CAN HIGH SCHOOL STUDENTS LEARN SOME OF THE CONCEPTS OF MODERN MATHEMATICS? by Nora Chell and W. F. Coulson

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The first part of this article deals with the question expressed in the title from the standpoint of what various writers have said in the literature. Later parts will deal with the same question but from the standpoint of the research which has been done.

To answer the question, one must first ask, "What is modern mathematics?". Some have defined modern mathematics as meaning modern in approach, while others have defined it as meaning modern content taught by modern approaches. Demsey (3:25) says, "As far as high schools are concerned the word modern applies more to approach than to content". By modern mathematics, Dr. Paul Beesack, assistant professor of mathematics at McMaster University (2:71) means, "axiomatic or postulational method which is characteristic of so much of the mathematical activity of this century". Rourke (11:225) states modern mathematics is "a means of broadening old ideas and introducing new ones to clarify, simplify, and unify our mathematical concepts". Zant (13:595) comments that modern mathematics does not imply a totally new program, but the "newer programs present essentially the same subject matter as before, but it is developed from a different point of view and use is made of a relatively small number of new concepts such as sets, algebraic systems, binary operations, axiomatic systems, etc.". According to May (5:93), modern mathematics includes "such topics as logic, theory of set, Boolean algebra, and set theoretic approaches to relations functions, and other topics of mathematics".

For this paper, modern mathematics will be defined to mean modern concepts taught from a modern approach. Modern content will refer to group and field theory, topology, symbolic logic, and set theory while modern approaches will refer to the axiomatic approach or postulational approach and the set-theory approach. The literature will be reviewed first considering those articles which deal with the average student, then the slow student, third the above average and gifted student, and fourth with respect to mathematics clubs.

Average Student

At St. Laurent High School, Montreal, experiments in modern mathematics were conducted with three groups of students: average, top, and students in mathematics clubs. According to Richardson (9:37), the average students in Grades X and XI benefitted from "the discussion of the postulational method and its use in mathematics".

Rourke (10:12) in his address to the Canadian Teachers' Federation Seminar in 1960 says -

> The boys at Kent School were using the set language in notation very well in Grade XII, so the next year we pushed it down to Grade XI, then down to Grade X and now, in Grade VIII we teach the idea of a relation as a set of pairs, because it fits very well there, with some of the work we are doing.

In his article, "Some Implications of Twentieth Century Mathematics for High Schools", Rourke (11:86) states -

> At the Grade X level, I made the definition of the function explicit and we talk the language of sets and set solutions. We don't get stuffy about it and we allow ourselves some elliptical expressions. We may, on occasion call $2x + 3$ a linear function, as indeed generations of mathematicians have done and likely will continue to do for some time. But we will keep in mind that in this elliptical talk, the function is really the set of ordered pairs (x,y) defined by the formula $y = 2x + 3$.

Zant (13:593) in describing an Oklahoma experimental program in modern mathematics used for regular classroom students states -

> The program was considered very successful. Teachers reported much more interest and understanding on the part of the students than did those in classes taught from the traditional program. Students seemed to be as proficient in the standard skills of mathematics as those taking traditional courses, though no particular effort was made to make comparisons.

A summer school course attended by teachers and superior students is described by Montague (7:22-23) in her article "A Demonstration Class in a National Science Foundation Summer Institute". The students formed a demonstration class and the topics discussed included: The Development of Mathematics, Modern Concepts in Geometry, Modern Concepts in Algebra and Some Aspects of Applied Mathematics. Montague says -

> Follow-up correspondence with teacher participants has revealed that many of the concepts presented by the lecturers were used in their regular classes the following year. The teachers felt that, even though the demonstration group was a highly selective one, the ease with which they accepted new concepts indicated that the average students could also learn some of them.

From the above statements, it seems as though the average high school student can learn some of the concepts of modern mathematics.

Slow Student

Allen (1:14) states that the "set serves to make feasible a broadened introduction to algebraic symbolism and the sentence, be it equation or inequation". He also states that the introduction of sets, including the set concepts of union and intersection motivated, in particular the disinterested and slow learners (1:20). For the slow learner the material was new and different. (If sets motivated the students, is it not inevitable that they must have learned some of the concepts?)

From the results of the St. Laurent experiment, Richardson (9:38) states -

> Sets of ordered pairs were used last year to present graphs to a group of students who were poor in algebra, and they showed good comprehension.

The opinions of two writers indicate that slow high school students can learn some of the concepts of modern mathematics.

Above Average and Gifted Students

In British Columbia an experimental program of Mathematics 91 was established in 1959-60, "to determine whether or not it is possible to incorporate new subject matter and new approaches to traditional subject matter into the regular course" (6:210).

The Chant Commission (12:257) reviewed the experimental classes and found -

> The results indicated that the classes could learn some additional concepts and also master the conventional material at a standard equivalent to that of other Mathematics 91 classes.

According to Richardson (9:38), advanced students were shown what is desirable in drawing up a system of postulates and discussed algebra from the viewpoint of the postulational method.

Rourke (11:85) states that the concept of the function (a set of ordered pairs (x, y) such that for each x there is exactly one y) as a kind of relation can be understood by many high school students, most certainly those capable of college study.

For the gifted child, Allen (1:22) says -

> . . . set terminology is felt to provide an unambiguous and succinct introduction to a number of concepts of beginning algebra.

Grossman (4:75-81) reports that at the National Science Foundation Summer Institute for high school students at Columbia, special topics such as Boolean algebra, groups, rings, integral domains, fields, and the development of the number system from Peano's axioms were taught.

Thirty-five students distinguished by their capacity and achievement participated in the 1961 course.

A similar report by Wilansky (12:250-254) states that gifted students were taught topics of linearity, group theory, and multiplication from a modernistic view of Lehigh University during the summer of 1960.

According to Montague (7:98-100), the demonstration class composed of gifted students at the National Science Foundation Summer Institute had no problems with the following topics: the development of mathematics, modern concepts in geometry, modern concepts in algebra, and some aspects of applied mathematics.

Nichols (8:100-103) describes a school held at Florida State University in 1958 in which the talented students were divided into two groups. Both groups studied the fundamental concepts of set theory. In addition, one group was given "instruction in development of the number system beginning with the notion of set, leading to the definition of a natural number, integer, etc. The students also learned to perform arithmetic calculations using various number systems." The students learned the new concepts very rapidly.

Articles written about modern mathematics and the above-average or the gifted child indicate that these students most definitely can learn some of the concepts of modern mathematics.

Mathematics Clubs

Allen (1:20) suggests that students can learn the concept of the set and the set approach to algebraic symbolism as an extracurricular activity in a mathematics club. He also says -

> The extracurricular programs which have been introduced in many of our high schools have revealed a number of occasions in which the interest of a pupil whose classroom performance has been mediocre was stimulated by an unfamiliar mathematical topic, a problem in change, an exercise in space perception, or the diagrammatic representation of the relationships between sets.

Richardson (9:37) reports that the concepts of modern mathematics were taught in the mathematics club at St. Laurent High School.

From the statements of these two writers, one gains the impression that at least one way to introduce the ideas of modern mathematics is through a mathematics club. This will take no time away from the authorized course and still present the interested students with some challenging topics in a challenging subject.

Conclusion

In each of the articles quoted here the authors indicated that high school students could learn some of the concepts of modern mathematics. Although traditionalists stated that they were definitely against modern mathematics, in no case, was there a statement suggesting that high school students could not learn some of the concepts of modern mathematics.

The writers recognize that many more articles could have been mentioned. Limitations of space made the problem of selection a difficult one. In the next article the writers will take a short look at the research. Is it possible to find evidence that is not merely an expression of opinion that high school students can learn modern mathematics?

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