EVALUATION OF SEEING THROUGH MATHEMATICS, by Mrs. M. Herchek

Editors' Note: Mrs. Herchek, a teacher at Allendale Junior High School in the Edmonton Public School system presents some of her ideas regarding STM.

As civilization moves into an era of ever increasing industrial complexity the following problems are foreseen.

1. The necessity of providing more mathematics to greater numbers of the student population.
2. The need for a greater number of creative mathematicians.
3. A greater number of people who can adapt existing mathematical models to special problems.
4. Citizens of a democratic society must become intelligent consumers of information stated in mathematical language. Consequently, (a) Schools must prepare themselves to teach more mathematics to more students, and (b) "Good" mathematics is easier to teach and learn than "bad" mathematics.

Mathematics developed slowly and over a long period of time. The best ideas found during this development led to mathematical concepts structured in a continuum where the student acquires and organizes information as it is related to a larger, "logical whole". When mathematics is viewed as a system the individual definitions, assumptions and properties take their places in contributing parts of the whole system.

Mathematicians of the past 150 years have learned different ways in which to ask the same questions. New number systems and algebras have been developed. They have learned to view problems in new ways and now enjoy more productive ways of finding solutions to problems.

If a student is guided to discover these relations for himself, to participate in the discovery of mathematical generalizations, he will enjoy the learning and will be able either to remember the details or work them out again when he needs them. This is the way "good" mathematics should be taught. Here the teacher's responsibility is clearly indicated - to try to see that each day's work

clarifies the student's mathematical thinking as he progresses in the program.

The SEEING THROUGH MATHEMATICS series contains basic concepts throughout. These are: (1) ideas of sets, (2) set operations, (3) conditions, (4) variables, (5) mathematical structure, (6) relation, (7) logic, (8) problem solving. These serve as "strands" that provide a broad perspective of the field of mathematics.

Clarity of language is important in communicating the ideas contained in the basic concepts and this precision of language is stressed in STM. Throughout, careful attention is given to the distinction between (a) the symbols as a means of communication and (b) the ideas expressed by these symbols. This distinction forms an integral part of the entire program. This precision of language eliminates the mechanical presentation as found in traditional texts, making it possible for a unified presentation of many types of conditions, such as conditions for equality, inequality and equivalence.

Another use of these basic ideas is to give the student a powerful and sound approach to problem solving. As he becomes skilled in structuring certain problems involving operations and their properties, he can extend these to solve more complicated problems as his knowledge increases.

Another use of conditions and variables is to unify geometric and algebraic ideas - the variable in a geometric condition may be replaced by points from a universe such as a line, plane or space. This "wedding" of point and number, one of the great unifying ideas in mathematics, enables the student to view certain algebraic conditions from a geometric point of view and vice versa.

As certain ideas occur and recur throughout mathematics, students discover that ideas give rise to basic structures or patterns. One such structure is the concept of the number systems. STM begins with the natural numbers, together with certain operations that satisfy the requirements of a number system. From the natural numbers

the rational numbers of arithmetic are developed and from them the rational numbers are developed. As it is difficult to use a definitional, constructive approach to define the system of real numbers, an axiomatic approach is used. The real numbers are characterized as a set of elements that satisfy the postulates of an ordered field.

In the STM program the concept of the mathematical system is constantly used. In particular, the role of logic in the development of a system is emphasized. Proof is introduced in an intuitive way in STM 1 and STM 2. In STM 2 deductive proof (in both algebraic and geometric situations) is studied. In STM 3 an entire unit is devoted to the study of logic and the nature of proof.

STM recognizes the importance of pure mathematics but does not neglect the "practical" problem solving. Each unit contains lessons on problem solving and on applied mathematics. As new types of conditions are introduced, these are applied to problem solving (a very wide application) so that problem solving becomes a vital and integral strand in the entire STM program. The problem solving approach is simple yet effective. It provides cohesion. It gives equal attention to equalities and inequalities - the inequalities lead to important applications in Grade IX, where linear programming problems that relate to systems of conditions are considered. STM is not organized around "social" arithmetic but it is included whenever appropriate.

The idea of relation as a set of ordered pairs is a recurring concept that is carefully and sequentially developed in STM. Problems concerning ratio and comparison are unified and simplified within the framework of proportional relations considered as sets of equivalent ordered pairs.

It is clearly evident that the two major themes in the STM program are
(1) Understanding of mathematical ideas (not memorization of number facts).
(2) Integration of mathematical ideas (not compartmentalization).
It is hoped that this will lead to the "thinking" rather than the "manipulating" student in mathematics.