WHAT IS ADDITION?

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What makes *modern math* different from the *traditional*? In a nutshell, it is that we attempt to clarify simple concepts and then build around them.

One such example is addition. Many textbooks and many teachers are not at all clear as to what the concept of addition is. Is it a sort of "joining together", a "combining action"? Is it a physical kind of action, or is it purely mental abstract? If the teacher pushes two groups of objects together to form one group, is this really the operation of addition? The answer is a definite "no".

We should not mention this except for the fact that when we ask students to add, for example, "a" and "b", they are completely bewildered. It is at these points that we separate the Black Sheep from the White in junior or senior high, and largely because the Black Sheep really were never taught the concept properly. Let's illustrate.

Three men stood at the edge of a pond. They saw a group of four mallards swimming around, and in a distance was a group of three pintails.

The first man was a Biologist. His mind was conceptualizing the difference in the species. As the two groups swam around, at one time they actually intermingled. Did this joining action constitute addition? Not at all. The biologist may have been thinking something similar to this:

M = X X X X	P = 0 0 0
A set of mallards	A set of pintails

No mathematical operation was taking place in his mind.

The seond man was a Hunter. He was thinking of a delicious Sunday meal for his family. What mathematics may have been going on in his mind when the two groups intermingled?

If his mental vision consisted of a platter of four baked mallards and three baked pintails, then, of course, no mathematics was going on in his head beyond that which the biologist was thinking. It is more likely, however, that the hunter was thinking that there would be one duck per each member of the family. In this case, we can program his math thoughts as follows:

(1) $M = \{X\ X\ X\ X\}$,	$P = \{0\ 0\ 0\}$	Disjoint sets
(2) $M = \{\overset{d}{\cancel{X}}\ \overset{d}{\cancel{X}}\ \overset{d}{\cancel{X}}\ \overset{d}{\cancel{X}}\}$,	$P = \{\overset{d}{\cancel{0}}\ \overset{d}{\cancel{0}}\ \overset{d}{\cancel{0}}\}$	Mallards, Pintails $\longrightarrow$ ducks
(3) $MUP = \{d\ d\ d\ d\ d\ d\ d\}$		Union of disjoint sets
$\quad\quad \updownarrow\ \updownarrow\ \updownarrow\ \updownarrow\ \updownarrow\ \updownarrow\ \updownarrow$		One-to-one corresondence
(4) $\quad = \{a\ b\ c\ d\ e\ f\ g\}$		with members of family

The third man was a Warden. His chief concern was to get a game count of water fowl in his district. He took out a little book and put down a mark. What math went on in his head?

The first three steps above were probably the same as for the hunter, but he likely went one step farther, omitting (4).

(5)	$MUP = \{1\ 1\ 1\ 1\ 1\ 1\ 1\}$	Physics to math, identity element of multiplication
(6)	$= 4 + 3$	Addition algorithm
(7)	$= 7$	Simplification (Arabic)

We should note well that the addition has really taken place in step (3) of the hunter's train of thought, except that he did not simplify as did the warden, because he did not need to simplify. It is important to understand that the two groups did not have to come together. The abstract idea of addition could have taken place regardless of the group placement.

The common fallacy in our teaching is to think that the physical combination was necessary and that addition is really step (7). This step is merely simplification. The algorithm 4 + 3 is the expression of addition as having taken place.

If children are clearly taught this concept of union of disjoint sets, they will not be bewildered by the algorithms "a + b". In other words, "a" add "b" is "a + b", period.

The implications for teaching addition in this manner will be, of course, that we need not waste page after page in textbooks and teacher's time in pictures of physical joining or separation. Hard to believe? Well, think about it.

"He who helps a child helps humanity with a distinctness, with an immediateness, which no other help given to human creatures in any other stage of their human life can possibly give again."

- Phillips Brooks