THE ALBERTA TEACHERS' ASSOCIATION

MATHEMATICS COUNCIL

NEWSLETTER

Volume IV, Number 2, April, 1965

Contents

JOINT SESSIONS OF THE MAA AND NCTM, by J.M. Cherniwchan

A STRAIGHT EDGE AND COMPASS CONSTRUCTION, by William J. Bruce and Edgar Phibbs

EVALUATION OF SEEING THROUGH MATHEMATICS, by Mrs. M. Herchek

EVALUATION OF EXPLORING MODERN MATHEMATICS, by H.L. Larson

ANNOUNCEMENTS: SUMMER SESSION SHORT COURSES, EDMONTON AND CALGARY

BOOK TITLES

SOME PUZZLES TO PONDER

Newsletter Editors:

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JOINT SESSIONS OF THE MAA AND NCTM, by J.M. Cherniwchan

Editors' Note: John Cherniwchan, a teacher at Salisbury High School and a former president of the MCATA, attended a joint meeting of the Mathematical Association of America and the National Council of Teachers of Mathematics in January, 1965 as a representative of the MCATA.

The day was devoted mainly to the presentation and examination of views as to what should make up the mathematics curriculum at the twelfth grade level. Individuals presenting these views were in the main university personnel; I surmise that the stand taken by these people was determined to a large extent by what they, as college teachers, would like to see included in the twelfth grade curriculum. I missed hearing any discussion of program or programs in mathematics for a student not college bound.

The first session concerned itself with the twelfth grade program from the college viewpoint with emphasis on competence, content and continuity. Today there is a considerable divergence in preparation for college, as well as a considerable range in the ability of students that enter college. Frequently there is somewhat of a gap between high school mathematics and college freshman mathematics. (In view of the current changes occurring in the high school mathematics program in Alberta, what follows will be of some interest. Some of the colleges in the United States are now receiving students who have gone through the new mathematics curriculum. Several comments pointed out that these students showed generally a lack of competence in algebraic manipulation.)

There appeared to be two schools of thought with respect to the inclusion of calculus in the Grade XII program.

Professor A. Black of New York University reported that some 10,000 high school students were presently receiving instruction in calculus. He felt calculus was a natural outgrowth of algebra and should be taught relatively soon after algebra. Teachers are probably better prepared to teach calculus than linear algebra or probability. Teacher readiness is a matter of mental attitude and intellectual honesty. Some texts in calculus for high schools are such that a teacher willing to work, will be able to teach the subject. An example of this is the SMSG text. Professor A.P. Mattuck from MIT felt that the emphasis on rigour is being carried too far. If calculus is to be applied, it is the techniques that are important. He pointed out that despite the "pure" approach prevalent today, the examinations the students write tend almost invariably to be applications of standard calculus formulas.

Several people took up the argument for the other side. There seemed to be general agreement that a better foundation for calculus could be laid at Grade XII level. Courses in Elementary Functions, Analytic Geometry and Inequalities are desirable. The Limit Concept, the Delta-Epsilon theorems are somewhat too sophisticated for the level of maturity that high school students have. Professor D. Richmond from Williams College delivered a very interesting paper on pre-calculus mathematics. He demonstrated in his paper how by using two inequalities it is possible to solve a good number of calculus problems. His contention was that this sort of thing could be done at high school level, and calculus taught in a traditional way in college. However, there were people that took objection to his approach. His retort was: "with too much purity, there cannot be much fertility".

Another session was devoted to the question of Probability and Statistics for the twelfth year student. It is desirable to continue the inclusion of units on data processing in secondary school mathematics courses. The notion of sets is here to stay; however, Probability should not be taught through set theory, and should be taught before statistics. A course in Probability and Statistics should be an elective for superior students.

A representative from Rand Corporation talked about the role of computers in secondary schools. A good number of reasons were quoted for the inclusion of computer courses, but many of these, as he himself admitted, were pretty weak. Judging from the reaction of my immediate neighbors in the auditorium, not too many took him seriously.

What were my general impressions? The controversy between those that argue that Calculus be included in, and those that want it excluded from the secondary school curriculum will not die soon. It should be remembered that individual school systems in the United States have a great deal of autonomy in curriculum design. I suspect there will be schools teaching Calculus for some time. However, the general trend, I believe, will be to give a better base for calculus. Elementary Functions, Analytic Geometry and Probability and Statistics will be included in the twelfth year program as electives, and probably as semester courses.

I arrived for the joint sessions the evening before in time to preview some films that are being readied for distribution. One of them I would like to recommend very highly: "What is Teaching? A Demonstration by George Polya". There is much in this film for every mathematics teacher.

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The theme of the 1965 MCATA Annual is to be ELEMENTARY MATHEMATICS. Members are invited to submit articles dealing with the general topic or any specific sub-topic. In this way, understandings and teaching techniques can be shared. As many articles as possible will be printed in the Annual or in the subsequent Newsletter.

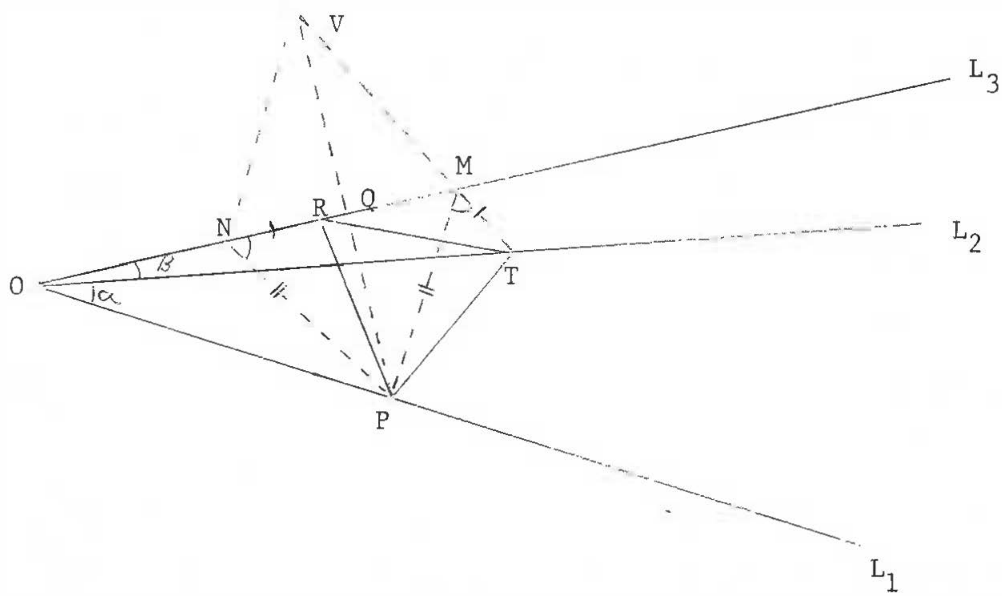
Submissions must be mailed to Professor Wm. F. Coulson, Faculty of Education, University of Alberta, Edmonton, by June 30, 1965.

A STRAIGHT EDGE AND COMPASS CONSTRUCTION, by William J. Bruce and Edgar Phibbs

Editors' Note: Professors Bruce and Phibbs of the University of Alberta, Edmonton, found a construction in plane geometry rather challenging and they pass their discoveries along in an article.

Problem: L_1, L_2 and L_3 are three rays emanating from O. P is any point on L_1. Locate T and R on L_2 and L_3, respectively, so that $\triangle$ PRT is equilateral.

Solution I. Consider the general case with rays L_1, L_2 and L_3 such that $\alpha = \beta$ as shown.



Construction:

1. Construct $PQ \perp L_3$.

2. Construct $\angle MPQ = \angle NPQ = 30^\circ$.

3. Construct the equilateral $\triangle$ NVM.

4. Produce VM to cut L_2 at T.

5. Construct NR = MT.

6. $\triangle$ PRT is equilateral.

Proof:

1. $\triangle$'s PNR and PMT are congruent by construction (s.a.s.)

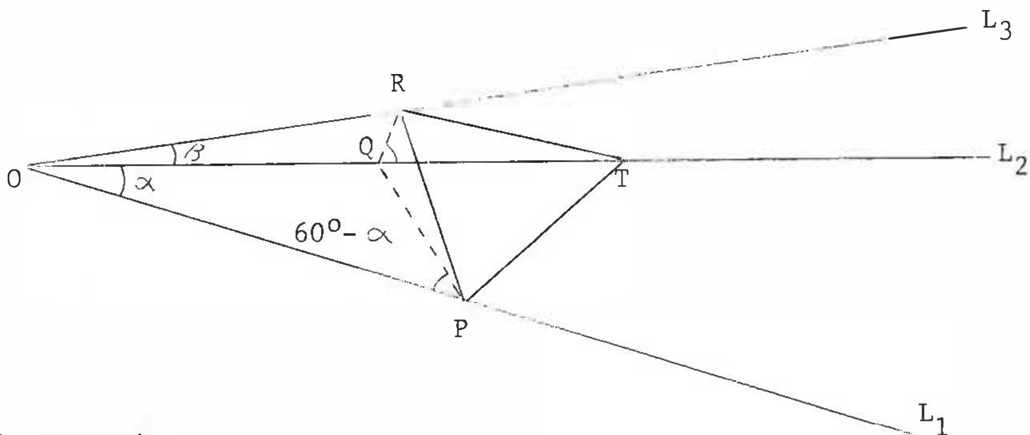
2. Hence PR = PT and $\angle NPR = \angle MPT$.

3. $\angle RPT = 60^\circ$ since $\angle NPM = 60^\circ$ by construction and equal angles NPR and MPT are subtracted therefrom and added thereto respectively.

4. Hence $\blacktriangle$ PTR is equilateral.

Note: If VN is produced to cut L_2 and steps corresponding to the above are followed, another such equilateral triangle results but with T on the opposite side of PR.

Solution II. Consider the general case with rays L_1, L_2 and L_3 such that $\alpha = \beta$ as shown.



Construction:

1. Construct $\angle$ OPQ $= 60^o - \alpha$.

2. Construct $\angle$ RQT $= 60^o$.

3. Draw a circle through P, Q and R cutting OL_2 at T.

4. $\triangle$ PRT is equilateral.

Proof:

1. $\angle$ PQT $= 60^o$.

2. $\angle$ TRP $= \angle$ PQT $= 60^o$. (Subtended by the same chord TP)

3. $\angle$ TPR $= \angle$ TQR $= 60^o$. (Subtended by the same chord TR)

4. Hence $\triangle$ PTR is equilateral.

Note: If $\angle L_1PQ$ is constructed to be $60^\circ + \alpha$, another such equilateral triangle results but with T on the opposite side of PR.

Special Cases and Remarks

When $\alpha + \beta \leq 60^\circ$ two triangles are possible without considering the backward extension of the rays. If $\alpha + \beta = 60^\circ$, one vertex lies at O. If in addition to this, $\alpha = \beta$, the construction is trivial.

When $60^\circ < \alpha + \beta \leq 120^\circ$ two triangles are possible by permitting the backward extension of the rays. In this case, one triangle lies on the forward extension, the other on the backward extension. If $\alpha + \beta = 120^\circ$, one vertex lies at O. Again, if in addition to this, $\alpha = \beta$, the construction is trivial.

For $120^\circ < \alpha + \beta < 360^\circ$ two triangles are generally possible by permitting the backward extension of the rays. If $\alpha = \beta = 90^\circ$, the construction is trivial with one triangle on the backward extension. If $\alpha = \beta = 120^\circ$, the construction is also trivial with three triangles possible. One of these triangles is on the forward extensions and the other two on the backward extensions.

EVALUATION OF SEEING THROUGH MATHEMATICS, by Mrs. M. Herchek

<u>Editors' Note:</u> Mrs. Herchek, a teacher at Allendale Junior High School in the Edmonton Public School system presents some of her ideas regarding STM.

As civilization moves into an era of ever increasing industrial complexity the following problems are foreseen.

1. The necessity of providing more mathematics to greater numbers of the student population.
2. The need for a greater number of creative mathematicians.
3. A greater number of people who can adapt existing mathematical models to special problems.
4. Citizens of a democratic society must become intelligent consumers of information stated in mathematical language. Consequently, (a) Schools must prepare themselves to teach more mathematics to more students, and (b) "Good" mathematics is easier to teach and learn than "bad" mathematics.

Mathematics developed slowly and over a long period of time. The best ideas found during this development led to mathematical concepts structured in a continuum where the student acquires and organizes information as it is related to a larger, "logical whole". When mathematics is viewed as a system the individual definitions, assumptions and properties take their places in contributing parts of the whole system.

Mathematicians of the past 150 years have learned different ways in which to ask the same questions. New number systems and algebras have been developed. They have learned to view problems in new ways and now enjoy more productive ways of finding solutions to problems.

If a student is guided to discover these relations for himself, to participate in the discovery of mathematical generalizations, he will enjoy the learning and will be able either to remember the details or work them out again when he needs them. This is the way "good" mathematics should be taught. Here the teacher's responsibility is clearly indicated - to try to see that each day's work clarifies the student's mathematical thinking as he progresses in the program.

The SEEING THROUGH MATHEMATICS series contains basic concepts throughout. These are: (1) ideas of sets, (2) set operations, (3) conditions, (4) variables, (5) mathematical structure, (6) relation, (7) logic, (8) problem solving. These serve as "strands" that provide a broad perspective of the field of mathematics.

Clarity of language is important in communicating the ideas contained in the basic concepts and this precision of language is stressed in STM. Throughout, careful attention is given to the distinction between (a) the symbols as a means of communication and (b) the ideas expressed by these symbols. This distinction forms an integral part of the entire program. This precision of language eliminates the mechanical presentation as found in traditional texts, making it possible for a unified presentation of many types of conditions, such as conditions for equality, inequality and equivalence.

Another use of these basic ideas is to give the student a powerful and sound approach to problem solving. As he becomes skilled in structuring certain problems involving operations and their properties, he can extend these to solve more complicated problems as his knowledge increases.

Another use of conditions and variables is to unify geometric and algebraic ideas - the variable in a geometric condition may be replaced by points from a universe such as a line, plane or space. This "wedding" of point and number, one of the great unifying ideas in mathematics, enables the student to view certain algebraic conditions from a geometric point of view and vice versa.

As certain ideas occur and recur throughout mathematics, students discover that ideas give rise to basic structures or patterns. One such structure is the concept of the number systems. STM begins with the natural numbers, together with certain operations that satisfy the requirements of a number system. From the natural numbers the rational numbers of arithmetic are developed and from them the rational numbers are developed. As it is difficult to use a definitional, constructive approach to define the system of real numbers, an axiomatic approach is used. The real numbers are characterized as a set of elements that satisfy the postulates of an ordered field.

In the STM program the concept of the mathematical system is constantly used. In particular, the role of logic in the development of a system is emphasized. Proof is introduced in an intuitive way in STM 1 and STM 2. In STM 2 deductive proof (in both algebraic and geometric situations) is studied. In STM 3 an entire unit is devoted to the study of logic and the nature of proof.

STM recognizes the importance of pure mathematics but does not neglect the "practical" problem solving. Each unit contains lessons on problem solving and on applied mathematics. As new types of conditions are introduced, these are applied to problem solving (a very wide application) so that problem solving becomes a vital and integral strand in the entire STM program. The problem solving approach is simple yet effective. It provides cohesion. It gives equal attention to equalities and inequalities - the inequalities lead to important applications in Grade IX, where linear programming problems that relate to systems of conditions are considered. STM is not organized around "social" arithmetic but it is included whenever appropriate.

The idea of relation as a set of ordered pairs is a recurring concept that is carefully and sequentially developed in STM. Problems concerning ratio and comparison are unified and simplified within the framework of proportional relations considered as sets of equivalent ordered pairs.

It is clearly evident that the two major themes in the STM program are

(1) Understanding of mathematical ideas (not memorization of number facts).

(2) Integration of mathematical ideas (not compartmentalization).

It is hoped that this will lead to the "thinking" rather than the "manipulating" student in mathematics.

EVALUATION OF EXPLORING MODERN MATHEMATICS, by H.L. Larson

Editors' Note: Mr. Larson is Superintendent of Schools at Athabasca. He is very interested in the introduction of new mathematics programs.

In evaluating the Exploring Modern Mathematics texts for use in Alberta schools, we might note the following.

1. Universality. By this we mean that the materials and methods are complementary to the other commercial texts now appearing on the market. Whenever a text is premised upon the research results of any of the great studies such as U.M.Ma.P., or S.M.S.G. we can be reasonably assured that it is structurally sound, insofar as research to date is able to take us. The development of each concept in EMM permits teacher or student to make comparisons with many other materials on the market. This aids in what is known as the spiral development of concept-building.

2. Language. The teacher of traditional mathematics finds much of the rigorous language and symbolism of modern mathematics rather distasteful, due probably to the fact that he must literally step down the ladder of understanding to acquire facility with a terminology beneath his intuitive level. The EMM texts are very gentle in their approach to rigour.

3. Teacher Individuality. All good experienced teachers have a considerable storehouse of techniques, especially in drill and review work. In using EMM, teachers will still be able to use many of these with only moderate changes in emphasis. An easier transition from the past to the present is bound to reflect in attitudes of both teachers and students.

4. Supplementary Reading. Provision for individual student differences through mathematical literature is a keystone for success in modern mathematics. EMM builds its concepts in such a way that the student feels certain that other rationales are worth examining. EMM has numerous historical and biographical sketches which stimulate students.

5. Problem Solving. At the Grade VII level, EMM is less concerned with problem solving, choosing instead to stress the basic understandings which are required later. At the Grade VIII and IX levels more stress is given to applied problems.

Traditionally, problem solving has been a means to an end, that end being competence in computation to a large extent. The application of problems, per se, is however largely wasted on Grade VII students. The efficiency of learning computation or mathematics principles via problem solving is questionable. The Grade IX level of EMM is delightfully full of applied problems to cheer the teacher who leans toward the pragmatic point of view.

6. Structure. With current interest in structural approaches to learning being so high, we must view all modern texts in the light of this development. It would appear that as a result of the research in the decade of the fifties, in mathematics instruction, we have come a long way toward achieving a structural approach. What is yet to be discovered is where, in the classroom, we might best develop certain topics. Bruner's summary of the work done in structuring raises this question of sequence and grade level. Piaget on the other hand has given some promise of a type of educational research that will give us some answers as to what extent major concepts should be developed at certain periods in student growth. The more flexible a text is, the longer it will be useful. EMM qualifies very well in this respect.

CONTINUING PROFESSIONAL EDUCATION FOR TEACHERS, Faculty of Education, Department of Extension, University of Alberta, Edmonton, Summer, 1965 (excerpt)

Effective on September 1, 1965, new textbooks for Grade VII Mathematics and Math 10 have been authorized by the Department of Education. Many new ideas are incorporated in the material found in these texts. In order to assist teachers who will be teaching these courses during the 1965-66 school year, two three-week short courses are being offered by the Department of Extension. The seminar approach will be used in both. In each of these courses, the seventh grade teachers will discuss the seventh grade texts. Similar opportunities will be provided for the teachers of Math 10 to discuss the Math 10 text. Teaching techniques as well as mathematical content will be analyzed. The following mathematical topics will be discussed in some depth as background for the material of the appropriate grade level: set language and operations, number systems, numeration systems, relations, functions, deductive reasoning, and Euclidean Geometry.

Each of the two courses will be offered at the beginning of the summer and again at the end, according to the following timetable:

Session I: July 5 to July 23, Monday through Friday
8:00 a.m. to 11:00 a.m. and 12:00 noon to 2:00 p.m.
Staff: Principal instructor for Session I is Dr. S.E. Sigurdson, Department of Secondary Education, U of A.

Session II: August 9 to August 27, Monday through Friday
8:00 a.m. to 11:00 a.m. and 12:00 noon to 2:00 p.m.
Staff: Principal instructor for Session II is Dr. A.L. Dulmage, Head, Department of Mathematics, U of A.

Each instructor will be assisted by other university staff members and by junior and senior high school mathematics teachers.

Text: The Grade VII or the Grade X text and Mervin L. Keedy, A Modern Introduction to Basic Mathematics, Addison-Wesley Publishing Company, 1963.

CONTINUING PROFESSIONAL EDUCATION FOR TEACHERS, Faculty of Education, Department of Extension, University of Alberta, Calgary, Summer, 1965 (excerpt)

New programs in modern mathematics in Grade VII and X will be launched in Alberta schools in September, 1965. In order to prepare teachers for these programs, this intensive, 3 week, all-day institute has been planned. There will be a balance of theory and practice. Group instructions will cover a unified presentation of the mathematical ideas which form around specific texts and will work through problems and projects with the help of a qualified leader. The aim of the course is to provide practical realistic preparation for teaching the new programs.

Chief Instructors: Dr. S.A. Lindstedt, Department of Curriculum and Instruction, and Dr. P. Lancaster, Department of Mathematics, University of Alberta at Calgary.

Date: Three weeks - July 5 to July 23, 1965. Classes will be held all day long Monday to Friday inclusive commencing at 9:00 a.m. A lecture will be presented each morning and afternoon, with the remainder of the time devoted to workshops.

Textbook: Grade X - "Secondary School Mathematics" by MacLean, Mumford, et al.
Textbook: Grade VII - "Seeing Through Mathematics, Book 1" by Hartung, VanEngen, et al. or "Exploring Modern Mathematics, Book 1" by Keedy, Jamieson, et al. (all available through the university bookstore).

Fee: $90

TITLES OF BOOKS TO BE CONSIDERED FOR YOUR PROFESSIONAL LIBRARY

For the Elementary Teacher

Grossnickle and Brueckner; Discovering Meanings in Elementary School Mathematics; New York, Holt, Rinehart and Winston, Inc.; 1963.

Topics in Mathematics for Elementary School Teachers; Twenty-Ninth Yearbook, National Council of Teachers of Mathematics; Washington, D.C.; 1964

For the Secondary Teacher

Brixey and Andree; Fundamentals of College Mathematics, Revised Edition, New York, Holt, Rinehart and Winston, Inc.; 1961

Dadourian; How to Study; Reading, Mass., Addison-Wesley; 1958.

SOME PUZZLES TO PONDER

1. The number 45 is the sum of four different numbers so chosen that if you subtract 2 from the first, add 2 to the second, multiply the third by 2 and divide the fourth by 2, you get the same number each time. What are the four different numbers?

2. If you had the opportunity of accepting a position for five years, which of the following contracts for payment of salary would you prefer: (a) a starting salary of $5,000 per year with an increase of $1,000 per year after each year. (b) a starting salary of $5,000 per year with an increase of $250 per year after each 6 months. Assume payments to be made at the end of each six month period.