members varied greatly in experience and training, the instructors were faced with a difficulty in the presentation of their lessons. I believe that the learning situation would have been somewhat better if the class had been divided into two sections, one for the uninitiated and the other for practising teachers of mathematics.

President T. Rempel and his executive, as well as the lecturers who supported this project so generously, deserve commendation for a truly professional venture in inservice training.

- E. S. Gish, Eastwood Elementary-Junior High School

SERIES, SETS AND SUCH

Editor's Note: It is hoped that this column may provide an outlet for contributors who may wish to report on methods of using "Modern mathematics" ideas in their teaching procedures. There are no holds barred. Please send to the editor your reactions to these ideas or send in your ideas. The only stipulation for publication is that your copy is intelligible to the editor, or to someone who might be qualified, in his opinion, to judge if it is intelligible! Following is an algorism by Professor Bruce and some barnyard mathematics involving types of simultaneous equations which might lead us to believe we had no solution and the acceptance of the spoken or unspoken language of sets in its solution.

The Square Root Algorism

We illustrate the thinking behind the mechanical square root method by example which is generally applicable in this case. Other methods are also available but the following is thought to be suitable at an elementary level. Consider

$$\sqrt{1290}.$$

We desire a number whose square is 1290. In this case, we know that the number is somewhere between 30 and 40 so start with 30 as our first trial. We must have

$$1290 = (30 + n_2)^2$$
$$= 900 + 2(30)n_1 + n_1^2,$$

where we desire n_1 as large as necessary. Thus n_1 must be chosen such that

$$2(30)n_1 + n_1^2 \leq 390$$

$$\text{or } n_1[2(30) + n_1] \leq 390.$$

(Note: 1290 - 900 = 390)

By trial, we find

$$n_1 = 5 \text{ and } n_1[2(30 + n_1] = 325 < 390$$

(Note: 6 would be too big.)

Hence, we must have

$$1290 = (35 + n_2)^2,$$

where again we desire n_2 as large as necessary.

We could have started this process by choosing 35 in the first step. Now we have

$$1290 = 1225 + 2(35)n_2 + n_2^2$$

$$\text{or } n_2[2(35) + n_2] = 65.$$

Thus n_2 must be chosen such that

$$n_2[2(35) + n_2] \leq 65$$

By trial, we find

$$n_2 = .9 \text{ and } n_2[2(35) + n_2] = 63.81 < 65$$

Hence, we must have

$$1290 = (35.9 + n_3)^2$$
$$= 1288.81 + 2(35.9)n_3 + n_3^2$$

Thus n_3 must be chosen such that

$$2(35.9)n_3 + n_3^2 \leq 1.19$$

$$\text{or } n_3[2(35.9) + n_3] \leq 1.19$$

By trial,

$$n_3 = .01 \text{ and } n_3[2(35.9) + n_3] = .7181 < 1.19.$$

This procedure may be continued as desired. If we stop at this point we have established $\sqrt{1290} = 35.9$, correct to one decimal.

If we strip off the algebra and arrange in a somewhat compact form we get

```
                      30 + 5 + .9 + .01
                  30 |1290
                     | 900
         2(30) + 5 |390
                   |325
         2(35) + .9 |65
                    |63.81
     2(35.9) + .01 |1.19
                   |_.7181
                     .3719
```

Now make this more compact by simplifying and dropping decimals to obtain the usual format

```
          35.91
   30 |1290.0000
      | 900
   65 |390
      |325
  709 |6500
      |6381
 7181 |11900
      |_7181
        3719
```

- William J. Bruce

Barnyard Mathematics

Somewhere early in my life as an algebra student, I was told that if I was to successfully solve an equation of one variable I required one equation; two variables, two equations; three variables, three equations, etc. However, the analysis associated with "modern mathematics" sometimes modifies such a concept. I have forgotten the precise numbers involved, but it does not matter since the attack on the problem would be similar for the numbers I use. It illustrates equally well how restrictions on the variables may creep in unnoticed.

The problem is that a farmer buys \$100 worth of chickens, pigs and horses. The price of each chicken is 50 cents, each pig \$3, and each horse \$10. If he buys altogether 100 animals, how many of each kind is he buying? If x, y, and z represent the number of chickens, pigs, and horses respectively, then we can express the relations between these numbers by the equations:

$$(1)\ 1/2x + 3y + 10z = 100$$
$$(2)\ x + y + z = 100.$$

Somewhere in beginning algebra we have been led to expect that if we are to determine the values of three variables we must have at least three simultaneous equations which meet certain conditions. Hence we are likely to throw up our hands and assume that this problem cannot be solved completely. Nevertheless, let's see how far we can push this until we come to a blank wall. The equivalent statements are:

$$(1)\ x + 6y + 20z = 200$$
$$(2)\ x + y + z = 100.$$

By subtraction, we have

$$5y + 19z = 100,$$

$$\text{or } y = \frac{100 - 19z}{5} = (20 - 3z) - \frac{4z}{5}.$$

Without further study we may merrily assume that this is as far as we can proceed; that y has an unlimited number of possible values since it is dependent on the values for z.

The Unspoken "Open Sesame" - Part of our assumption is correct - the part that states a dependency of y on the values for z. But this isn't saying much, since by virtue of our equations, all the variables are related to one another. The "open sesame" that slides back the walls of the cave for us lies in an unspoken agreement: the assumption that the variables must be restricted to non-negative integers (that is, live, whole pigs). In the light of this assumption, let's examine the expression for y, namely,

$$(20 - 3z) - \frac{4z}{5}.$$

Since z is now understood to be a non-negative integer, we examine this expression to see when y may also be a non-negative integer. The part in parentheses we shall worry about later since this will be an integer for integral values of z.

Two-Faced Solution - If 4z/5 is to be an integer, 5 must be a factor of z. Another way of saying this is that z must be a multiple of 5; z could be a member of the set (5, 10, 15, . . .). However, examination of equation (1) shows x and y are integral and $x + 6y > 0$ if and only if $z = 5$.

$$\therefore y = \left(20 - 3z - \frac{4z}{5} \,\middle|\, z = 5\right).$$

If z = 5, y = 1, x = 94; this makes a possible solution (x, y, z) = (94, 1, 5). We need try no further since z larger or equal to 10 will give negative values for y, as we can see from the expression in the parentheses. Hence (94, 1, 5) is the only possible solution.

It may be implicit in the problem that the purchase must include at least one of each kind of animal. In that case, the farmer purchases 94 chickens, 1 pig, and 5 horses. Only this solution will check with the conditions stated in the problem.

- J. E. Holditch