THE ALBERTA TEACHERS' ASSOCIATION

MATHEMATICS COUNCIL

NEWSLETTER

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Contents

Newsletter Editor: Mary Beaton
Faculty of Education, University of Calgary

DEAR FRUSTRATED TEACHER,

Educators in Alberta are being faced with a new kind of freedom and, consequently, must be prepared to accept the accompanying responsibility. I, too, heard the rumors, read the handbook and fearfully felt I was being launched into an unfamiliar orbit. Openly I asked, "What is this academic option? How will it affect the existing program? Where will I find a curriculum and texts?" Secretly I wondered if I had the courage to free my students to explore any mathematical world that seemed inviting to them. Could I provide the necessary stimulation and environment?

The first glimpse of this new horizon was a little frightening, but by looking at the broader spectrum I began to see the opportunities provided by the option. Now we could explore a whole galaxy of topics as well as experiment with new techniques.

Initially I felt I must do some serious thinking about the purpose of offering a mathematics option or, for that matter, of teaching mathematics at all. Students of today will not find society as we know it. We must try to produce capable citizens for tomorrow - citizens who can live in a highly mechanistic, rapidly changing, increasingly demanding world. The brief period they spend in school can serve only as a primary launch vehicle for projecting them into an increasingly complex society. As teachers we must bear our share of the responsibility to help them reach this destination.

This sounds great, doesn't it? Now I should offer some ideas. Recently I had the opportunity of attending the NCTM Annual Meeting in Minneapolis. Of the many worthwhile sessions attended, those that illustrated the changing techniques and philosophy of teaching mathematics seemed to generate the most enthusiasm. Just to gain admittance to any of the sessions led by Dr. Viggo Hansen or Miss Edith Biggs was no small undertaking!

Dr. Hansen, San Fernando Valley State College, Northridge, California, discussed the steps in planning, developing and funding a multicomponent mathematics laboratory. Those of us who were fortunate enough to get into this session were able to participate in a simulated math lab situation. Slides of operational labs revealed happy, enthusiastic children actually enjoying mathematics.

Here was a practical idea. The cost would be minimal and the approach would be different. If students enjoy it as much as we did, your problem as a teacher will be to get them to leave at the end of a period.

The best news of all is that Dr. Hansen has accepted an invitation to speak to Alberta teachers this fall. He will be attending the Annual Meeting of the Mathematics Council in Edmonton, September 26 and 27. I plan to hear him again and hope to see you there.

Miss Edith Biggs, Her Majesty's Inspector of Schools, London, England, led three sessions. Miss Biggs emphasized the new approach to mathematics beautifully by her opening statement that mathematics is like a kiss - one couldn't really judge its merits until one has participated in it. *Participate* is certainly the key word!

Interpreting and recording statistical data was the topic of one session. Such fascinating questions as "How do you compare to an elephant?" and "Are you a Square?" proved to contain some interesting mathematics. Two thoughts struck me as we worked with these problems: first, how stilted we all had become from so many years of being passive receivers rather than active participators; secondly, what a wealth of ideas emerged as we, participators in the lab, gradually became so involved that we forgot our fears of appearing ridiculous or being wrong.

Unless you had the opportunity to attend some of these sessions, a detailed description would be of little value. However, there is an open invitation to enjoy the previously mentioned session along with many others. Your admittance depends on your ability to beg, buy or borrow a copy of *Freedom to Learn*. Addison-Wesley (Canada) Ltd. publishes the book and the authors are - you guessed it: Edith Biggs and James MacLean (the latter is Assistant Superintendent of Curriculum, Ontario Department of Education). Do you recall the 1968 MCATA Conference in Red Deer? Perhaps you were among those who heard Mr. MacLean speak on this topic.

Whether or not you heard either of the authors speak, I know you will enjoy *Freedom to Learn*. I hope my excitement over these two sessions has at least pricked your curiosity.

Sincerely,

Bernice Andersen

Mrs. Andersen, secretary of MCATA, is mathematics consultant with the Secondary Department of the Calgary School Board.

The *Journal of Experimental Education*, Fall, 1968, contains a reprint of the book *Research and Development Toward the Improvement of Education*. The issue is divided into five parts: Conditions and Processes of Learning, Subject Matter, Content and Sequence, Instructional Methods and Teacher Behavior, and the Current Scene. Of particular interest to the mathematics teachers are Chapter 5, "Curriculum Research in Mathematics", by E.G. Begle; "Strategies for Concept Attainment in Mathematics", by M.F. Rosskopf; and "Socrates, A Computer Based Instructional System in Theory and Research", by L.M. Stolurow.

OPEN SEARCH IN ELEMENTARY MATHEMATICS EXPERIENCES

T.P. Atkinson

Mr. Atkinson, former editor of the MCATA Newsletter, is Associate Professor in Elementary Education at the University of Alberta.

> The aim of elementary school mathematics is to foster continuous and maximum development of each child's potentialities in terms of the affective domain, the cognitive domain and the psychomotor domain.[1]

The teacher is challenged to be aware of the behaviorial domains and the child's need for growth in each of them. My purpose is to examine Bloom's plan for classifying educational objectives within the cognitive domain[2], to examine a modification of Bloom's taxonomy suggested by Avital and Shettleworth[3], and to make some suggestions that the teacher can follow. In a sense my purpose, on a small scale, is the same as that stated by Sanders[4], who used the social studies as the vehicle for the presentation of his ideas.

> The objective of this book is to describe a practical plan to insure a varied intellectual atmosphere in a classroom. The approach is through a systematic consideration of questions that require students to use ideas, rather than simply to remember them.[5]

Bloom's taxonomy postulates six major levels of educational objectives, the categories being named Knowledge, Comprehension, Application, Analysis, Synthesis, Evaluation. The categories are assumed to be arranged sequentially, in that performance in any given category depends upon mastery of related materials in the preceding categories. It is also understood that by virtue of different degrees of experience, two persons may place an objective into different categories.

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[1]Department of Education, Government of Alberta, *Elementary Mathematics*, September 1969. Edmonton, Alberta: The Queen's Printer, 1969.
(This is a preliminary statement in the process of publication, excerpts of which will form the mathematics section of the Program of Studies for the Elementary School.)

[2]Bloom, Benjamin S. (ed.), *Taxonomy of Educational Objectives*, Handbook I: Cognitive Domain. New York: David McKay Co., 1956.

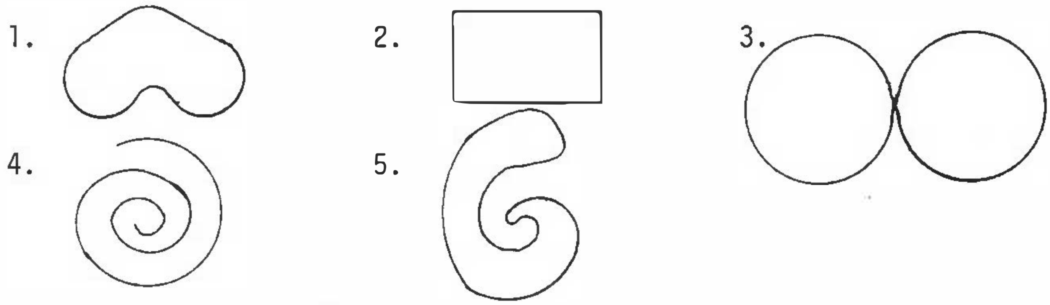
[3]Avital, Samuel M., and Shettleworth, Sara J., *Objectives for Mathematics Learning*. The Ontario Institute for Studies in Education, Bulletin No. 3, 1968.

[4]Sanders, Norris M., *Classroom Questions: What Kinds?* New York: Harper & Row, 1966.

[5]Sanders, *op. cit.*, p.2.

A set of multiple-choice questions, each applicable at some stage in the elementary school, is presented to illustrate the first five categories. The level Evaluation is omitted because it is difficult to illustrate with mathematics as the vehicle.

Knowledge — Which of the pictures does not show a closed curve?



Comprehension — If 16 x 16 = 256, which product is equal to 25,600?

1. 160 x 160   2. 16 x 160   3. 256 x 10

4. 160 x 16   5. 530 x 520

Application — A merchant sold canned corn at 2 cans for 35 cents on Friday and at 3 cans for 50 cents on Saturday. How much would a shopper save if she bought 12 cans at the cheaper rate?

1. 20 cents   2. 10 cents   3. nothing

4. 15 cents   5. 5/6 of 1 cent

Analysis — A gardener has two plots of land of equal area. One is square and the other is rectangular with its length four times its width. What is the ratio of the two perimeters?

1. 5/4   2. 5/1   3. 1/1

4. 3/4   5. 3/1

Synthesis — Examine the pattern in these number sentences.

1 + 3 = 4
1 + 3 + 5 = 9
1 + 3 + 5 + 7 = 16
1 + 3 + 5 + 7 + 9 = 25

What is the last number in a set of this kind whose sum is 400?

1. 20   2. 39   3. 41

4. 72   5. 100

Avital and Shettleworth identify three levels of mathematical thinking which they associate with objectives that fit the five levels in the taxonomy. In tabular form they are[6]

Thinking Process	Taxonomic Level
1. Recognition, recall	1. Knowledge
2. Algorithmic thinking, generalization	2. Comprehension, application
3. Open search	3. Analysis, synthesis

It is the third thinking process that gives me the title to my presentation.

Among the mathematical experiences provided for elementary school children, all three thinking processes must be developed and practised. It is essential that children rocognize and recall mathematical concepts, facts, terms and symbols. They must be able to develop, understand and use algorithms; they should be asked to generalize from data. However, we teachers fail to provide enough opportunities for open search. We create the impression that in mathematics there is one correct answer for a question and the pupil's sole responsibility is to learn techniques for producing that answer. We attach considerable importance to the one technique we think is the best and tend to downgrade all others.

In open search the pupil should be permitted, encouraged, forced to use his ingenuity, and praised when he does so. In the synthesis example quoted earlier, the child who recognizes the pattern and simply writes the terms of the series, keeping a running total until the sum reaches 400, is using all three thinking processes. Do not worry about more sophisticated techniques; they will be developed in due course.

In open search there may be assumptions to be made, differences of opinion to be discussed, incomplete information to be supplemented. Consider the problem:

> A box contains steel bars, some weighing 4 pounds each and others weighing 6 pounds each. The total weight of the box and its contents is 50 pounds. If you removed all of the six-pound bars, what would the box and its contents weigh?

Let two or three alert children at any grade level from two up wrestle with the problem. If they have not been too severely conditioned by the mathematical training they have encountered to date, their solutions should be interesting.

It is difficult to provide a group of teachers with problems that meet the dual criteria of being applicable to the elementary school and demanding the open search. I have prepared three activities for you to consider. You will find these in a future issue of the Math Bin.

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[6]Sanders, *op. cit.*, pp. 6,7.

" A DAY OF MATH" IN EDMONTON

Sponsored by the Edmonton Area Elementary Regional
of the Mathematics Council

Saturday, May 3, 1969, was the second annual "Day of Math"; it took place at Elmwood School in Edmonton. Lynn Fossum, president of the Edmonton Elementary Regional, extended greetings to those in attendance. The keynote speaker was Tom Atkinson, an associate professor at the Faculty of Education, University of Alberta. We have reproduced in this Newsletter (page 4) his paper on the topic he chose to present: "Open Search in Elementary Mathematics Experiences." Following the Annual Meeting of the Edmonton Elementary Regional of the Mathematics Council, a smorgasbord was enjoyed at the Golden Gate restaurant.

In the afternoon a workshop display included teacher-prepared mathematics activities. Demonstrators assisted visitors in constructing activities of their choice.

Victor Comptometer Company representatives demonstrated electric calculators for use in the elementary school. Addison Wesley Company displayed mathematics texts and supplementary materials. Various aids were also displayed by Moyer Division of Vilas Industries.

The Edmonton Elementary Regional - the largest regional of the Mathematics Council and one of the most active regionals of any specialist council - was organized late in 1967. Membership is automatic to members of the Mathematics Council in the Edmonton area. The purpose of the Regional is to improve the teaching of mathematics at the elementary school level through inservice sessions, workshops and seminars. For further information call

Mr. Lynn S. Fossum
429-5621, local 317
or 434-2671 (residence)

CURRENT RESEARCH IN ELEMENTARY SCHOOL MATHEMATICS SUMMARIZED

The Association for Supervision and Curriculum Development of the NEA has released the third edition of its research monograph entitled *Elementary School Mathematics: A Guide to Current Research.*

Similar to prior editions, the authors attempt to identify questions of primary concern to school personnel, to examine the literature for relevant studies, and to summarize the studies with the object of determining their applicability to making a change for the better in the classroom.

This 128-page booklet can be obtained, for $2.75, from the Association of Supervision and Curriculum Development, 1210 Sixteenth Street, N.W., Washington, D.C. 20036.

THE MARCH SYMPOSIUM OF MATHEMATICS AT THE UNIVERSITY OF CALGARY

On March 8 the Department of Mathematics, University of Calgary, sponsored a Joint Senior High School - Mathematics Department Symposium on the campus. Dr. B.G. Wilson, Dean of the Faculty of Arts and Science, welcomed the participants. A panel discussion on the topic "The Content and Teaching of Beginning Calculus" followed. The panel members were Mrs. Carol Armour, mathematics specialist; A.A. Gibb, associate dean of the Faculty of Education; A.S.B. Holland, associate professor of mathematics; and J.E.L. Peck, professor of mathematics (computing science). Another highlight of the program was a talk by R.K. Guy, head of the Mathematics Department, entitled "Seeing Mathematics".

After a luncheon in the Blue and Gold Dining Room, the following three group meetings were held:

- <u>Geometry Symposium,</u> "Prerequisite Geometry for Calculus and Mathematics Majors"
- <u>Computing Science</u>
- <u>Applied Mathematics Symposium,</u> Problems in teaching matrices, vectors, linear algebra and trigonometry

A NOTE OF APPRECIATION

From the Henry Wise Wood Senior High School Mathematics Staff Participating in the Symposium

The members of the Mathematics Department of this school who attended the symposium wish to express their gratitude for the most pleasurable and informative meeting. It is felt that such a meeting has greatly enlightened the members and has clarified many questions. Meetings of this caliber are much too rare. It certainly would be of great value to many high school instructors if such symposia were held at least twice a year. Most unfortunately many questions have since come to mind - questions which, according to Henry Wise Wood members, could make a great difference to course planning and effective teaching. As a group, the members feel that the meeting was worthwhile, most informative and certainly much too short for the number of activities scheduled. The only disappointing item was the fact that each member was unable to attend all of the functions. We certainly hope that there will be another symposium in the near future, and one which will allow all participants to attend all the functions.

Joan Haden
Helen Tywoniuk
Jim Oliver
Dino Pagnucco

Plan to attend the MCATA Annual Mathematics Conference & Business Meeting next fall!

Friday evening & Saturday, Sept. 26 & 27/69

Barnett House, Edmonton

INTERESTING IDEAS, INFORMATION, INSPIRATION

SPEAKERS -

- *Dr. Julius Hlavaty, president of the National Council of Teachers of Mathematics, New Rochelle, New York. (Dr. Hlavaty is well known for his wit and humor, as well as the high quality of his presentations. He spoke at the first Annual Meeting of MCATA 10 years ago.*
- *Dr. Bruce Harrison, The University of Calgary.*
- *Dr. Viggo Hansen, San Fernando Valley State College, Northridge, California.*
- *Mrs. Joan Kirkpatrick, The University of Minnesota (returning to the University of Alberta).*
- *Ron Radomsky, Calgary Public School Board.*

You need not be a member of the Mathematics Council, ATA, to attend. Registration fee is $3 for non-members.

A detailed program will be mailed to MCATA members and to all schools in September.

RELATED ACTIVITIES

Werner Liedtke

Mr. Liedtke, a former teacher in Edmonton, obtained his M.Ed. at the University of Alberta in 1968. He is at present in his first year of a doctoral program in Elementary Mathematics Education at the University of Alberta.

The following questions will probably sound familiar to mathematics teachers. *Is drill necessary? How much drill is needed?* These questions are frequently asked, and they are answered in varying ways. One argument often presented is that in order to teach mathematics in the upper elementary grades, the pupils have to know the basic facts. If they do not know these facts, some means must be found for teaching them.

A possible method of presenting or reviewing basic facts might be in the form of presenting numerous related activities. These activities could be a game, puzzle, or a search for some pattern or relationship. In this way pupils can apply what they have learned or what has been presented to them, and at the same time they practise mathematical skills.

As an example, let us consider some multiplication facts. Suppose the discussion is centered around the basic facts dealing with 7, 8, and 9. The following activities could be presented to the pupils.

The Adding Wizard: Ask one of the pupils to dictate four (or more) numerals consisting of four (or more) digits. Write these numerals on the board. For higher grades it might be necessary to state the following restriction $1,100 < n_i < 9,000$ for $i = 1,\ldots,4$. Their given numerals might look similar to this:

1,263
4,569
3,276
2,892

Now tell the pupils that you want to show them how quickly you can add, but first you are going to match these numerals with four others. ("Adding four is too easy, I'll make the problem more difficult.") Each digit is matched in such a way that the sum of any two digits is nine. This results in the following four numerals:

8,736
5,430
6,723
7,107

Since there are four nines in each column, the sum 39,996 can be written on the board without much hesitation. Is the answer correct? Most students will want to check it, and they add the columns willingly.

Depending on the grade level, one of the following things could be done next: Why is the answer correct? You could show them the numerals that were matched. How were they matched? Tell the students that you arrived at the sum by using the 9 - times table. Ask them to find out how many nines divide into that sum.

Once the matching pattern is discovered, the following problems could be presented to the class: For a similar trick, how many numerals would your friend have to write down and how many would you have to write down to yield a sum of 29,997? Suppose you have the sum 4,995 in mind. How many digits would there have to be in the numerals your friend writes down? How many of these numerals would you ask him to write down? Similarly, how many for 59,994; 7,992; and so on. Could the same procedure be used for multiplication facts with the 8 and 7 (or even 6)? What restrictions would you have to state or make before your friend dictates his numerals to you?

Finally, a suggestion for a homework assignment that almost every pupil will complete: Ask your students to "impress" their friends or parents by showing them how quickly they can add. (Challenge Dad to an adding match for 10¢).

An example of related activities was presented. There exist many others. Is this drill? Well, it is practice with a purpose and perhaps with the same or even better results.

THE ONTARIO JUNIOR MATHEMATICS CONTEST

The total number of competitors in the 1969 Ontario Junior Mathematics Contest was 17,171, representing 791 schools. The *second highest score* in Canada was made by *L.W. Tu* of Harry Ainlay Composite High School in *Edmonton*. *Five other Alberta students made the Canadian Honour Roll*. These were R. *Liknaitzky*, Jasper Place Composite High, Edmonton; *R.J. Nowakowski*, St. Francis High School, Calgary; *D.N. Williams*, McNally Composite High School, Edmonton; *D.E. Anderson*, Paul Kane High School, St. Albert; and *L.R. Custead*, William Aberhart High School, Calgary. The individual champion is Brian Calvert of G.A. Wheable Secondary School, London, Ontario.

In the Canadian Team Competition, the championship was won by Sir Winston Churchill Secondary School, Vancouver. The Harry Ainlay Composite High School team from Edmonton was fifth in the Canadian Team Honour Roll, which included 41 schools across Canada.

Eighty-one students were listed on the 1969 Alberta Provincial Honour Roll for the Junior Contest. Congratulations are extended to all those who participated in the contest.

A COORDINATED REVIEW OF RECENT RESEARCH CONDUCTED IN THE DEPARTMENT OF ELEMENTARY EDUCATION UNIVERSITY OF ALBERTA RELEVANT TO MATHEMATICS EDUCATION

Part III

W.G. Cathcart and W.W. Liedtke

There are some practical applications of Piaget's theory for mathematics education. Piaget says that thinking is an internalized action which has been developed as a result of physical actions. Therefore, it is important that children be given an opportunity to actively explore and manipulate physical objects in their environment. This implies that manipulative materials should be an important part of the mathematics program.

A recent study by Scherer[1] (University of Alberta, 1968) examined the role of manipulative materials in problem-solving at the Grade III level. Four classes were given concrete materials consisting of 50 counting sticks, 50 little tile squares, and 50 rectangles. Four other classes were chosen as a control group. Verbal problems presented consisted of the six types of additive and subtractive problems as outlined by the *Seeing Through Arithmetic* program. The experimental group used the concrete materials to represent the problem situations, while the control classes only discussed the problems. The treatment covered eight lessons at the rate of one lesson a day.

Scherer found that the use of manipulative materials was no more effective in developing ability to solve verbal problems than the use of discussion and printed materials. However, a number of methodological weaknesses may account for this lack of significant gain by the experimental group. The major weakness was the short duration of the experiment. Eight days may be an inadequate length of time in which to observe change. Furthermore, Grade III students may have passed the age when concrete materials are most useful, at least for the type of problems they were working with. We still must maintain with Almy[2] (1966) that Piaget's theory strongly suggests the importance of learning through activity. Certainly this is an area that requires more research.

From the research conducted in the Department of Elementary Education, a number of practical applications have arisen out of the concept of conservation. There is the need for teachers to be aware of the child's status with respect to conservation as it may be a clue to the cause of many difficulties. For example, teachers could be guilty of classifying a child as dull when, in

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[1]R. Scherer, "Manipulative Materials in the Teaching of Problem Solving". Unpublished Master's thesis, University of Alberta, Edmonton, 1968.

[2]M. Almy, with E. Chittenden and P. Miller, *Young Children's Thinking*. New York: Teachers College Press, Teachers College, Columbia University, 1966.

actual fact, he does not yet conserve the properties of the subjects that are being examined. This suggests that there may be a close relationship between achievement in mathematics and conservation.

Reimer[3] (University of Alberta, 1968) gave a test of conservation of number, quantity, and length to a group of 81 Grade I subjects. He found that both intelligence and conservation were closely related to achievement on the *Seeing Through Arithmetic* test; furthermore, achievement on this test was highly predictable for conservers - that is, conservers generally had high achievement scores. However, the achievement of non-conservers could not be predicted on the basis of their conservation scores.

Another implication of the principle of conservation is that it may be affected by experiences unique to certain children. Liedtke[4] (University of Alberta, 1968) studied the conservation of distance and length in a sample of 50 bilingual and 50 monolingual Grade I students. A bilingual student was defined as a student who could converse as well in French as in English. Liedtke found that the concept of conservation of length was developed to a greater extent in the bilinguals, who also demonstrated a greater ability to make linear measurements. Perhaps there exist other unique experiences in our cultural setting that may affect the ability of young children to converse.

A practical application arising out of Pelletier's[5] (1966) study of Grade I children's concepts of linear measure is that the ability to verbalize measurement terms or identify measuring instruments is no assurance that the child understands the underlying concepts of linear measures. The implication is that before we teach a child to use a ruler to mark off inches, we need to be sure he understands the underlying principles such as transitivity, segmented length, and subdivisions.

A somewhat similar implication results from Brace's[6] (1963) study of the preschool child's concept of number. He found that young children tend to confuse spatial relationships with number. Therefore, they must be given experiences with, and manipulation of, physical objects so that they can abstract the true concept of number apart from the physical properties of the objects. A variety of objects needsto be used in developing the concept of number so that number will not get confused with a particular shape, color,

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[3]A. Reimer, "A Study of First Grade Mathematics Achievement and Conservation." Unpublished Master's thesis, University of Alberta, Edmonton, 1968.

[4]W.W. Liedtke, "Linear Measurement Concepts of Bilingual and Monolingual Children." Unpublished Master's thesis, University of Alberta, Edmonton, 1968.

[5]J.D. Pelletier, "A Study of Grade One Children's Concept of Linear Measurement." Unpublished Master's thesis, University of Alberta, Edmonton, 1966.

[6]A.T. Brace, "The Pre-School Child's Concept of Number." Unpublished Master's thesis, University of Alberta, Edmonton, 1963.

or size. This may point to one major weakness in teaching devices such as Cuisenaire rods, at least in the early grades.

Another study done in mathematics education in this department that does not fit the Piagetian framework deserves mention.

Seward[7] (University of Alberta, 1966) attempted to determine if exposure to the mathematical concept of ratio in Grades V, VI, and VII affected students' success with tasks involving verbal analogies. Seward formed five groups on the basis of the amount of exposure to ratio instruction. The groups were

1. ratio instruction in Grades V, VI, and VII

2. ratio instruction in Grades VI and VII

3. ratio instruction in Grades V and VI

4. ratio instruction in Grade VII only, and

5. no ratio instruction.

Each group was subdivided into three groups on the basis of intelligence, making a total of 15 groups. The only significant difference in achievement on a verbal analogies test was within the five groups of average intelligence. Using a Newman-Keuls comparison of ordered means, Seward found that the group who had ratio instruction in Grades V, VI, and VII, and the group who had it in Grade VII only did significantly better than the group who had no formal ratio instruction. A non-statistical examination of the data enabled Seward to conclude that the study of ratio enables students of average intelligence to perform almost as well as high intelligence students on a verbal analogies test. In contrast, the average student who had no ratio experience did little better than the student of lower intelligence.

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[7]R.K. Seward, "Relationship of Mathematical Ratios to Verbal Analogies." Unpublished Master's thesis, University of Alberta, Edmonton, 1966.

SOME NEW MATH IN OLD RUTS

H.L. Larson

Mr. Larson is assistant superintendent of schools in Red Deer.

One of the pedagogical dangers we face in promoting new methods of teaching mathematics is that of falling into narrower and deeper ruts than our predecessors did. The temptation to immortalize a particular method that appears at first to be "the" means to an end is indeed very great. In so doing we may lose sight of many implications of the new method as well as shut out other refreshing ideas for handling the same problem.

A good example is the so-called rate-ratio method of handling percentage problems. One of the authorized texts for Alberta has accented this method to such an extent that many teachers and students alike have almost made a fetish of it. Mentioned below are some of the implications:

1. Whenever percentage problems appear, especially in the physical sciences, students begin to set up a proportional equation - almost without thinking. Unfortunately some problems do not lend themselves easily to this form of equation, and many students guess rather wildly the position of the "unknown" symbol.

2. The computational bogey of "cross multiply" is still very much with us, despite certain refinements (Comparative or Trichotomy laws of order in Rational Numbers) which only a few teachers seem to be able to get across to their classes. We still find many students "cross-multiplying" in algorithms of binary operation as well as proportional equations.

3. It would appear that many students also are so well "trained" in solving proportional equations that when they meet other types for the first time, they have no method of attack at all - for example, the $d = rt$, $V = lwh$ or $I = prt$ types. Teachers of science are especially critical of students who have been steeped in this rather singular modus operandi.

To briefly conclude what could be a rather long article, may I suggest a comparison between the rate-ratio-proportional method and *another* method in handling one of the more difficult of the three types of percentage problems.

Problem. If I still have \$50 left after having started with \$200, what % did I save?

Rate-Ratio Method. We may reduce this to the form of \$50 = n% of \$200. Thence $\frac{50}{200} = \frac{n}{100}$. Or we may symbolize the problem directly by thinking "50 per 200 is equal to n per 100". The computation to close this sentence is as follows:

$$\frac{50}{200} = \frac{n}{100}$$

$$200n = 5000$$

$$n = \frac{5000}{200} = 25$$

However simple this may seem, we find many students getting the "answer" n = 25%. This seems ample proof that the total operation has been mechanical. If the teacher insists that the student interpret the answer in the light of the original problem, one hears all kinds of agonizing sounds coming from students. In fact some teachers do not seem to realize that the "answer" n = 25% is indeed very wrong.

Another Method.

$50 = n\%$ of $\$200$

$50 = n \cdot \frac{1}{100} \cdot 200$ — % is a special fraction $\frac{1}{100}$

$50 = n \cdot \left(\frac{1}{100} \cdot 200\right)$ — Assoc of mult.

P F F — Multiply but don't reduce yet

$50 = n \cdot \frac{200}{100}$ — Using inverse form equation

P F F

$50 \div \frac{200}{100} = n$

$50 \times \frac{100}{200} = n$ — Inverse of division

$25 = n$ — Simplification

Most of the steps in the reasoning are shown here, but the student soon learns the shortcuts, based upon solid deductvie logic.

Note that the meaning of % is utilized in step 2; consequently, there is no repetition in the final result. Another advantage of this method is that only the basic *field* laws are used, and this is in keeping with the modern psychology of the "power of conceptual thinking".

However, the most important implication is that this method provides a modus operandi for a whole range of equations such as

$$V = lwh$$

$$150 = 10.w.3 \qquad \text{Given V, l and h. Find w}$$

$$150 = 10.3.w \qquad \text{Comm. of Mult.}$$

$$\text{P} \quad \text{F} \quad \text{F}$$

$$150 = 30 \,.\, w$$

$$\text{P} \quad \text{F} \quad \text{F}$$

$$150 \div 30 = w \qquad \text{Inverse form of equation}$$

etc.

May we conclude that while rate-ratio "method" of solving some problem situations is extremely useful, it can also be a narrow and even dangerous device if it becomes the only strategy.

INNOVATIVE MATERIALS AVAILABLE

Cynthia Parsons

Reprinted from *The Christian Science Monitor*, February 15, 1969

Computer-based Mathematics Instruction, Ventury Hall, Stanford University, Stanford, California.
Math drill from a computer. Visitors welcome with advance notice.

Computer Training and Use in Secondary Schools, Kiewit Computation Center, Dartmouth College, Hanover, New Hampshire.
Eighteen schools and 3,600 students learning to use the computer as an aid in math, natural science, and social science. Some free materials.

Education Development Center (EDC), 55 Chapel Street, Newton, Massachusetts.
Vast enterprise in curriculum development in the United States and abroad. Ranges from preschool to graduate school. Free annual report and catalogue.

Educational Facilities Laboratories (EFL), 477 Madison Avenue, New York, N.Y.
Tremendously inventive nonprofit corporation coming up with dramatic solutions to school and college building problems. Some free booklets.

Mount Hermon Summer Schools, Mount Hermon, Massachusetts.
Experimental programs for academically-talented boys and girls who have completed Grade X.

International Clearinghouse on Science and Mathematics Curricular Development, Science Teaching Center, University of Maryland, College Park, Maryland.
Prints a yearly report on all major math and science curriculum centers. Send 25 cents to cover cost of mailing.

Minnesota Mathematics and Science Teaching Project (MINNEMAST), 720 Washington Avenue SE, Minneapolis, Minnesota.
Coordinated science and math for K-6 teacher-training material. Free quarterly reports.

Nuffield Foundation Science Teaching Project, Pulton Place, Fulham, London, SW 6, England.
Science materials for teachers of 12 to 17-year-olds. Free reprints of working papers.

Nuffield Mathematics Teaching Project, 12 Upper Belgrave S., London SW 1, England.
Some very exciting math ideas and some excellent films for children age 5-13. Most of the materials is aimed at teaching teachers how to be better teachers.

Madison Project, Mathematics Department, Smith Hall, Syracuse University, Syracuse, N.Y.
Special materials,films, tapes, toys, worksheets, booklets for teaching Madison algebra and other math to younger children. Some free reprints.