THE ALBERTA TEACHERS' ASSOCIATION

MATHEMATICS COUNCIL

NEWSLETTER

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Contents

Newsletter Editor: Mary Beaton
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Members of MCATA join in thanking Mr. Bruns for his enthusiastic leadership of the organization in the past year.

A MESSAGE FROM THE PAST PRESIDENT

A.W. Bruns

As I look back at my enjoyable term of office as President of your Mathematics Council, let me say at the outset that I wish to thank the members of the 1968 Executive for their co-operation and willingness to help the mathematics teachers of this province to become better teachers of mathematics. Many hours have been spent at executive meetings discussing ways and means of reaching the math teachers. We have not always been successful in doing this because teachers do move from one part of the province to another, and the problem of communication is a very real one. I hope the incoming executive officers will be able to solve this problem.

It would be good to be able to say that we no longer need inservice training in mathematics. But changes will continue to take place; new ideas will continue to come forth from dedicated teachers on how to teach both the old and new concepts. Our pupils must be given an opportunity to keep up with many new developments in technology; our teachers need to keep up to date in the new mathematics of today's world through inservice training.

I feel that MCATA has played an important role in the continuing evolution of mathematics and in the inservice education of teachers. However, we need a strong organization of people interested in mathematics. We need members and teachers with imagination and innovative ideas who are willing to present their ideas to the Council through seminars, newsletters and other means. I invite the teachers of mathematics to support the Council and to encourage participation in the work of the Council. We should not sit back and let Joe do it, but we should unite to make 1969 a successful year for the incoming officers.

My congratulations to the new officers, and best wishes for a very successful and rewarding term of office.

THE PRESIDENT'S MESSAGE

Murray Falk

A question I often have been asked by MCATA members or prospective members is "What do I get for my $5 membership fee?" My immediate reaction has been to list some of MCATA's services to members:

- The *MCATA Newsletter* - four issues per year.
- The *MCATA Annual* - the current issue has gone to press.
- Sponsorship of regional councils in Calgary, Central Alberta, and Edmonton.
- Sponsorship of film programs in many areas of the province.
- Liaison between MCATA members and other organizations, primarily The National Council of Teachers of Mathematics and its affiliates, and The Canadian Association of Mathematics Teachers and its affiliates.
- The annual conferences and delegates sponsored at meetings of mathematics teachers in widely separated centers across North America have brought to this province transfusions of valuable ideas.

Mr. Falk was born in Didsbury and attended school in Calgary. A B.Ed. Mathematics Major from the University of Calgary, the President has been teaching in junior high schools since 1959. For two years he taught in DND, Zweibruecken, Germany. Since 1965 he has been teaching STM at Sir Wilfred Laurier Junior High School in Calgary. Mr. Falk, the former Vice-President of MCATA, helped to organize the Calgary Public Junior High Regional Council.

The list does not end here. MCATA has initiated and will continue to initiate other worthwhile projects. Examples of projects begun or envisaged are the study of the ideal mathematics classroom, the review of current new textbooks (reviews will appear in issues of this *Newsletter*), and the exchange of ideas for overhead projectuals.

I have not yet provided the real answer to the question. What you get for your membership is directly proportional to what you are willing to give. I see in MCATA an unlimited opportunity to share your ideas, your problems, and your solutions with your colleagues throughout the province. I see in MCATA an opportunity for mathematics teachers to meet in groups, large or small, for inspiration, imagination, and innovation. I see for MCATA a good year ahead! But MCATA cannot function without your support.

CONJUGATE MULTIPLICATIVE INVERSES

Lambertus Verbeck

Quite often in a classroom situation it is possible to take an ordinary problem and generalize it to the point of developing good creative effort on the part of students.

In working with complex numbers in an intermediate algebra class, the following textbook problem was encountered:

Find the multiplicative inverse of $\frac{1}{2} + \frac{\sqrt{3}}{2}i$.

The students applied the usual methods and found the inverse to be $\frac{1}{2} - \frac{\sqrt{3}}{2}i$. Someone in the class noticed that, unlike other complex numbers, the multiplicative inverse of $\frac{1}{2} + \frac{\sqrt{3}}{2}i$ was its conjugate.

The question then arose, "What kinds of complex numbers have conjugates for multiplicative inverses?" From working previous problems, the students were aware that this was not true of all complex numbers. What, then, were the characteristics of this particular number that led to this curious fact? Could a general complex number with these characteristics be found? How should the investigation proceed?

An examination of $\frac{1}{2} + \frac{\sqrt{3}}{2}i$ did not provide any clues. The following line of investigation was then suggested: Assume there is such a complex number and let it be $x + yi$. Then $(x + yi)(x - yi) = 1$. This leads to $x^2 + y^2 = 1$. The only requirement then is that $x^2 + y^2 = 1$. A check shows that $\frac{1}{2} + \frac{\sqrt{3}}{2}i$ satisfies this requirement since $(\frac{1}{2}) + (\frac{\sqrt{3}}{2})^2 = 1$.

The next question concerned the problem of generating more of these special complex numbers: How could we find more? After an investigation another complex number was produced: $\frac{3}{5} + \frac{4}{5}i$. Students then noticed that the numbers 3, 4, 5, are a Pythagorean triple. The proof that any Pythagorean triple would work followed easily. Since the Pythagorean relationship $a^2 + b^2 = c^2$ leads immediately to $\frac{a^2}{c^2} + \frac{b^2}{c^2} = 1$, we need only let $x = \frac{a}{c}$ and $y = \frac{b}{c}$ to satisfy our only requirement: $x^2 + y^2 = 1$.

At this point we decided to let $x = \frac{a}{c}$ and $y = \frac{b}{d}$ and to state in general terms what we had concluded. So far we had decided that any complex number of the form $\frac{a}{c} + \frac{b}{d}i$ would have its conjugate for a multiplicative inverse, provided that $c = d$ and $a^2 + b^2 = c^2$.

We now were able to find complex numbers of this special nature where both $\frac{a}{b}$ and $\frac{b}{d}$ were rational. But suppose $\frac{a}{c}$ or $\frac{b}{d}$ were not rational; then what would the relationship have to be? We decided to first investigate complex numbers of the form $\frac{a}{c} + \frac{\sqrt{b}}{c}i$. It is then easy to conclude that $a^2 + b = c^2$ is our requirement. Letting a and c be any integers such that $/c/ > /a/$, b is very easy to find: $b = c^2 - a^2$. For example, if we let $a = 5$ and $c = 7$, then $b - 7^2 - 5^2 = 24$. Thus $(\frac{5}{7} + \frac{\sqrt{24}}{7}i)(\frac{5}{7} - \frac{\sqrt{24}}{7}i) = 1$.

We concluded that we are permitted one irrational number, and complex numbers of the form $\frac{a}{c} + \frac{\sqrt{b}}{d} i$ have conjugates for multiplicative inverses when $c = d$ and $a^2 + b = c^2$.

Next we asked, "Are we permitted two irrational numbers?" If we can have two irrationals, then our requirement is $\frac{a}{c^2} + \frac{b}{c^2} = 1$ and $a + b = c^2$. Choosing $c>1$, it is easy to find at least one pair of a,b. For example, let $c = 15$: then $(\frac{1}{15} + \frac{\sqrt{224}}{15}i)$, $(\frac{\sqrt{2}}{15} + \frac{\sqrt{223}}{15}i)$, $(\frac{\sqrt{3}}{15} + \frac{\sqrt{222}}{15}i)$, etc., all have conjugate multiplicative inverses. Our third result was that complex numbers of the form $\sqrt{\frac{a}{c}} + \sqrt{\frac{b}{d}}\, i$ have conjugates for multiplicative inverses when $c = d$ and $a + b = c^2$.

An interesting sidelight might be developed at this point. One of the most famous of unsolved problems in the "Goldbach's Conjecture". In a letter to Euler written in 1742, Goldbach observed that every even integer, excepting 2, seemed representable as the sum of two primes. As yet this conjecture is unproved, but the Russian mathematician Schnirelmann showed that every positive integer can be represented as the sum of not more than 30,000 primes. Later Kloostermann reduced this number to 6. High school students are often surprised to learn that there is any unsolved problem remaining in mathematics, particularly one that is so simply stated.

Our final question was: "Is it necessary for c to equal d?" More generally if the product $(\sqrt{\frac{a}{c}} + \sqrt{\frac{b}{d}}i)(\sqrt{\frac{a}{c}} - \sqrt{\frac{b}{d}}i) = \frac{a}{c} + \frac{b}{d} = f$, here f is an integer, does $/c/ = /d/$? At this point it was necessary to prove the following theorem: If $\frac{a}{b} + \frac{b}{d} = f$, f an integer and $\frac{a}{c}$ and $\frac{b}{d}$ are in reduced form, then $/c/ = /d/$.

This is our final result. We may then characterize a complex number that has its conjugate for a multiplicative inverse as one of the form $\frac{a}{c} + \frac{b}{d}i$, where $a + b = c^2$, $/c/ = /d/$, a and b are real, and c is an integer such that $/c/ = /d/$.

In conclusion, it can be argued that our most general result could have been arrived at in one large step rather than a series of small steps. But this would have destroyed the exact learning situation we wished to create. We should develop the attitude in the student that perhaps each problem holds a hidden pattern that can be identified and pursued relentlessly to its most general form. In this way he has the chance to taste the exhilaration of a creative feat accomplished.

- *MTA News and Views*,
Vol.2, No.2, March 1968.
NSTU (Nova Scotia Teachers' Union)
Mathematics Teachers Association

The *Annual* will be mailed to members shortly.

NEW CURRICULA

ESM at St. Vincent de Paul School, Calgary

J.P. Hazelzet

Mr. Hazelzet specializes in the teaching of science and mathematics. Comments from other teachers who have experimented with the ESM or any other series would be welcomed by the editor of this *Newsletter*.

ESM (Elementary School Mathematics) is a promising, relatively recent mathematics program specifically designed for Grades K-8.

Developed in the United States (Ball State Teachers College, Ball State University) by and under the guidance of experts in the field of mathematics teaching, ESM has received wide acclaim from those who have taught the program. Pupils also have expressed their enthusiasm for this fresh approach to the exciting world of mathematical ideas.

"Drill for the mastery of skills must be used carefully and, primarily, as a follow-up to the introduction of concepts which are approached through an understanding of the overall structure of mathematics."

The key word in this quote is "structure". No longer should the emphasis in mathematics learning be on the mastery of isolated facts. Rather, the pupil's attention is focused upon fundamental concepts which form a solid base for a logical and sequential structure in mathematics. Pupils are encouraged to think rather than memorize!

ESM's stress on mathematical structure is at once clear from the kind of mathematical content it presents: previously thought-of isolated ideas and concepts as well as facts have been integrated, "put together", into meaningful (structured) wholes or relationships. As a result, a Grade IV pupil, for example, through ESM enounters content with which he, through programs other than ESM, may well deal in later grades. In short, the Elementary School Mathematics curriculum pushes the mathematical content down through the grades. However, it presents these "advanced" concepts in an understandable and meaningful way.

Although the program originated in the USA, it has been modified to meet the needs of Canadian elementary pupils. The informative and effective pictorial material as well as its accompanying text are Canada-oriented throughout. In Canada, ESM is published by Addison-Wesley (Canada) Limited, Don Mills, Ontario.

At St. Vincent de Paul School the teachers and pupils in Grades I to VI are enthusiastic about this new approach to mathematical teaching and learning. They would like to share their enthusiasm with you.

Noose Notes, ATA Calgary Separate School Local No.55

PRACTICAL BEGINNING FOR A CHILD-CENTERED MATHEMATICS PROGRAM

J.R. MacLean
Assistant Superintendent of Curriculum
Ontario Department of Education

A report of an elementary general session at the Annual Mathematics Conference, held in Red Deer - by Mary Beaton

Dr. MacLean gave an illustrated presentation including a statement of the objectives of a child-centered program. The mathematics teacher needs to diagnose the level of understanding reached by his pupils and use appropriate activities to raise this level. At the elementary level, many materials such as cupcake tins, home-made balances, a trundle wheel and ruler can be assembled and used by the children in solving real-life problems. In one classroom, the mirror from the nurse's room was used on a table for studying notions of symmetry. The overhead projector can be used effectively by the children after they have made their own transparencies.

Ontario teachers at a workshop

The oral approach should be emphasized. Each child should have an opportunity to report to a group or to the class on how he has solved a problem or how he has constructed a polyhedron.

The pictorial approach is also useful. In one class, the children were discussing their pets. After each child had drawn a picture of his pet, the group went on to tally the number of pets of each kind represented. This led to the development of a histogram.

A group of students constructing polyhedra

If each child's work is displayed, the material on the

The overhead projector used by a student in group discussion

bulletin board will be constantly changing.

Dr. MacLean emphasized that evaluation will frequently occur during discussion with individual pupils in the child-centered approach. Samples of work can be retained for parent examination.

Dr. MacLean has visited classes where children cannot wait to get to school in the morning and have to be driven out at four o'clock. He believes enthusiasm is the greatest generator of ideas.

AN ABSTRACT OF A STUDY OF THE RELATIONSHIP BETWEEN SELECTED ACTIVITIES FOR TEACHER PREPARATION AND STUDENT ACHIEVEMENT IN GRADE IX MATHEMATICS

Morgan Johnson, Calgary

Of interest to teachers, and to those groups concerned with the education of teachers, is a continuing assessment of the effectiveness of programs which are alleged to contribute to teacher effectiveness. As yet, conclusive answers to questions in this area appear to be far away. Mr. Johnson's research attempted to come to grips with certain aspects of this problem with reference to preparation for the teaching of a specific mathematics program. Some of his conclusions confirm, and some appear to contradict, other research findings in the same general problem area. Perhaps his most encouraging finding is the positive relationship between "teacher effectiveness" and participation in professional development activities.

A.A. Gibb, Associate Dean,
Faculty of Education,
University of Calgary

The purpose of this study was to survey a selected group of Alberta teachers of Grade IX mathematics, using *Seeing Through Mathematics III* as the primary text, to determine those activities most significantly related to competence. The activities surveyed were those which contribute to a teacher's: (1) academic qualifications, (2) inservice and professional development, (3) teaching experience. All of these were analyzed to determine their relation to teacher competence as measured by student achievement on a modern mathematics test.

The data regarding these teacher activities were collected by means of a questionnaire. The criterion measure of student achievement was a test designed by mathematics educators to test the specific objectives of the Grade IX mathematics course; therefore, it included cognitive domain items testing intellectual abilities and skills as well as knowledge. Students' IQ scores were used to control initial differences in ability among the students. Both the criterion and the covariate scores were mean scores for each teacher's students.

The general method of data analysis was multiple linear regression, using an analysis of covariance design, which was performed on the IBM 360. The teacher predictor variables were included individually with the covariate in full regression models. These were compared with a restricted model which included only the covariate as the predictor. The comparison was used as a measure of prediction efficiency of the particular teacher activity.

The findings indicate that knowledge of the number of years of teaching experience and knowledge of the number of relevant professional development activities in which a teacher has participated contribute toward the efficiency of prediction of criterion scores in a significantly positive way. Knowledge of the number of years of teacher training for which the teacher was being paid and knowledge of the total number of years which a teacher has taught *Seeing Through Mathematics I* or *II* did not significantly improve efficiency of prediction at the level chosen; however, they were close enough to merit consideration. Knowledge of the number of modern mathematics courses taken by a teacher was negatively related to student achievement. Detailed analysis of this phenomenon indicated that other variables studied were peculiarly distributed. Knowledge of other variables in this study did not lead to significant increases in efficiency of prediction of criterion scores.

Questionnaire results indicate that the teachers were well experienced, particularly in the teaching of *Seeing Through Mathematics I* or *II*, that they were active professionally, and that they were well trained.

Years of teaching experience and number of professional development activities are shown to be significantly related to teacher competence; years of training and experience with other courses in a series have been shown to make some contribution. In view of further possible revisions and changes, this study indicates that educators contemplating the updating and upgrading of junior high school mathematics curricula should be concerned with these particular teaching activities.

ON SOME PROBLEMS OF TEACHING APPLICATIONS OF MATHEMATICS

Dr. H.O. Pollak, director, Mathematics and
Statistics Research Centre, Bell Telephone
Laboratories, Murray Hill, New Jersey

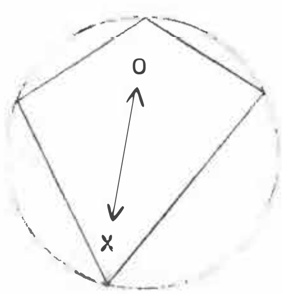
A report of a general meeting at the Annual Mathematics Conference, held in Red Deer - by Mary Beaton

Dr. Pollak examined some false folk theories about the applications of mathematics. He showed that practical applications are not limited to those involving classical analysis including calculus. On the contrary, all mathematics from elementary school level to the undergraduate level can be good applicable mathematics. He cited the application of probability to the study of population being at least as important as applications of calculus.

Another false folk theory is that mathematics can be applied only to classical physics. Although this application has had a two-thousand year record of tremendous success, over half of the applications to be found in the *Journal for the Society of Industrial and Applied Mathematics* are not in classical physics. Modern linear algebra can aid in the understanding of physical process. One should use the best mathematics available in applications. Linear algebra can indicate how accurate the pendulum formula for a grandfather's clock is if X is substituted for sin X. The clock would lose half a minute per day which is not good enough.

Dr. Pollak attacked another folk theorem, namely the idea that application problems will motivate pupils. He suggested to begin with a physical problem instead. Applications ought to be real, and the objective should be to find out how mathematics will help to understand the problem.

Also listed as a false folk theorem was the assumption that teaching with applications is a very different type of teaching. Dr. Pollak suggested that an example of letting studentsdiscover ideas for themselves could be a study of polygons in a circle. The class could first examine triangles and find that any triangle can be inscribed in a circle. After a study of quadrilaterals, it would become evident that some quadrilaterals can be inscribed in a circle whereas others cannot. The students could examine the sum of the opposite interior angles of an inscribed quadrilateral.



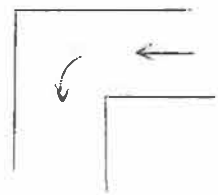
Let the students draw the diagonals; with some help, a few of the students may discover Ptolemy's theorem.

In the study of prime numbers, let the students consider whether or not they would want the number one to be prime. How do we use prime numbers? What about two? Should it be a prime?

Dr. Pollak suggested a method for helping elementary students to learn the multiplication facts. The child is asked to make up a problem whose answer is 14, then 15, and 16. When he is asked for a problem whose answer is 17, he is encouraged to analyze the difficulty in setting up such a problem. This approach can be a process of sowing seeds of understanding for later stages.

Dr. Pollak gave numerous examples of applications of mathematics in everyday life:

1. What is the largest two-dimensional object which can be taken around a square corner in a plane? This is an unsolved problem.



2. How should you rake up leaves - in parallel lines or towards a central point?

3. How large should a display counter in a five-and-ten store be?

4. How many items should be allowed in the express lane at a super market? It would be necessary to decide the purpose of the express lane. Is it to cut the waiting time of the customers? Is it to reduce the maximum wait? Is it because people refuse to wait longer than ten minutes? When you have decided the purpose of the express line, you then have a well-defined mathematics problem.

If mathematics is understood, it will be remembered better. Situations in the real world are never just like problems in a textbook. Students must know how to apply mathematics in a new situation. The purpose of mathematics is not to cover the subject but to uncover it.

MATHEMATICS COUNCIL, ATA

Table Officers, 1968-69

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