COMPARING METHODS OF PRESENTING MATHEMATICAL IDEAS IN JUNIOR HIGH SCHOOL, by L. Doyal Nelson

Editor's Note: We are not often able to examine a report of good research data in mathematics education, which applies closely to the problems which confront us in Alberta at the present time. Doyal Nelson's paper dealing with problems in junior high school in Minnesota is most apt and refreshing in that it deals with problems which we would like to have answered in evaluating the attitudes formed and actions taken in setting up the new junior high school curriculum. Doyal is an active member of the MCATA and has shown marked interest in executive matters.

The desire to improve the Alberta junior high school mathematics program will no doubt result in the authorization of mathematics texts for these grades which are strikingly different from those which are currently being used. Junior high school mathematics teachers are already aware of the nature of the content changes which characterize a modern program. However, any change in content brings with it a question of how the new material can best be organized to promote optimum learning efficiency on the part of the pupils.

Our limited knowledge of the exact nature of learning processes prevents us from finding a definitive answer to the question which would apply to the wide range of pupils who might study the material. However, there are ways of obtaining information about students in particular ability ranges. One way of shedding some light on the question is to use different methods of presenting identical mathematical ideas to comparable groups of pupils, then to compare their mastery of the ideas after a specified period of exposure to these ideas. If two mathematics textbooks are available with identical content but with different presentations, the problem is simplified. There are, in fact, such mathematics textbooks available at both the Grade VII and the Grade IX levels. These textbooks were prepared by the School Mathematics Study Group (SMSG). Certain units are identical in content and technical language but differ in the presentation of ideas.

The text Mathematics for Junior High School, Volume I was prepared especially for college-bound seventh-grade students. A modification in the presentation, organization and development of much of the material in this text was subsequently made for slower learning seventh grade pupils and is contained in the text called Introduction to Secondary School Mathematics, Volume I. According to an SMSG newsletter: "The changes and adjustments which were made were prompted by a desire to simplify the presentation and reduce the reading difficulty. Explanatory sections were shortened and exercises added to lead pupils through simple steps to appropriate conclusions."[1] These texts contain such topics as systems of notation, the system of whole numbers, rational numbers, factors and primes, non-metric geometry, and others commonly found in modern seventh grade texts.

In a similar manner the textbook Mathematics for High School, First Course in Algebra was prepared for college-bound ninth grade pupils and its content was subsequently modified for slower learning pupils. The modified version appears in the textbook called Introduction to Algebra. These textbooks contain topics such as the following: the system of real numbers, properties of operations and order, sets and sentences, polynomial and rational expressions, functions, and the like. For a complete picture of the nature of the content and the nature of the differences between the more difficult and the modified textbooks, the reader should refer to the textbooks themselves.[2]

In general, the textbooks for college-capable students develop ideas largely by exposition. Since the texts are intended for high-ability pupils, illustrations, examples and applications are kept at a minimum. The modified texts on the other hand, contain many illustrations and examples which would enable pupils to discover mathematical principles and relationships. Reading difficulty is reduced to a minimum and more problem examples are provided to enable the pupil to make use of the ideas as they are developed. This paper will describe an

[1] Newsletter No. 11, SMSG, Stanford University, 1962; p. 15.

[2] These textbooks are distributed in Canada by McGill University Press.

investigation which was conducted in an attempt to evaluate the two methods of presentation of mathematical ideas used in these textbooks.

Problem

It was the object of this investigation to study the following questions: What was the effect on the mathematics achievement of high ability students who used the SMSG texts designed for slower learning pupils? Was their mathematics achievement different from that of similar high-ability students who learned the same ideas from the texts designed for college-capable students?

Specifically, the basic hypotheses tested were as follows: (a) there was no difference in mathematics achievement, as measured by a standardized mathematics test, between high-ability Grade VII students who used the seventh grade SMSG text for college-capable students and those who used the text for slower learners; (b) there was no difference in mastery of ideas, as measured by various unit tests prepared specifically to test achievement in material covered by the texts, between high-ability Grade VII students who used the seventh grade SMSG text for college-capable students and those who used the text for slower learners. Similar hypotheses were tested for high-ability ninth grade pupils.

Design

This investigation was conducted in 14 schools in Minnesota during the 1961-62 school term. Each of the schools provided two mathematics classes - both of which consisted of either high-ability Grade VII pupils or high-ability Grade IX pupils. One of the classes in each school used the SMSG text for college-capable students at the appropriate grade level and the other class in the school used the SMSG text which had been modified for slower learners.

Seventh grade pupils for the two experimental classes in each school were selected from among those seventh grade students in the school who were above the mean in measured mental ability. Ninth grade pupils for the experiment were selected from among the top third in mental ability. Teachers and administrators in each school were asked to use official files and any other source of information to make up the two experimental classes so that their mean abilities were as nearly alike as possible. Six pairs of seventh grade, and eight pairs of ninth grade classes were obtained for the experiment.

One mathematics teacher from each school was assigned to teach both experimental mathematics classes. Teachers were asked to use a method of presentation in their instruction which conformed closely to the method of the particular textbook used. All other variables were controlled as nearly as possible. For example, teachers for the experiment were selected by the Minnesota National Laboratory and maintained close contact with members of the organization throughout the course of the experiment. Once each month all experiment teachers met with officials of the laboratory where they discussed instructional problems and problems associated with the administration of the experiment. Teachers were constantly reminded to keep a record of time spent in developing ideas, time spent reviewing ideas, standards of scoring tests, use of motivating devices, and so forth, as nearly the same for the two groups being compared as possible.

These teachers were not typical junior high school mathematics teachers. All of them had had previous experience teaching experimental programs in junior high school mathematics and all but one had taken formal courses designed to help them teach modern ideas in junior high school. Their mathematics background varied from 24 to 67 college quarter credits.

Nothing in the design of this experiment would guarantee that a student who was classified as high-ability in one school would be so classified in another. Thus it was necessary to consider each pair of classes as a separate experimental situation or replication for the purpose of testing the hypotheses. In the six pairs of seventh grade classes there were 285 students and in the eight pairs of ninth grade classes there were 460 students.

Execution of the Test

During the course of the experiment various tests were administered and the results of these tests were used to test the hypotheses which have already been stated. In order to get measures of pre-experiment mathematics achievement, STEP Mathematics, Form 3B was administered to all seventh grade participants and STEP Mathematics, Form 2B to all ninth grade participants during September, 1961. A series of five unit tests designed to measure achievement in topics specifically treated in the SMSG textbooks was administered to pupils at each grade level during the course of the year as the topics were completed. The five unit tests administered to seventh grade students covered the following topics: non-metric geometry, factors and primes; rational numbers and fractions; decimals, ratio and percent; measurement; area and volume; parallels, polygons, prisms, circles, statistics, and graphs. The five at the ninth grade level covered the topics: sets, sentences and variables; open sentences and properties of operations; real numbers, multiplication and addition of real numbers; properties of order, subtraction and division of real numbers; factors, exponents, radicals and polynomials. These tests were especially prepared by a panel of test experts for SMSG and have been widely used to evaluate student achievement. An estimate of their reliability was obtained from the data of this experiment and shows coefficients of the same general order as reported for the STEP tests.

At the termination of the experimental period in May, 1962, STEP Mathematics, Form 3A was administered to seventh grade students and STEP Mathematics, Form 2A to ninth grade students. The unit test scores and the scores on the final STEP tests were used as criteria of mathematics achievement.

Statistical Treatment

Analysis of covariance and regression analysis were used to treat the data collected. The covariable at each grade level was the score on the pre-experiment STEP test. It was thus possible to partial out any differences in pre-experiment mathematics achievement of the groups being compared. Since each pair of classes in each school made up a separate experimental situation, there were six replications at the seventh grade level and eight at the ninth grade level. There were 36 tests of the hypotheses for Grade VII; one for each of the six pairs of classes when the set of final STEP scores were used as criterion and five for each of the six pairs of classes when the unit tests were used as criteria. There were 48 tests of the hypotheses for Grade IX.

Normally, when analysis of covariance is used, the means of the groups being compared are adjusted and an appropriate test used to determine if the adjusted means are significantly different. However, this procedure is based on the assumption that the slopes of the regression lines involved are not different. In this study it was important that achievement as measured by the criterion tests could be compared over the whole range of the covariable. The hypothesis tested in each case was as follows:

If S_1 and S_2 are slopes of the regression lines of the groups

involved, then I_1 and I_2 are the intercepts on the criterion axis.

$$H_o: \left(\begin{array}{l} S_1 = S_2 \\ \\ I_1 = I_2 \end{array} \right.$$

An appropriate test of this hypothesis was devised. Whenever the hypothesis of no difference was rejected, scatter diagrams of the data involved were drawn to permit a more complete analysis of the nature of the differences.

Results

Of the 36 tests of the hypotheses for Grade VII, 30 were accepted and 6 rejected. Of the 6 rejections, five occurred in one school. The scatter diagrams for the five rejections indicated that there was a tendency for the high-ability students in this school who were low achievers in mathematics at the beginning of the experiment to achieve better if they used the modified SMSG mathematics text rather than the one for college-capable students. There was a tendency, though not so marked, for those who had been high achievers at the beginning of the experiment to achieve better on the criterion tests if they used the SMSG text for college-capable students than if they used the modified version of this text. Although the differences were not great enough to be statistically significant, the same trend was noted for the remaining criterion test for this school. In fact, in 14 of the remaining cases where the hypothesis was accepted the same trend was noted. In the one case where the hypothesis was rejected, unit test one was involved and the pupils who used the modified version of the SMSG text tended to achieve better than those who used the more difficult version over the whole range of the covariable.

When the final STEP mathematics test scores were used as criteria of achievement, there were no significant differences found between the Grade IX groups being compared in any of the eight schools. However, of the 40 tests of the hypothesis when the unit test scores were used as criteria, there were 11 rejections. Analysis of the scatter diagrams in the cases where the hypothesis was rejected revealed a trend similar to that found for Grade VII, that is, those students who had been low achievers at the beginning of the experiment tended to achieve better on the unit tests if they used the modified text rather than the text for college-capable students. There were, however, notable cases where this trend was not found. For example, in one school where there were three rejections of the hypothesis on the basis of unit tests, students who used the modified text tended to perform better regardless of their pre-experiment achievement level. This tendency was most marked for the highest achievers. Generally speaking, using the text for college-capable pupils did not appear to give any decided advantage to Grade IX students even if they were high achievers.

Conclusions and Recommendations

1. The number of times the hypothesis that there was no difference between the mathematics achievement of high-ability junior high school pupils who used the SMSG text for slower learners and those who used the text for college-capable students was rejected, indicated an effect which could not be entirely attributed to chance.

2. In the cases where the hypothesis was rejected, the most common tendency was for the lower-achieving high-ability pupils to score better on the criterion tests if they used the text for slower

learners. This tendency, in general, decreased for the high-achieving high-ability pupils and there was some evidence to indicate that the highest achievers among them might gain some small advantage from using the text designed for college-capable students.

3. The variability in the nature of the differences as revealed by regression analysis would indicate that there are factors other than the method of presentation of the material which affect the mathematics achievement of high-ability pupils at this level. These factors are undoubtedly associated with the teacher, the pupils, the school, or various combinations of these.

4. The modified versions of the SMSG texts at both the Grade VII and Grade IX levels tended to favor the lower achievers among the high-ability pupils involved. There can be little doubt that the modifications in presentation of material made by the School Mathematics Study Group would assist lower-ability students in learning mathematics. It would appear that teachers of mathematics would be well advised to adopt methods of presenting mathematics material which would permit students to discover mathematical principles and relationships for themselves. Their method of presentation should also include an adequate number of significant situations which would permit the student to use the principles which he has discovered. Clarity and simplicity of language used in communicating mathematical ideas seems to be essential at this level.

5. The term "high ability" in this investigation was not rigorously defined. There is a need for more information about the comparative performance of students within carefully defined ranges of ability when different methods of presenting mathematics material are compared. There is a need also to compare methods of presentation of specific, well-defined topics in mathematics for pupils of varying ability and achievement.

One might question the need of providing a more difficult version of a mathematics textbook for high-achieving, high-ability pupils. Among the high-ability students in this investigation only the highest achievers seemed to find the treatment which involved the use of the SMSG text for the college-capable advantageous from the standpoint of mathematics achievement. This advantage was slight and the number of students involved was small. It might be argued that, at the junior high school level, teachers and textbook writers should constantly search for better ways of making the presentation of mathematics ideas as simple as possible. It would appear, for example, that reading difficulties and vague verbalizations should not be allowed to interfere with the acquisition of fundamental ideas in mathematics, that examples should be most carefully selected to promote pupil discovery of significant mathematical principles and relationships, and that a variety of problem situations should be provided to enable students to appreciate the significance of the ideas included in the program.

GUIDELINES FOR REVISED JUNIOR HIGH SCHOOL MATHEMATICS CURRICULUM

> Editor's Note: A bulletin has been prepared by the Junior High School Mathematics Subcommittee designed to assist teachers and administrators in providing a more suitable program for students who have completed the STA course. The procedures outlined are considered beneficial for students who have followed other programs as well. Junior and senior high mathematics teachers will find it especially enlightening insofar as considerable information is given as to content of the new junior high school curriculum. The material suggests procedures for use of the Winston Text from a "modern" point of view, outlines a unit on numbers and gives an excellent annotated bibliography. Below is the text of the final section on the guidelines for the revised junior high school mathematics section, together with a skeletal bibliography provided by the subcommittee for those who would care to investigate further on their own.

These guidelines regarding content for revised junior high school mathematics curriculum were prepared by the Junior High School Mathematics Subcommittee, April, 1962.

1. Sets - The concept of sets should permeate the course wherever