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MCATA NOTES

SECOND REPORT ON NCTM SUMMER CONVENTION, by H. L. Larson

Editor's Note: This is the second article by Mr. Larson. It deals with a lesson taught by Dr. Oscar Schaaf, which provides food for thought.

NCTM Venture into Student Text Field

Dr. Oscar Schaaf, of the University of Oregon and a well-known writer of SMSG materials, was given the assignment during the summer of 1963 of heading up a group of writers for the National Council of Teachers of Mathematics.

For some time, the NCTM executive has felt that too little attention has been given to the student in high school who is unlikely to major in a mathematics or science field. These students should not be deprived of the type of mathematics that will stimulate logical thinking. A grant of \$40,000 was given for this project - a first in this type of endeavor. The materials prepared by Dr. Schaaf and his group of writers at Eugene, Oregon are being used this winter in several classrooms. These will be revised in 1964 and offered for sale, perhaps in the fall of 1964.

In the light of this great need in Alberta, teachers may be interested in sampling these materials. One may secretly hope that they may be suitable for the large number of "C" and "B" students from Grade IX who need a new foundation before proceeding with other mathematics courses. The following is a resume of the demonstration lesson given by Dr. Schaaf at the convention. His students were an unselected group somewhat below average, juniors in high school.

Introduction - The teacher took a few moments to clearly outline the nature of the problem. It was one of the "pretend" type but aroused immediate mathematical interest.

Mrs. Jones has a large pie. She expects between 22 - 29 guests. What is the <u>least number</u> of straight cuts she must make in order to make sure that each guest will get one piece of pie? (The size and shape of each piece is immaterial.)

Class Discussion - How would you begin to tackle this problem? Yes, we can represent the pie with a circle and make some cuts. The students tried this on paper. When they came to the third cut something strange happened. Some students found that they had 6 pieces of pie and others had 7. The teacher then developed the problem on the blackboard up to that point. The students with diverging opinions on the third cut were asked to explain.

The following diagrams illustrate this development.



At this point, the class interest mounted. The teacher was careful not to "explain". Some students began to speculate what would happen on the fourth cut. They seemed anxious to find one that would give them a maximum number of "pieces".

One student "discovered" a generality. He stated that no new cut should intersect where other cuts had already crossed. Thus fortified, the class proceeded to illustrate two more cuts.



four cuts 11 pieces



five cuts 16 pieces

At the fifth cut it was evident that there was a limit to the physical drawing of the continuous cutting, even though imagination was possible. Was there any other way to determine the number of pieces beyond the fifth cut? Yes, there was! One student suggested setting up a table of values:

| Cuts   | 0 | 1 | 2 | 3  | 4  | 5  | 6 | 7 | 8 |
|--------|---|---|---|----|----|----|---|---|---|
| Pieces | 1 | 2 | 4 | _7 | 11 | 16 | ? | ? | ? |

The question was then raised as to how to find the answers from such a table? (Isn't this an ideal example of induction?)

The teacher then went to the textbook material which introduced various examples of "rate pairs". Only a few will be shown here. Find n.

| Τ. | 6, | 9   | II. | 2,  | 7  | III. | 60,  | 30 | IV. | 2, | 2 | , γ | 1, | 13 |
|----|----|-----|-----|-----|----|------|------|----|-----|----|---|-----|----|----|
|    | 8, | 12  |     | 15, | 20 |      | 100, | 50 |     | 3, | 4 |     | 2, | 16 |
|    | 4, | 6   |     | 3,  | 8  |      | 20,  | 10 |     | 4, | 8 |     | 3, | 20 |
|    | n, | 1.5 |     | 6,  | n  |      | n,   | 40 |     | 5, | n |     | 4, | n  |

After the students "discover" various patterns, they should be able to do the question at hand. Actually this is as far as Dr. Schaaf was able to get in the time available. But we got the message!

Author's Note: The new book which the National Council of Teachers of Mathematics will be publishing subsequent to the 1964 revision will be called Experiences in Mathematics Discovery. Concepts will be developed with a minimum of teacher "explanation". Exercises will develop each concept very gently. Practical implications will be stressed. Watch NCTM literature for announcements. Some of the chapter headings are: Patterns, Formulae and Graphing; Arrangements and Selections; Intuitive Geometry; A New Look at Whole Numbers; Ratio and Percent; Directed Numbers; A New Look at Fractions.

### REPORTS ON EDMONTON REGIONAL COUNCIL WINTER SEMINAR

Editor's Note: Edmonton regional council president Ted Rempel reports that the seminar is proving popular and that the enthusiasm of the participants is very encouraging. "We started out with about 150 people and now have about 120 attending regularly", he says. Here are the views of a mathematics teacher and a principal who have attended. Guidance and criticism is eagerly sought by those who are organizing these programs.

## Inservice Program in Modern Mathematics

# A Teacher's Opinion

Evaluation of such a program cannot be summed up by such phrases as, "It was worthwhile", or "I got a lot out of it". The lectures to date have been most competently presented, although the material has not been unfamiliar. However, they have not been repetitious or boring by any means. It has been satisfying to be able to follow the lectures and at times to know what was coming next. This only helps to reinforce these new concepts that are being introduced in modern mathematics. Those teaching S.T.M. in Grade VII would have found the idea of sets neatly summarized. A few new ideas about number and numeration systems helped in the actual teaching of these concepts. The remaining programs are not going to be entirely new but if missed would leave one with a feeling that perhaps something really worthwhile has been left out of his own knowledge of that concept. On the whole, the program is interesting, refreshing, enlightening and most decidedly reinforcing to the teacher of modern mathematics.

- D. Sandulak, Allendale Junior High School

## A Principal's Opinion

Speaking from the point of view of a principal who has not specialized in the field of mathematics, the seminar sponsored this year by the Edmonton regional group of the MCATA has provided me with a clearer understanding of the new concepts and the different approaches that are currently being introduced at all levels in many Alberta schools. If a principal proposes to share intelligently in the evaluation of new courses, he must have a fairly intimate knowledge of the latest curricular developments. This seminar, I feel, has given me the necessary background for participating in discussions with the members of my staff who are presently involved in teaching modern mathematics.

I would, however, like to offer a suggestion in regard to grouping members for instructional purposes. Since our large class of some 60

members varied greatly in experience and training, the instructors were faced with a difficulty in the presentation of their lessons. I believe that the learning situation would have been somewhat better if the class had been divided into two sections, one for the uninitiated and the other for practising teachers of mathematics.

President T. Rempel and his executive, as well as the lecturers who supported this project so generously, deserve commendation for a truly professional venture in inservice training.

- E. S. Gish, Eastwood Elementary-Junior High School

#### SERIES, SETS AND SUCH

Editor's Note: It is hoped that this column may provide an outlet for contributors who may wish to report on methods of using "Modern mathematics" ideas in their teaching procedures. There are no holds barred. Please send to the editor your reactions to these ideas or send in your ideas. The only stipulation for publication is that your copy is intelligible to the editor, or to someone who might be qualified, in his opinion, to judge if it is intelligible! Following is an algorism by Professor Bruce and some barnyard mathematics involving types of simultaneous equations which might lead us to believe we had no solution and the acceptance of the spoken or unspoken language of sets in its solution.

## The Square Root Algorism

We illustrate the thinking behind the mechanical square root method by example which is generally applicable in this case. Other methods are also available but the following is thought to be suitable at an elementary level. Consider

V1290.

We desire a number whose square is 1290. In this case, we know that the number is somewhere between 30 and 40 so start with 30 as our

first trial. We must have

$$1290 = (30 + n_2)^2$$
  
= 900 + 2(30)n\_1 + n\_1^2,

where we desire  $n_1$  as large as necessary. Thus  $n_1$  must be chosen such that

$$2(30)n_1 + n_1^2 \le 390$$
  
or  $n_1(2(30) + n_1) \le 390$ .  
(Note: 1290 - 900 = 390)

By trial, we find

 $n_1 = 5 \text{ and } n_1 (2(30 + n_1) = 325 4 390)$ 

(Note: 6 would be too big.)

Hence, we must have

$$1290 = (35 + n_2)^2,$$

where again we desire  $n_2$  as large as necessary.

We could have started this process by choosing 35 in the first step. Now we have 2

$$1290 = 1225 + 2(35)n_2 + n_2^2$$
  
or  $n_2(2(35) + n_2) = 65$ .

Thus n<sub>2</sub> must be chosen such that

$$n_2(2(35) + n_2) \le 65$$

By trial, we find

$$n_2 = .9$$
 and  $n_2(2(35) + n_2) = 63.81 < 65$ 

Hence, we must have

$$1290 = (35.9 + n_3)^2$$
  
= 1288.81 + 2(35.9)n\_3 + n\_3^2

Thus  $n_3$  must be chosen such that

$$2(35.9)n_3 + n_3^2 \le 1.19$$
  
or  $n_3(2(35.9) + n_3) \le 1.19$ 

By trial,

$$n_3 = .01 \text{ and } n_3(2(35.9) + n_3) = .7181 < 1.19.$$

This procedure may be continued as desired. If we stop at this point we have established  $\sqrt{7290} = 35.9$ , correct to one decimal.

If we strip off the algebra and arrange in a somewhat compact form we get

$$30 + 5 + .9 + .01$$

$$30 | 1290 | 900 | 900 | 325 | 2(30) + 5 | 390 | 325 | 2(35) + .9 | 65 | 63.81 | 2(35.9) + .01 | 1.19 | .7181 | .7181 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .719 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .719 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181 | .7181$$

Now make this more compact by simplifying and dropping decimals to obtain the usual format

- William J. Bruce

#### Barnyard Mathematics

Somewhere early in my life as an algebra student, I was told that if I was to successfully solve an equation of one variable I required one equation; two variables, two equations; three variables, three equations, etc. However, the analysis associated with "modern mathematics" sometimes modifies such a concept. I have forgotten the precise numbers involved, but it does not matter since the attack on the problem would be similar for the numbers I use. It illustrates equally well how restrictions on the variables may creep in unnoticed.

The problem is that a farmer buys 100 worth of chickens, pigs and horses. The price of each chicken is 50 cents, each pig 3, and each horse 10. If he buys altogether 100 animals, how many of each kind is he buying? If x, y, and z represent the number of chickens, pigs, and horses respectively, then we can express the relations between these numbers by the equations:

> (1) 1/2x + 3y + 10z = 100(2) x + y + z = 100.

Somewhere in beginning algebra we have been led to expect that if we are to determine the values of three variables we must have at least three simultaneous equations which meet certain conditions. Hence we are likely to throw up our hands and assume that this problem cannot be solved completely. Nevertheless, let's see how far we can push this until we come to a blank wall. The equivalent statements are:

(1) 
$$x + 6y + 20z = 200$$
  
(2)  $x + y + z = 100$ .

By subtraction, we have

5y + 19z = 100,

or 
$$y = \frac{100 - 19z}{5} = (20 - 3z) - \frac{4z}{5}$$
.

Without further study we may merrily assume that this is as far as we can proceed; that y has an unlimited number of possible values since it is dependent on the values for z. The Unspoken "Open Sesame" - Part of our assumption is correct - the part that states a dependency of y on the values for z. But this isn't saying much, since by virtue of our equations, all the variables are related to one another. The "open sesame" that slides back the walls of the cave for us lies in an unspoken agreement: the assumption that the variables must be restricted to non-negative integers (that is, live, whole pigs). In the light of this assumption, let's examine the expression for y, namely,

$$(20 - 3z) - \frac{4z}{5}$$

Since z is now understood to be a non-negative integer, we examine this expression to see when y may also be a non-negative integer. The part in parentheses we shall worry about later since this will be an integer for integral values of z.

Two-Faced Solution - If 4z/5 is to be an integer, 5 must be a factor of z. Another way of saying this is that z must be a multiple of 5; z could be a member of the set (5, 10, 15, . . .). However, examination of equation (1) shows x and y are integral and x + 6y > 0 if and only if z = 5.

, y = 
$$(20 - 3z - \frac{4z}{5})/z = 5)$$
.

If z = 5, y = 1, x = 94; this makes a possible solution (x, y, z) = (94, 1, 5). We need try no further since z larger or equal to 10 will give negative values for y, as we can see from the expression in the parentheses. Hence (94, 1, 5) is the only possible solution.

It may be implicit in the problem that the purchase must include at least one of each kind of animal. In that case, the farmer purchases 94 chickens, 1 pig, and 5 horses. Only this solution will check with the conditions stated in the problem.

- J. E. Holditch

COMPARING METHODS OF PRESENTING MATHEMATICAL IDEAS IN JUNIOR HIGH SCHOOL, by L. Doyal Nelson

Editor's Note: We are not often able to examine a report of good research data in mathematics education, which applies closely to the problems which confront us in Alberta at the present time. Doyal Nelson's paper dealing with problems in junior high school in Minnesota is most apt and refreshing in that it deals with problems which we would like to have answered in evaluating the attitudes formed and actions taken in setting up the new junior high school curriculum. Doyal is an active member of the MCATA and has shown marked interest in executive matters.

The desire to improve the Alberta junior high school mathematics program will no doubt result in the authorization of mathematics texts for these grades which are strikingly different from those which are currently being used. Junior high school mathematics teachers are already aware of the nature of the content changes which characterize a modern program. However, any change in content brings with it a question of how the new material can best be organized to promote optimum learning efficiency on the part of the pupils.

Our limited knowledge of the exact nature of learning processes prevents us from finding a definitive answer to the question which would apply to the wide range of pupils who might study the material. However, there are ways of obtaining information about students in particular ability ranges. One way of shedding some light on the question is to use different methods of presenting identical mathematical ideas to comparable groups of pupils, then to compare their mastery of the ideas after a specified period of exposure to these If two mathematics textbooks are available with identical ideas. content but with different presentations, the problem is simplified. There are, in fact, such mathematics textbooks available at both the Grade VII and the Grade IX levels. These textbooks were prepared by the School Mathematics Study Group (SMSG). Certain units are identical in content and technical language but differ in the presentation of ideas.

The text <u>Mathematics</u> for Junior <u>High</u> School, <u>Volume I</u> was prepared especially for college-bound seventh-grade students. A modification in the presentation, organization and development of much of the material in this text was subsequently made for slower learning seventh grade pupils and is contained in the text called Introduction to Secondary School Mathematics, <u>Volume I</u>. According to an SMSG newsletter: "The changes and adjustments which were made were prompted by a desire to simplify the presentation and reduce the reading difficulty. Explanatory sections were shortened and exercises added to lead pupils through simple steps to appropriate conclusions."<sup>1</sup> These texts contain such topics as systems of notation, the system of whole numbers, rational numbers, factors and primes, non-metric geometry, and others commonly found in modern seventh grade texts.

In a similar manner the textbook Mathematics for High School, First Course in Algebra was prepared for college-bound ninth grade pupils and its content was subsequently modified for slower learning pupils. The modified version appears in the textbook called Introduction to Algebra. These textbooks contain topics such as the following: the system of real numbers, properties of operations and order, sets and sentences, polynomial and rational expressions, functions, and the like. For a complete picture of the nature of the content and the nature of the differences between the more difficult and the modified textbooks, the reader should refer to the textbooks themselves.<sup>2</sup>

In general, the textbooks for college-capable students develop ideas largely by exposition. Since the texts are intended for high-ability pupils, illustrations, examples and applications are kept at a minimum. The modified texts on the other hand, contain many illustrations and examples which would enable pupils to discover mathematical principles and relationships. Reading difficulty is reduced to a minimum and more problem examples are provided to enable the pupil to make use of the ideas as they are developed. This paper will describe an

<sup>1</sup>Newsletter No. 11, SMSG, Stanford University, 1962; p. 15.

<sup>2</sup>These textbooks are distributed in Canada by McGill University Press.

investigation which was conducted in an attempt to evaluate the two methods of presentation of mathematical ideas used in these textbooks.

### Problem

It was the object of this investigation to study the following questions: What was the effect on the mathematics achievement of high ability students who used the SMSG texts designed for slower learning pupils? Was their mathematics achievement different from that of similar high-ability students who learned the same ideas from the texts designed for college-capable students?

Specifically, the basic hypotheses tested were as follows: (a) there was no difference in mathematics achievement, as measured by a standardized mathematics test, between high-ability Grade VII students who used the seventh grade SMSG text for college-capable students and those who used the text for slower learners; (b) there was no difference in mastery of ideas, as measured by various unit tests prepared specifically to test achievement in material covered by the texts, between high-ability Grade VII students who used the seventh grade SMSG text for college-capable students and those who used the text for slower learners. Similar hypotheses were tested for highability ninth grade pupils.

#### Design

This investigation was conducted in 14 schools in Minnesota during the 1961-62 school term. Each of the schools provided two mathematics classes - both of which consisted of either high-ability Grade VII pupils or high-ability Grade IX pupils. One of the classes in each school used the SMSG text for college-capable students at the appropriate grade level and the other class in the school used the SMSG text which had been modified for slower learners.

Seventh grade pupils for the two experimental classes in each school were selected from among those seventh grade students in the school who were above the mean in measured mental ability. Ninth grade pupils for the experiment were selected from among the top third in mental ability. Teachers and administrators in each school were

asked to use official files and any other source of information to make up the two experimental classes so that their mean abilities were as nearly alike as possible. Six pairs of seventh grade, and eight pairs of ninth grade classes were obtained for the experiment.

One mathematics teacher from each school was assigned to teach both experimental mathematics classes. Teachers were asked to use a method of presentation in their instruction which conformed closely to the method of the particular textbook used. All other variables were controlled as nearly as possible. For example, teachers for the experiment were selected by the Minnesota National Laboratory and maintained close contact with members of the organization throughout the course of the experiment. Once each month all experiment teachers met with officials of the laboratory where they discussed instructional problems and problems associated with the administration of the experiment. Teachers were constantly reminded to keep a record of time spent in developing ideas, time spent reviewing ideas, standards of scoring tests, use of motivating devices, and so forth, as nearly the same for the two groups being compared as possible.

These teachers were not typical junior high school mathematics teachers. All of them had had previous experience teaching experimental programs in junior high school mathematics and all but one had taken formal courses designed to help them teach modern ideas in junior high school. Their mathematics background varied from 24 to 67 college quarter credits.

Nothing in the design of this experiment would guarantee that a student who was classified as high-ability in one school would be so classified in another. Thus it was necessary to consider each pair of classes as a separate experimental situation or replication for the purpose of testing the hypotheses. In the six pairs of seventh grade classes there were 285 students and in the eight pairs of ninth grade classes there were 460 students.

#### Execution of the Test

During the course of the experiment various tests were administered and the results of these tests were used to test the hypotheses which

have already been stated. In order to get measures of pre-experiment mathematics achievement, STEP Mathematics, Form 3B was administered to all seventh grade participants and STEP Mathematics, Form 2B to all ninth grade participants during September, 1961. A series of five unit tests designed to measure achievement in topics specifically treated in the SMSG textbooks was administered to pupils at each grade level during the course of the year as the topics were completed. The five unit tests administered to seventh grade students covered the following topics: non-metric geometry, factors and primes; rational numbers and fractions; decimals, ratio and percent; measurement; area and volume; parallels, polygons, prisms, circles, statistics, and graphs. The five at the ninth grade level covered the topics: sets, sentences and variables; open sentences and properties of operations; real numbers, multiplication and addition of real numbers; properties of order, subtraction and division of real numbers; factors, exponents, radicals and polynomials. These tests were especially prepared by a panel of test experts for SMSG and have been widely used to evaluate student achievement. An estimate of their reliability was obtained from the data of this experiment and shows coefficients of the same general order as reported for the STEP tests.

At the termination of the experimental period in May, 1962, STEP Mathematics, Form 3A was administered to seventh grade students and STEP Mathematics, Form 2A to ninth grade students. The unit test scores and the scores on the final STEP tests were used as criteria of mathematics achievement.

#### Statistical Treatment

Analysis of covariance and regression analysis were used to treat the data collected. The covariable at each grade level was the score on the pre-experiment STEP test. It was thus possible to partial out any differences in pre-experiment mathematics achievement of the groups being compared. Since each pair of classes in each school made up a separate experimental situation, there were six replications at the seventh grade level and eight at the ninth grade level. There were 36 tests of the hypotheses for Grade VII; one for each of the six pairs of classes when the set of final STEP scores were used as criterion and five for each of the six pairs of classes when the unit tests were used as criteria. There were 48 tests of the hypotheses for Grade IX.

Normally, when analysis of covariance is used, the means of the groups being compared are adjusted and an appropriate test used to determine if the adjusted means are significantly different. However, this procedure is based on the assumption that the slopes of the regression lines involved are not different. In this study it was important that achievement as measured by the criterion tests could be compared over the whole range of the covariable. The hypothesis tested in each case was as follows:

If  $S_1$  and  $S_2$  are slopes of the regression lines of the groups involved, then  $I_1$  and  $I_2$  are the intercepts on the criterion axis.

 $H_{o}: \begin{array}{c} (S_{1} = S_{2} \\ (I_{1} = I_{2} \end{array})$ 

An appropriate test of this hypothesis was devised. Whenever the hypothesis of no difference was rejected, scatter diagrams of the data involved were drawn to permit a more complete analysis of the nature of the differences.

#### Results

Of the 36 tests of the hypotheses for Grade VII, 30 were accepted and 6 rejected. Of the 6 rejections, five occurred in one school. The scatter diagrams for the five rejections indicated that there was a tendency for the high-ability students in this school who were low achievers in mathematics at the beginning of the experiment to achieve better if they used the modified SMSG mathematics text rather than the one for college-capable students. There was a tendency, though not so marked, for those who had been high achievers at the beginning of the experiment to achieve better on the criterion tests if they used the SMSG text for college-capable students than if they used the modified version of this text. Although the differ-

ences were not great enough to be statistically significant, the same trend was noted for the remaining criterion test for this school. In fact, in 14 of the remaining cases where the hypothesis was accepted the same trend was noted. In the one case where the hypothesis was rejected, unit test one was involved and the pupils who used the modified version of the SMSG text tended to achieve better than those who used the more difficult version over the whole range of the covariable.

When the final STEP mathematics test scores were used as criteria of achievement, there were no significant differences found between the Grade IX groups being compared in any of the eight schools. However, of the 40 tests of the hypothesis when the unit test scores were used as criteria, there were 11 rejections. Analysis of the scatter diagrams in the cases where the hypothesis was rejected revealed a trend similar to that found for Grade VII, that is, those students who had been low achievers at the beginning of the experiment tended to achieve better on the unit tests if they used the modified text rather than the text for college-capable students. There were, however, notable cases where this trend was not found. For example, in one school where there were three rejections of the hypothesis on the basis of unit tests, students who used the modified text tended to perform better regardless of their pre-experiment achievement level. This tendency was most marked for the highest achievers. Generally speaking, using the text for college-capable pupils did not appear to give any decided advantage to Grade IX students even if they were high achievers.

### Conclusions and Recommendations

1. The number of times the hypothesis that there was no difference between the mathematics achievement of high-ability junior high school pupils who used the SMSG text for slower learners and those who used the text for college-capable students was rejected, indicated an effect which could not be entirely attributed to chance.

2. In the cases where the hypothesis was rejected, the most common tendency was for the lower-achieving high-ability pupils to score better on the criterion tests if they used the text for slower

learners. This tendency, in general, decreased for the high-achieving high-ability pupils and there was some evidence to indicate that the highest achievers among them might gain some small advantage from using the text designed for college-capable students.

3. The variability in the nature of the differences as revealed by regression analysis would indicate that there are factors other than the method of presentation of the material which affect the mathematics achievement of high-ability pupils at this level. These factors are undoubtedly associated with the teacher, the pupils, the school, or various combinations of these.

4. The modified versions of the SMSG texts at both the Grade VII and Grade IX levels tended to favor the lower achievers among the highability pupils involved. There can be little doubt that the modifications in presentation of material made by the School Mathematics Study Group would assist lower-ability students in learning mathematics. It would appear that teachers of mathematics would be well advised to adopt methods of presenting mathematics material which would permit students to discover mathematical principles and relationships for themselves. Their method of presentation should also include an adequate number of significant situations which would permit the student to use the principles which he has discovered. <u>Clarity</u> and simplicity of language used in communicating mathematical ideas seems to be essential at this level.

5. The term "high ability" in this investigation was not rigorously defined. There is a need for more information about the comparative performance of students within carefully defined ranges of ability when different methods of presenting mathematics material are compared. There is a need also to compare methods of presentation of specific, well-defined topics in mathematics for pupils of varying ability and achievement.

One might question the need of providing a more difficult version of a mathematics textbook for high-achieving, high-ability pupils. Among the high-ability students in this investigation only the highest achievers seemed to find the treatment which involved the use of the SMSG text for the college-capable advantageous from the standpoint

of mathematics achievement. This advantage was slight and the number of students involved was small. It might be argued that, at the junior high school level, teachers and textbook writers should constantly search for better ways of making the presentation of mathematics ideas as simple as possible. It would appear, for example, that reading difficulties and vague verbalizations should not be allowed to interfere with the acquisition of fundamental ideas in mathematics, that examples should be most carefully selected to promote pupil discovery of significant mathematical principles and relationships, and that a variety of problem situations should be provided to enable students to appreciate the significance of the ideas included in the program.

### GUIDELINES FOR REVISED JUNIOR HIGH SCHOOL MATHEMATICS CURRICULUM

Editor's Note: A bulletin has been prepared by the Junior High School Mathematics Subcommittee designed to assist teachers and administrators in providing a more suitable program for students who have completed the STA course. The procedures outlined are considered beneficial for students who have followed other programs as well. Junior and senior high mathematics teachers will find it especially enlightening insofar as considerable information is given as to content of the new junior high school curriculum. The material suggests procedures for use of the Winston Text from a "modern" point of view, outlines a unit on numbers and gives an excellent annotated bibliography. Below is the text of the final section on the guidelines for the revised junior high school mathematics section, together with a skeletal bibliography provided by the subcommittee for those who would care to investigate further on their own.

These guidelines regarding content for revised junior high school mathematics curriculum were prepared by the Junior High School Mathematics Subcommittee, April, 1962.

1. Sets - The concept of sets should permeate the course wherever

applicable, that is, wherever the use of set terminology and set operations clarify or simplify the presentation of a concept.

2. Numeration Systems - A discussion of numeration systems to other bases gives students a greater understanding of the base-10 system, develops an appreciation of the history of number development and provides exercises that challenge the mind and imagination of students. For the above reason the inclusion of the topic of numeration systems is considered desirable. The development computational facility in other bases should be regarded as enrichment which can be deleted without interfering with the continuity of the course.

3. <u>Geometry</u> - The point set approach is considered to be acceptable. The study of geometry at the junior high school level will continue to include an intuitive development of basic geometric relationships.

4. Number Systems - The study of the elements, operations and laws of operation of the natural, rational, integral, and real number systems is judged to be desirable to generalize or systematize arithmetic operations with whole numbers and fractions and to give meaning to algebraic operations.

5. Problem Solving - Problem solving is considered to be an important aspect of the curriculum. An approach is favored which stresses: (a) the statement of the problem situation in the form of a mathematical sentence followed by computation and then an interpretation relating the answer to the original problem situation; (b) the use of a ratio or a rate-pair approach to all problems to which it can be applied; (c) problems involving inequalities as well as problems involving equations; and (d) solution procedures which include graphing.

6. <u>Measurement</u> - The topic is judged to be an important one for junior high school mathematics. Some emphasis on the process of measurement is desirable.

7. Logic - If algebraic or geometric proof is introduced towards the end of the junior high school, it is considered desirable to precede

such a section by a brief treatment of strategies useful in proving a statement.

8. Permutation, Combination, Probability, Statistics, and Series - A very brief intuitive treatment of the one or more of the above topic may be included on an optional basis.

9. Conventional Topics - Topics such as reviews of previously learned concepts, application of percent to interest, merchandising, etc. and statistical graphs will continue to receive emphasis but in the cases of the application of percent to business, the amount of emphasis will be reduced.

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- (4) Johnson, Donovan A., and William H. Glenn Exploring Mathematics On Your Own Series, Webster Publishing Co., Pasadena, 1960. (Distributed in Canada by Longmans Green.) (\$1.05 each). Individual titles follow:
  - a. Number Patterns, 47 pp.
  - b. Understanding Numeration Systems, 56 pp.
  - c. Sets, Sentences and Operations, 63 pp.
  - d. Invitation to Mathematics, 64 pp.
  - e. Short-Cuts in Computing, 46 pp.
  - f. The Pythagorean Theorem, 48 pp.
  - g. The World of Measurement, 64 pp.
  - h. Fun With Mathematics, 43 pp.
  - i. Computing Devices, 55 pp.
- (5) Keedy, Mervin L., Richard Jamieson, and Patricia Johnson: Exploring Modern Mathematics - Book I, Holt Rinehart and Winston Inc., New York, 1963, 438 pp.

- (6) Marks, John L., James R. Smart and Richard E. Purdy: Ginn Modern Mathematics Packets for Junior High Schools, Ginn and Company, Toronto, 1962. (\$2.50 per packet).
- (7) School Mathematics Study Group: Mathematics for the Junior High School, Vol. 1 (revised edition), Yale University, 1960.
  a. Parts One and Two, Student's Texts (\$3 less 30% discount).
  b. Parts One and Two, Teacher's Commentary (\$3 less 30% discount).
- (8) Rosskopf, Myron F., Robert L. Morton, Joseph R. Hooten, and Harry Sitomer: Modern Mathematics for Junior High School, Silver Burdette Company, Chicago, 1961, 424 pp. (\$4.15).
- (9) Van Engen, Henry, Maurice L. Hartung, Harold C. Trimble, Emil Berger and Ray W. Cleveland: Seeing Through Arithmetic, Book 1, Parts 1 and 2, W. J. Gage and Co., Toronto, 1961 and 1962. Part 1-244 pp., Part 2-255 pp. (\$2.25 each).

## NOTICE OF PAYMENT OF MCATA FEES

Fees paid now will keep your membership in Good Standing until August 31, 1965. Send to:

> Mrs. Jean Martin Box 277 Ponoka, Alberta

#### MCATA NOTES

1. Following President T. P. Atkinson's trip to Hay River, teachers from that area have been invited to attend our annual conference and seminars for one year on a complimentary membership basis.

2. Further steps have been taken to obtain guidance in the payment of honoraria to those engaged in seminars.

3. There were 326 paid-up members and 20 complimentary memberships as of February 29, 1964. Bank Balance - \$2,209.89.

4. Regionals will be supplied with MCATA paper at cost and may affix their own stamp for their own uses.

5. MCATA officers will be elected by distributed ballot in 1964.

6. The University of Alberta is launching a series of non-credit short courses for teachers in 1964. Mathematics is not included among the offerings for 1964. Expectations are that these offerings will be subsequently extended to mathematics.

7. No newsletter appeared in January and February owing to the editor's hospitalization during those months.

8. One-week midsummer seminars are proposed at Red Deer, Calgary and Edmonton, based on similar material to the winter inservice seminar conducted by the Edmonton regional MCATA. Topics suggested were: (a) Sets, (b) Number systems, (c) Enumeration systems, (d) Problem solving, and (e) Non-metric geometry. Presently-appointed directors for the seminars are: T. P. Atkinson, 6551 - 112A Street, Edmonton; G. P. Jepson, 2316 - 9 Avenue N.W., Calgary; and H. L. Larson, Box 609, Ponoka.

## NOTICE REGARDING MCATA CONSTITUTIONAL CHANGES

Notice of motion will be mailed to all paid-up members. Mailing of notice is through: Mrs. Jean Martin, Box 277, Ponoka, Alberta

Editor - J. E. Holditch, 11035-83 Avenue, Edmonton