A COORDINATED REVIEW OF RECENT RESEARCH CONDUCTED IN THE DEPARTMENT OF ELEMENTARY EDUCATION UNIVERSITY OF ALBERTA RELEVANT TO MATHEMATICS EDUCATION

W.G. Cathcart and W.W. Liedtke

How do children acquire concepts? What limitation does development put on the capabilities of logical thought? What are the characteristics of the development of thought at any given age?

The answers to these questions would certainly aid in providing learning experiences and materials that are meaningful and beneficial for the learner in that they could meet his needs at various stages of development.

Increasing interest in the study of children's thought processes and the manner in which they acquire certain concepts becomes evident when some of the recent research projects undertaken in the Department of Elementary Education in the field of elementary school mathematics are reviewed.

Perhaps the greatest quantity of work in the field of cognitive development has been conducted by the Swiss psychologist Jean Piaget, who has investigated the growth of children's concepts in a variety of areas. A simple ratio reflects the influence of Piaget's work on the studies done here. Since 1963, eight out of ten investigations into teaching and learning mathematics in the elementary school have been based on his work.

Piaget has concerned himself not only with the learner but also with the substance to be learned and its structure or logical organization. His theory and the results of his investigations provide a guide to some of the experiences necessary before certain mathematical concepts are formed or acquired. His method presents a way of evaluating an individual's stage of cognitive development.

Intellectual development, according to Piaget, is the result of four contributing forces: maturation, experience, socialization, and equilibration. These interacting factors contribute to the development of the intellect, and the results of his experiments indicate that a child becomes capable of 'adult thought' or reaches maturity at the age of 11 or 12. Characteristics of the development of thought before the age of about 12 are described by Piaget in terms of limitations or in terms of 'things' children cannot do. These limitations give rise to three distinct developmental stages. They are sensori-motor (0-2 years), the pre-operational (2-7 years), and the concrete operational stage (7-11 years). (Baldwin, 1967, pp.167-309).[1]

The stages of most interest to people concerned with elementary education are the pre-operational and concrete-operational. The transition from the former to the latter stage is marked by the acquisition of various conservation concepts or the realization that properties of nature remain invariant under certain changes and transformations. The ages at which individuals acquire conservation concepts vary somewhat. However, from Piaget's

experiments a certain order is apparent: quantity (continuous and discontinuous) and number (6-7 years), mass and length (7-8 years), area and weight (9-10 years), and finally volume (11-12 years). (Piaget, 1957, pp.16-17).[2]

In his book *The Child's Conception of Number*, Piaget (1952)[3] describes the child's knowledge of, and stages of development in, such concepts as conservation of quantity, cardinal and ordinal correspondence, the properties of cardinal and ordinal number, the notion of series, the ability to classify, and the relation of classes to number. He tested what he called ordinary school children of primary school age and used a combination of observation, interview, and recording as his methods.

Brace (University of Alberta, 1963)[4] replicated some parts of the work described above. His study was designed to determine to what extent preschool children have developed some of the basic concepts of number. He tested 124 preschool children on the following specific concepts:

1. rational counting
2. comparisons - group relations
3. conservation of number
4. cardinal number
5. ordinal number
6. place value as applied to the decimal system of notation

In addition, Brace attempted to clarify the nature of the relationships between the child's knowledge of basic number concepts and other variables such as the ability to count, sex, influence of older siblings, socio-economic status or environment, and chronological age.

The most important finding of his investigation was the existence of a tremendous gap between the child's ability to count and his knowledge of the basic ideas of number. The relationship of counting to knowledge of cardinal number and the conservation of number was found to decrease with the age of the child, being less significant in older preschool children. That of counting to the child's concept of ordinal number increased with the age of the child. He found that chronological age and environment (socio-economic status) contributed significantly to the knowledge of number concepts. Sex and the influence of older siblings were not related to the understanding of basic number concepts.

In his book *The Child's Conception of Geometry*, Piaget (1960)[5] made a number of suggestions about how concepts develop out of simple behavior patterns, and how a basic repertoire of geometric concepts is formed. He described how such notions as distance, length, change of position, ideas of measurement, and conservation of distance, length, area, and volume, lead to the construction of Euclidean space.

Pelletier (University of Alberta, 1966)[6] subsequently used some of Piaget's tests to investigate Grade I children's concepts of linear measurement.

His sub-tests dealt with reconstructing relations of distance, conservation of length, measurement, and subdividing a straight line. Mathematics instruction of the 120 subjects tested was based on three different programs, and each program treated the topic of measurement differently. The three mathematics programs studied by these children were: *Numbers We Need* (Brownell and Weaver), *Seeing Through Arithmetic* (Hartung et al.), and a program based on Cuisenaire materials. In addition, Pelletier considered the simultaneous effects of sex, mental ability, and socio-economic status.

The results of the study indicated no significant difference between the three groups tested. Many of the subjects were able to verbalize measurement terms, but they did not comprehend the underlying concepts. Such words as 'distance', 'length' and 'measure' were not understood, and many subjects could not conserve length. Although no child had achieved complete mastery of the relevant concepts, children with higher mental ability were clearly superior in their understanding of linear measurement. Differences in sex and environment did not contribute significantly to the development of the concept of linear measurement.

Closely related to the construction of Euclidean space are the concepts of direction and scale. In his book *The Child's Conception of Space*, Piaget (1963)[7] deals with the child's knowledge of space and with intellectual development, that is, the development of intelligence as it operates in connection with spatial relations.

Towler (University of Alberta, 1965)[8] selected four spatial concepts that he considered necessary for map reading and interpretation as well as for the graphing in mathematics. They were:

1. the concept of a reference system,
2. the concept of distance,
3. the concept of direction, and
4. the concept of scale

He examined the development of these concepts in elementary school children. In addition, he sought to clarify the relationships between the development of the concepts and sex, socio-economic status, chronological age, intelligence, and grade level. He individually tested 120 children from Grades I to VI.

The results of his study in general indicated a development of thought processes similar to those described by Piaget. However, the development of the concepts of a general reference system and scale occurred later in children studies by Towler than in those studied by Piaget. Towler found that intelligence, chronological age, and grade level were significantly correlated with the development of the spatial concepts under investigation, while sex and socio-economic status were not.

In the elementary school mathematics program, attempts are usually made to develop and deepen concepts of time. A comprehensive report on the stages of development of time sense in young children has been made by Piaget

(1955)[9]. The results of his experiments led him to the conclusion that the development of time concepts implies the progressive structuring of perceptual data by a sequence of logical operations such as seriation and duration.

Newman (University of Alberta, 1967)[10] investigated the factors which influence the elementary school child's understanding of time duration and the stages by which such understanding is acquired. He tested 192 elementary school children from Grades I to VI on the following specific aspects of the concept of duration: understanding of verbal comparison of duration, understanding of graphic comparisons of duration, and ability to use measures of duration. Newman found that intelligence, age, grade, and socio-economic status were related in a positive way to a child's understanding of time duration. He found that comparisons of duration were generally understood at the Grade III level and measures of duration at the Grade IV level. The ability to use measures of duration was closely linked with the ability to understand verbal comparisons of duration. Grade I and II students understood verbal comparisons better than graphical comparisons of duration, but from Grade III onward both were understood equally well.

(To be continued)

Footnotes

[1]Baldwin, A.L., *Theories of Child Development*. New York: John Wiley and Sons, Inc., 1967, pp.167-300.

[2]Piaget, J., *Logic and Psychology*. New York: Basic Books, Inc., 1957.

[3]_____, *The Child's Conception of Number*. London: Routledge and Kegan Paul, 1952.

[4]Brace, A.T., "The Pre-School Child's Concept of Number". Unpublished Master's Thesis, University of Alberta, Edmonton, 1963.

[5]Piaget, J., B. Inhelder and A. Szeminska, *The Child's Conception of Geometry* (translated by E.A. Lunzer). New York: Basic Books, Inc., 1960.

[6]Pelletier, J.D., "A Study of Grade One Children's Concept of Linear Measurement." Unpublished Master's Thesis, University of Alberta, Edmonton, 1966.

[7]Piaget, J., and B. Inhelder, *The Child's Conception of Space*. London: Routledge and Kegan Paul, 1963.

[8]Towler, J.O., "Spatial Concepts of Elementary School Children". Unpublished Master's Thesis, University of Alberta, Edmonton, 1965.

[9]Piaget, J., "The Development of Time Concepts in the Child", *Psychopathology of Childhood*. P.H. Hook and J. Zubin (eds.). New York: Grune & Stratton Inc., 1955, pp.34-44.

[10]Newman, W.O., "Children's Understanding of Time Duration". Unpublished Master's Thesis, University of Alberta, Edmonton, 1967.