- IDEAS for Computation
- IDEAS for Problem Solving
- IDEAS for Geometry
- IDEAS for Measurement

The IDEAS section has been a feature of the *Arithmetic Teacher* since 1971. This collection has been selected from those activities appropriate for students in grades 4 through 8. The selections have been reprinted just as they originally appeared in the journal.

On one side of each page you will find the Pupil Activity Sheet; the teacher directions are on the reverse. This booklet has been perforated so that the pages can be easily removed and reproduced for classroom use. We suggest that you make a file of these pages or punch them for storage in a loose-leaf binder. Copies should be kept in the same file or binder so that you can use them when they are needed.

This volume has been topically arranged so that IDEAS for computational skills, for example, appear in one section, IDEAS for problem solving are grouped in another section, and so on. Suggested grade levels appear in the teacher directions for each IDEAS sheet.

IDEAS from the *Arithmetic Teacher* ________ (Quantity) @ \$5.40 . . . . \$________ (Subtotal)

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4th International Congress on Mathematical Education

Berkeley, California, U.S.A.

August 10-16, 1980

You are invited to participate in the 4th International Congress on Mathematical Education. The Congress offers participants an opportunity to become better informed about current issues and developments in mathematical education at all levels. There will be many scheduled and unscheduled informal events so that participants may gain from personal contacts.

The official Congress languages are English, Spanish, and French. At all times, simultaneous translations of at least one session, including all plenary sessions, will be available.

A report of the proceedings of the Congress will be sent to all Full Members.

The participation fee for Full Members will be listed in the Second. Announcement. There will be a lesser fee for Associate Members (accompanying a Full Member but not taking part in the work of the Congress). These fees include a choice of planned excursions and cultural events. Optional excursions and activities will be at extra cost. The fee will *not* include travel, accommodations, or board. Those interested in participating should complete the "Request for Information" form on the following page and return it to the official Congress address:

ICME IV
Mathematics Department
University of California
Berkeley, CA 94720, U.S.A.

The Second Announcement and applications for registration and accommodations will be sent to respondents at a later date.

Communications should be mailed to the official Congress address. Mark "ATTN: PROGRAM" on requests about program. Mark "ATTN: LOC" on requests concerning exhibits, special interest groups, and local conference operations.

Programs

The scientific program will examine a broad spectrum of problems in mathematical education at all levels and for every variety of learner. Special emphasis will be given to questions of universal primary education, of research, technology, applications, the profession of teaching, and the relationship of language and mathematics. Preliminary plans include four plenary session invited addresses, at least fifteen other main invited speakers (one hour each) and about sixty panels and debates (one to two hours). Many of the main speakers will be followed by a panel on the same topic. There will be about twenty three-lecture series (called "miniconferences") on individual topics. Of special interest are proposed miniconferences in honor of the late Edward G. Begle on critical variables in mathematical education, and several expository series on topics which have become so fundamental, applicable, and pedagogically accessible that they could be considered worldwide for inclusion in the mathematics curriculum.

Short Communications

Following the pattern of the Third Congress, at Karlsruhe, participants are invited to present short communications via "Poster Sessions," where papers selected for presentation can be displayed and authors will be available for discussion. Posters may be submitted on all topics; especially solicited are posters on teaching mathematics to the handicapped and on problems and curricula in mathematical education in the participating countries. Details will be in the Second Announcement.

Projects

Selected research and development projects in mathematical education throughout the world will be invited to make presentations.

Exeter Working Groups and Karlsruhe Study Groups

Existing working and study groups will be given the opportunity to meet, to continue their work, and to tie in with the rest of the program.

National and International Organizations and Journals

Meetings for national and international organizations of mathematics teachers and for editors of mathematical education journals will be scheduled during the Congress.

Films

Presentations of mathematical films are being organized. Forward your recommendations (*not* the films) to the official Congress address, marked "ATTN: FILMS."

Exhibitions

Publishers and firms producing teaching aids for mathematical education may present their books and materials. Teacher-made and student-made mathematics exhibits will also be displayed.

PLEASE COPY THIS ANNOUNCEMENT and distribute it to your colleagues and to other members of our profession.

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- Secondary • Secondaire • Secundaria ☐
- College • Lycée • Universitaria ☐
- Teacher Training • Formation d'Enseignants • Entrenamiento de Profesor ☐
- Supervision • Direction • Supervisión ☐
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Monograph Review

Math Monograph No. 5: Calculators in the Classroom.

K. Allen Neufeld, editor. November 1977. 165 pp., $5.00.
Publication of the Mathematics Council of The Alberta Teachers' Association, Edmonton

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This *Monograph* presents a variety of papers concerning the use of calculators by educators in Britain, the United States, and Canada. Some of the articles are original publications; others are reprints from other journals. They are organized with respect to five themes: opinions, specifications, research, activities for junior and senior high, and activities for elementary and junior high.

Part one, "Opinions," contains the following articles: "Calculators in the Classroom: Proceedings of a Symposium Sponsored by Rockwell International," "Calculating Machines in Schools," "Computational Skill is Passé," "The Influence of Calculators on Mathematics Curriculums," and "Calculators - a Review."

The first paper is a synopsis of comments made by a panel of university-affiliated educators as they discussed the promises and potentials of electronic calculators as teaching aids in arithmetic classes in middle grades (VI-IX). It also includes the question-and-answer session that followed the formal presentation.

In the second article the authors review priorities of arithmetical and mathematical education in light of the increased availability of calculators, make recommendations regarding the extent to which the use of calculating machines should be encouraged at various stages in education, and consider the types of machines most appropriate for school use.

The third article, which appeared in the *Mathematics Teacher*, consists of seven issues posed to a sample of teachers, mathematicians, and laymen. Included are their responses, given in percentage form, along with some of their positions and justifications.

In the fourth article, after a discussion of the effects of calculators on the curriculum, the author concludes that "educators will not be allowed to decide the issue of whether students will or will not use calculators. Students will use them.... But if curricular materials are designed to take full advantage of the power of the calculator as an educational tool, then perhaps student use of calculators will lead to increased mathematical achievement. Students may even find mathematics more interesting and more useful." (p.34)

In the last article in this section the author reviews some of the recent literature concerning the use of calculators in the classroom. She concludes that "the issue of calculator use in schools continues to be debated and the questions arising are being investigated by educational researchers.... But curriculum changes will not appear overnight, they will only occur if and when teachers are convinced of the calculator's potential." (p.39)

Part two, "Specifications," contains two articles: "So You Want to Buy a Calculator" and "Specifications for Electronic Calculators." Both articles include a discussion of machine features that are felt to be most desirable for use in the classrooms. Thus they provide some guidelines that specify what to look for in a basic machine.

Part three, "Research," contains six articles: "Survey of the Use of Hand-Held Calculators in Mathematics Classes in the Secondary Schools of British Columbia," "The Effect of the Use of Desk Calculators on Achievement and Attitude of Children with Learning and Behavior Problems," "The Use of the Mini-Calculator in the Classroom," "Achievement and Attitudes of Ninth-Grade Students Using Conventional or Calculator-Based Algorithms," "Achievement and Attitude of Low-Achieving Ninth Graders," and "Pocket Calculator Experiment with Fifth and Sixth Graders."

Although the reader must keep in mind the limitations of each study, this research does provide the classroom teacher with some insight into the various ways the calculator can be used and some information as to which children may benefit from the use of this technological instrument. It is up to the teachers to study this research and consider implications for their students.

Part four, "Activities - Junior and Senior High," contains the following articles: "Using Electronic Calculators," "Programmable Calculators and Mini-Computers in High School Mathematics," "Some Uses of Programmable Calculators in Mathematics Teaching," "The Pocket Calculator as a Teaching Aid," and "The Hand Calculator in Secondary Mathematics." These articles contain an assortment of interesting ideas and activities for use in the secondary schools. Such topics as number patterns, trigonometry, functions and limits, programming, flow charting and simulations are explored.

Part five, "Activities - Elementary and Junior High," contains the following articles: "Exciting Excursions in Number Theory with an Electronic Calculator," "Experiences with the Hand-Held Calculator in Teaching Computation, Problem-Solving, and Fractions," "Games with the Pocket Calculator," "The Hand-Held Calculator," and "Problem-Solving Practice via Statistical Data." These articles contain high interest projects, games, and activities for the elementary and junior high students. Such activities as "Calculator Tales - Jaws," "Towards a Million," and "Target," are but a few of the many ideas discussed which should generate high interest and promote student involvement.

In the foreword the editor of the *Monograph* states, "You are encouraged to sample the opinions and make up your own mind, peruse the specifications and buy appropriately, study the research and select activities which will supplement and enrich the mathematics curriculum for your students." (p.3) This reviewer feels that this book fulfills its purpose. If you are looking for an informative resource book on calculators that will provide interesting food for thought, then this might be just what you are looking for.

Reviewed by Norma M. Molina, California State University, Fullerton, California

in the Arithmetic Teacher *(October 1979, pp.49-50).*

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In all, 2455 students participated. Overall, the experimental group enjoyed the word problems and their textbook more than the control group, and the study indicated that they found these materials more interesting than most. Data indicate that the experimental materials can be used successfully in a variety of school situations, comparing favorably with traditional first-year algebra materials.

Here is a first-year algebra course that—

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- is no more difficult than standard courses;
- can be implemented with no special teacher training or changes in the school curriculum.

Trends in the Mathematics Curriculum

by *Marlow Ediger*
Professor of Education
Northeast Missouri State University
Kirksville, Missouri

Teachers and supervisors need to study, appraise, and ultimately implement selected relevant trends to improve teaching-learning situations in the curriculum area of mathematics. Sources for gathering data on these major trends include the following:

1. College- and university-level textbooks in teacher training programs of mathematics education.
2. Reputable textbooks for students (including teacher's manuals) utilized in the school/class setting.
3. Knowledgeable resource persons in the area of mathematics teaching.
4. Recent professional periodical articles, pamphlets, and brochures directly related to improving the mathematics curriculum.
5. Observations made regarding significant innovative methods of teaching mathematics in the public school setting.
6. Use of diverse audio-visual materials to obtain new content.

Relevant Trends

The following relevant trends could be considered important in developing appropriate objectives, learning experiences, and evaluation procedures in ongoing units of study, and therefore in developing a more meaningful and interesting mathematics curriculum.

1. Students should have ample opportunities to acquire knowledge inductively. To emphasize inductive procedures in teaching and learning situations, the teacher should be proficient in asking sequential questions. Students may then be guided to develop relevant concepts and generalizations through the use of discovery methods. Deductive means of teaching, such as meaningful explanations, need not be slighted. Within the framework of inductive instruction, the teacher will need to clarify content discussed in the school/class setting, which involves deductive approaches in assisting students to achieve major understanding and skills objectives.
2. Students must be provided with a variety of learning activities to assist them in achieving optimal development in the mathematics curriculum. Too frequently, no doubt, reputable series of textbooks provide the major resource for students learning mathematics. In addition to recommended series of textbooks, the teacher should also utilize markers, filmstrips, films, slides, transparencies, pictures, and simulated materials to provide for individual differences.
3. Students should have ample opportunities to attach meaning to structural ideas in ongoing units of study. The commutative and associative properties of addition and multiplication, the distributive property of multiplication over addition, the identity elements of addition and multiplication, and the property of closure should be stressed adequately in the mathematics curriculum. Structural

ideas, for students to attach meaning to, need to be significant and must be understood. A variety of learning activities could make these major generalizations more meaningful to individual learners.

4. Balance between and among understanding, skills, and attitudinal objectives need to be stressed in teaching-learning situations. Too often only achievement of understanding content or subject matter is emphasized. These are significant ends to attain. However, students should be given opportunities to practice or utilize content or subject matter. What has been learned previously in terms of facts, concepts, and generalizations may be applied in problem-solving situations, thus emphasizing skills objectives in the mathematics curriculum. A further result of achieving understanding and skills objectives should be desirable student attitudes. Interesting, challenging, and purposeful learning experience for successful achievers can do much to guide their development in the attitudinal or affective dimension. Having an adequate self-concept as well as a desire to learn will aid students in achieving these objectives.

5. The use of diverse techniques is required in appraising achievement. Not all techniques evaluate or measure the same facet of achievement, nor is any one technique of evaluation a perfect approach. Thus, a variety of evaluation methods must be utilized in appraisal. The following approaches may then be used to evaluate progress in mathematics achievement: teacher-written test items, rating scales, checklists, teacher observation, sociometric devices, student self-appraisal, anecdotal records, assessing of students' products and problem-solving abilities. Intellectual, social, emotional and physical growth must be evaluated to ensure sequential progress.

6. Students should be given opportunities to engage in solving realistic problems in the mathematics curriculum. Within a stimulating environment, they need to identify problems, select content related to solving the problem area, develop a possible answer or answers and evaluate the quality of the solution. Critical thinking is required to choose delimited problems and analyze related content in order to arrive at solutions. Creative thinking may well be inherent in problem-solving situations, for example, learners developing unique, novel solutions to problems. Situations in society demand that individuals become proficient in problem-solving, critical thinking, and creative thinking. Therefore, objectives in the mathematics curriculum must stress student proficiency in these skills.

In Summary ...

Teachers and supervisors need to study significant trends in the mathematics curriculum by using a variety of reference sources. Ultimately, they may implement revised, agreed-upon objectives, learning opportunities, and appraisal techniques in the school curriculum.

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INFORMATION

The National Council of Teachers of Mathematics serves YOU through . . .

ARITHMETIC TEACHER, published monthly from September through May, features articles for preservice and in-service elementary school teachers and teacher-educators. Special sections include reproducible activity sheets for classroom use, practical teaching activities, reviews of books and instructional materials, and reports on educational research having implications for classroom practice.

MATHEMATICS TEACHER, published monthly from September through May, emphasizes practical ways of helping teachers in secondary schools, two-year colleges, and teacher-education colleges teach more effectively. Regular features include "Activities," for duplication and use in junior high school classes; "Sharing Teaching Ideas," offering brief practical tips for teachers; and "New Products, New Programs, New Publications," which highlights the latest educational materials and programs.

NCTM'S NEWSLETTER is distributed to all members in September, December, March, and May. It provides current information about Council activities and services.

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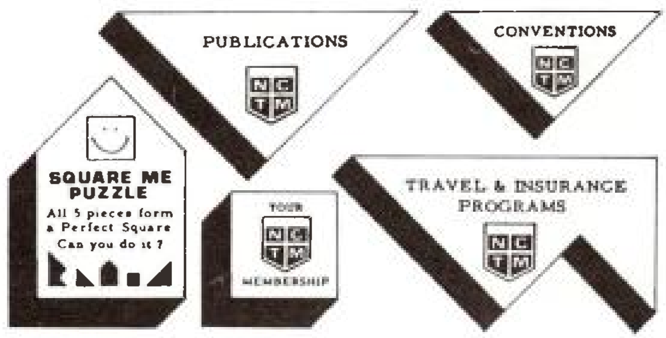

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+ - x ÷ + - x ÷ + - x ÷ + - x ÷ + - x

÷ THE 1980 FACTS ÷

+ - x ÷ + - x ÷ + - x ÷ + - x ÷ + - x

(An activity sponsored by The NCTM-1980 Seattle Meeting)

Use the digits 1, 9, 8, 0 in sequence to make true sentences.

Example: 1 + 9 + 8 + 0 = 18

Fill in the □ with +, -, or x to solve these. You will need to use ( ).

1. 1 □ 9 □ 8 □ 0 = 11
2. 1 □ 9 □ 8 □ 0 = 2
3. 1 □ 9 □ 8 □ 0 = 10
4. 1 □ 9 □ 8 □ 0 = 17
5. 1 □ 9 □ 8 □ 0 = 80
6. 1 □ 9 □ 8 □ 0 = 72

Now use +, -, x and ÷ with digits in any way:

Examples:

81 ÷ 9 + 0 = 9

19 + 8 - 0 = 27

What numbers can you make?

+ - x ÷ + - x ÷ + - x ÷ + - x ÷ + - x ÷ + - x ÷ + - x ÷ + - x ÷ + - x ÷ + - x ÷ +

1980 = 2 x 2 x 3 x 3 x 5 x 11

Using these factors of 1980, cross out the numbers in each row so that the product of the remaining factors is equal to the product on the right.

							Product	
Example:	~~2~~	2	3	3	~~5~~	11	198	(See that 2x3x3x11 = 198)
	2	2	3	3	5	11	60	
	2	2	3	3	5	11	99	
	2	2	3	3	5	11	330	
	2	2	3	3	5	11	180	
	2	2	3	3	5	11	132	
	2	2	3	3	5	11	220	

+ - x ÷ + - x ÷ + - x ÷ + - x ÷ + - x ÷ + - x ÷ + - x ÷ + - x ÷ + - x ÷ + - x ÷ +

The National Council of Teachers of Mathematics (NCTM) unveils "The Curriculum of the 1980s" at the 58th Annual Meeting - April 16-19, 1980, The Seattle Center, Seattle, Washington.

+ - x ÷ + - x ÷ + - x ÷ + - x ÷ + - x ÷ + - x ÷ + - x ÷ + - x ÷ + - x ÷ + - x ÷ +

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