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Jim Kean, President of MCATA, was editor of the Annual for 1968-69. Jim is Head of the Mathematics Department at Lord Beaverbrook High School, Calgary.

## FROM THE PRESIDENT

DEAR MCATA MEMBERS,

I would like to take this opportunity to thank all members of the 1968-69 Executive Committee for their efforts and contributions in making last year one of the most successful Mathematics Council history.

The annual conference was an example of the thought and effort that went into the activities of last year.

I hope that in this new year the Math Council will continue to grow as it has in the past. Participation of members in the Council's activities and programs will be the main theme of the 1969-70 term. I want to see strong regional councils become the major activity of MCATA. Plans are already being made to organize two more regional councils in the Peace River area and in Calgary. Members in other areas who are interested in promoting regionals should send a letter to the Executive Committee.

in this is	5 S U 8
FROM THE PRESIDENT - James A. Kebo	1
THE NEW NEW MATHEMATICS - Dr. Julius H avaty	2
A MATHEMATICS LABORATORY IN GRADES VII AND VIII - Lames H. Yance	9
TEACHING MATHEMATICS A letter to the Globe and Mail - R.A. Staat	11
CANADIAN UNIVERSITY SERVICE OVERSEAS	12
REFERENCES FOR MATHEMATICS 15 ANO 25 - Marshal Eye	14
CURRICULUM NEWS	14
BOOK REVIEW - A.S.B. He land	15
TO KEEP YOU INFORMED	16

James A. Kean

Newsletter Editor: Mary Beaton Faculty of Education, University of Calgary



Dr. Julius H. Hlavaty

The highlight of the 1969 Annual Math Conference in Alberta was the following address by Dr. Hlavaty, president of the National Council of Teachers of Mathematics. It was recorded by Jim Kean, president of MCATA.

Today I should like to talk about three things: first, very briefly, about the old mathematics (the bad old days); second, where we are today (probably labelled the new mathematics); and, last, "Where do we go?" (and that would be the new, new mathematics).

You have heard the old days described often enough. Frankly, sometimes (and I took part in this campaign myself) the criticisms of the old mathematics instruction and curriculum were exaggerated for propaganda purposes and for other reasons. Nevertheless, it is a fact that however poor the instruction was, or however poor the mathematics, or however poor the textbooks, we did manage to train enough mathematicians, teachers of mathematics, physicists, and scientists. To be sure, we need more of them now.

What was the old mathematics? It was drill and more drill, and we shoved decimal points around and inverted fractions, but we didn't really know what it was all about. We borrowed, and we carried, and although we borrowed, we never gave back anything; and then we drilled and we drilled some more. This was the essence of arithmetical instruction. In algebra, we were in the groove by that time for doing the kinds of things that were called for in algebra. There, we always kept changing signs - whenever you saw a sign, you changed it immediately. And we played a sort of put-and-take game. You picked up something here and put it over there (and, of course, you had to change the sign right away). We solved problems, most of them supposedly very practical; they always dealt with these three fellows: A, B, and C - they worked themselves to death! They raced, they filled and emptied cisterns, they built, they mixed things (fortunately they never drank the stuff that resulted from the mixing) and, of course, poor C was always the low man on the totem pole. One of your great Canadian humorists, Stephen Leacock, wrote an essay - some of you perhaps know it - about those great characters, A, B, C. We drilled and drilled incessantly. You couldn't leave a thing unsimple anywhere. When I began teaching algebra, I used to teach my youngsters to unsimplify fractions. As a matter

of fact, in many cases is this not exactly what you need to do if you are going to get anywhere at all? Take a simple problem like one-half and one-third. Your first job is to unsimplify those fractions, isn't it? Then you can go ahead.

What did we do in geometry? First, we sprang a great surprise on our students (in the States, usually in the tenth grade.) We told them: "You know, there's such a thing as logic!", as if they had not been reasoning all their lives, right up to that moment - and reasoning logically! Parents know that a three-year-old can out-argue them on purely logical grounds, nine times out of ten! Yet in teaching geometry we pretended that young people didn't know anything about logic - we had to tell them. And what did we use this logic for? To prove an endless series of absolutely obvious things that anybody with half an eye knows are so. You put up a triangle with two equal sides and ask them, "What about those angles?" Anybody can see that they are equal, but we have to prove that!

In trigonometry the situation was not any better. We solved oblique triangles, for months on end: It had to be done to so many decimal places, accurately, using tables and so on. Again, in geometry and trigonometry we drilled and drilled, kept going over, incessantly, the same problems, presumably in the expectation that this would give some kind of skill.

The calculus was not exempt from this. Daily you did any number of derivatives and integrals. You had not the slightest idea what a derivative was, or what it was good for, or what an integral could mean.

What were the results of this? First, for the pupils: The pupils were stultified in any creative or imaginative mathematical interest, especially those who did not resist us sufficiently. Those who resisted (you know, the troublemakers in your class who always ask you questions) were not completely spoiled by this kind of education - they survived it. And my impression is that you were among the people who survived this kind of education. You were among the troublemakers in your class, who eventually learned some mathematics in spite of what was prescribed. The results for the teachers were, in my opinion, even more dangerous and stultifying, because we stopped living intellectually. And after a few years of teaching we were not getting the thrill and the joy that mathematics can and should always give, because, after a while, we were tired of turning these crankhandles and getting out the same old proofs. We ourselves did no more thinking in mathematics. Then, the dull things we were repeating became so obvious to us that we could not understand why those kids did not see it the first time we taught it.

#### WHAT ARE SOME OF THE CHANGES ALREADY MADE OR AT LEAST ON THE WAY?

In arithmetic, I think we have begun to give some meaning to a number. We know by now that it is not the thing you scribble on the board. It's something else, but it is a something that has real existence in our minds, and it is a very useful kind of symbol. Secondly, we have some idea now of what we mean by an operation. An operation is an association. It is a mapping of pairs of numbers into single numbers. This is a very difficult idea to develop, but

we are beginning to develop it now in our elementary instruction. We are also beginning to give some reason so that youngsters know why they do some of the things that they do in arithmetic when they find a sum or a product.

You never knew why you shoved a decimal point this way or that way. When my students did addition, they would start in the right-hand column, first units, then tens, then hundreds - I would stop them and say "Wait a minute, why are you doing this problem backwards?" And they would look at me, again in surprise, "What are you talking about - backwards?" "Don't you read this way, from left to right?" I would ask. "Why suddenly do you read numbers from right to left?" They didn't know. Do you? I will let you in on a secret. It is one of those historical accidents: When the Italian merchants stole the Hindu-Arabic system of numeration from the Arabs, they lifted out of the books, in Arabic, the algorithms for all the operations; and, of course, the Arabs write *everything* backwards, including the numbers.

We are at least beginning to raise some of these questions and to discuss them, and students are beginning to see some sense in this world of arbitrary rules and regulations. If taught early, they can understand that the number system somehow or other evolves under our direction in response to expanding needs. First, the whole numbers help us answer questions such as "How many?" Then, in investigating problems about "How large?" or "How much?" we need fractions and decimals. Then with the problem of going this way or that way, signed numbers come in. Children are, I think, learning that these are not arbitrary but natural developments, necessary evolutions in response to expanding needs. Probably that realization is much more important than some of the algorithms which we used to work at so long and so diligently.

#### WHAT PROGRESS HAVE WE MADE?

We have introduced the concept of what a variable is. Now we use the notion of a set in a conscious way. The notion of sets is one of the things that tie together all of mathematics.

In algebra now, even in the grade schools, youngsters get some idea of open sentences (you may prefer to call them something else) and immediately, from the very beginning, not only equations but inequality. Some of you may recall your own first experience in the calculus, and what a chilling experience it was to use the first inequality there. Never in your life had you seen an inequality before! Through eight years of elementary school and four years of high school you had never met things that were not equal. No wonder, then, that when you actually used these inequalities in calculus, it was a traumatic experience, and many people did not survive it! That's why the calculus used to wipe out even more potential mathematicians than geometry - and that's saying a lot.

From seeing this notion of the number system as an evolving thing, we do get into large and substantial mathematical structures. I think the first great surprise is the real number system. You sense that here you have a complete world - it is closed; there seems to be no way out of it. Up to that time, there was always something new you had to do. With just the non-negative numbers, you can ask impossible problems and invent the negative numbers, and so on.

In geometry the reform movement and the revolution probably had least success, for a number of reasons. Perhaps it was because we had concentrated on the least exciting part of geometry - that is, the logical, axiomatic development of Euclid's geometry. Euclid's Elements is not a geometry textbook. It is a very modern text in integrated mathematics. It contains all the arithmetic, number theory, algebra, and geometry known in Euclid's day. Unfortunately we picked out the dullest theorems of all the 13 books, squashed them into six books, and said, "This is Euclid". We left out the beautiful proof of the infinity of primes; we left out algebraic discussions; we left out formulas of perfect numbers, we left out all the really elegant things in Euclid, and then we were brash enough to say, "This is geometry". What was the result? The reform movement tried to perfect the monster that we had created. It was said, "Euclid made all kinds of mistakes." And, of course, he did, for in his age he did not know what we know about real numbers. He had only the clumsiest numeration system. So he really could not cope with some of the problems. There are a great number of logical fallacies and faults in geometry. You know, for example, that by Euclid's own theorems you can prove that every triangle is isosceles, and you cannot disprove that proof by Euclid alone. In the reform movement, those involved wondered: "How do we eliminate these logical flaws in Euclid?" Therefore, all the changes made in geometry were only to perfect the logical structure. That made geometry even worse - because now, if you're going to talk about continuity, and you want to do it properly, you start at the beginning of September and you don't reach isosceles triangles until the following Easter! Some of the new texts do fall into precisely this trap - they want to be organized, overexact; I think students in Grade X or even in Grade XII don't want to know that much about the foundations of geometry. They do want to know about geometry. Nevertheless, we did improve the geometry; we have introduced, to a considerable extent, coordinate methods into geometry instruction, and we have introduced space geometry in our discussions with plane geometry, both of which I think are sound ways of expanding what we used to call plane geometry.

I am now going into some of the good things that we have done with trigonometry. We have moved away from the old computational trigonometry and went into the analytical trigonometry which is more important today and much more exciting. We have started talking not only about functions of angles but also about functions of numbers. We have brought in, for example, the wrapping functions as a beautiful way of introducing young people to what the trigonometric functions were. Even in calculus, there have been a great many improvements, and instruction has moved from just grinding out derivatives and integrals into trying to show that claculus is one of the greatest achievements of mankind that calculus gave man a tool to analyze the most constant thing in the world, in life, namely, *change*.

The present textbooks, both commercial and experimental, are very much better than textbooks ever were, either in secondary or elementary schools. I think teaching is better than it has ever been, partly because of the all-out teachers' involvement in the last 10 years. We have found it necessary to start learning once again; the thing that can pep up teaching more than anything else is active learning activity on the part of the teacher every single day. Maybe this is over-stating it, but any day that I did not learn something new in the classroom, I considered a failure. We perform our teaching task well only when we ourselves keep learning and remain excited about what we are doing. We have now more teachers who know more about mathematics and we have more mathematicians who know more about education.

Because the reform movement was a joint venture of teachers, mathematicians, and education specialists, all of us have learned something that we did not know before. We have increased our knowledge of our subject matter, and the mathematicians have begun to appreciate and understand some of the educational problems. This is significant, because when we get to the next stage about "Where next?" we have a resource pool of people who will be able to work towards the answer more expertly.

To be altogether fair, there have been exaggerations in this movement, just as there were in the old. The old mathematics was never really quite so bad as I pictured it a few minutes ago. The new mathematics is not quite so beautiful as I have described it. When we discovered sets, we just went wild. It was like a new toy; every new book had two chapters on sets at the beginning; we spent three months on sets and then never mentioned sets again in the rest of the mathematics curriculum. We discovered scales of notation - wonderful! Such a delight, such a new thing to do! Addition and multiplications in scale seven notations! "Oh, great!" Here again we went overboard, and many books just went wild!

I still remember a teacher who, starting to teach SMSG mathematics, fell in love with non-decimal numeration systems, especially three. This could justifiably take perhaps a couple of lessons. This man spent a month and a half on it, and when I last saw him, he was having the class translate the log tables into base three numeration:

Yet despite these exaggerations, a fair balancing of what has happened would show that we have advanced in our mathematics teaching.

#### WHERE DO WE GO NOW?

Now we do have a number of tentative directions. Europeans, looking at what we were doing, said, "You have a good idea, but why not really reorganize mathematics?" And their reform movement spurted ahead (they are about five years in advance of us right now). Let me explain: Secondary education, in particular, in almost all European countries has been and is a highly selective process. Only five to 10 percent of the school population is in secondary schools. Therefore, they have fewer teachers - and, therefore, those fewer teachers can be expected to fulfill very high requirements, and so can the highly selected pupils. If you looked at texts written in the last four years in Europe, you would not recognize them as secondary school texts. The Europeans have reorganized their mathematics instruction to fit into what mathematics is today. Some attempts are being made here in this direction. The Cambridge Conference Report attempted just this.

What kinds of mathematics would be indicated for the new era? I would

like to direct your attention to at least a few actual experiments. One is the UICSM (University of Illinois Commission on School Mathematics), the University of Illinois program. This was the very first approach to what the new mathematics ought to look like. Second, SMSG (School Mathematics Study Group), is going through what they call "the second round". Some of you may have heard rumors about it; they are writing a series of experimental units for Grade VII - but with this new view of a reorganization of the secondary school curriculum. Third, there is a global operation under Professor Howard Fehr of Teachers' College at Columbia - global in its view about mathematics and what should be taught, but not attempting to impose or introduce a brand-new program on all the schools in the country.

What is essential in this new program? First, new ideas will be introduced early. Perhaps some of you shudder when things like matrix algebra turn up in the third or fourth grade. Naturally, if you think of taking a semester course in matrix algebra, as in graduate school, this is not what is meant at all for the fourth grade! However, some of the ideas in matrix algebra - just a square or rectangular array of numbers to present a single fact - can be grasped by the young. A fraction is a very simple illustration of a matrix: two-thirds - that's two and three - there is a matrix. You have used two numbers to represent a single idea. Now consider the box score of a baseball game which includes runs, hits, and errors. We have a matrix consisting of six numbers. Nothing hard about that! Kids understand. This is what is meant by introducing matrix algebra early. Here are six numbers arranged in a rectangle to communicate a single idea. Such ideas would be introduced early, and after a while, at some stage, they would begin to be organized into a consistent and logical whole. These things would be organized now, not in the categories as we had them: arithmetic, algebra, geometry, trigonometry, and so on, but mathematics, mathematics, mathematics.

As for geometry, it properly always did begin, and should begin consciously, with the kindergarten. Kids know a lot of geometry then. They have been bumping into shapes and they are conversant with sizes - they know a lot of intuitive geometry. Geometry means much more than proving theorems from a set of axioms. It means looking at shapes, moving them around; looking at them through mirrors, trying to fit them (sometimes they do fit, sometimes they don't); turning them over; folding paper. That's geometry. None of this occurred in geometry as we learned it ourselves. Yet this is geometry. These are real genuine experiences with real genuine things.

Two major topics must enter early into the curriculum, and they must persist right through the curriculum: probability and statistics, and computers. Probability is perhaps the newest of the major mathematical sciences. It transfuses all thinking today in applications and in pure mathematics. For the first time in this century we have a very substantial tool to deal with phenomena of uncertainty. Where do we teach this? We start it in kindergarten, in the first grade, by collecting data, organizing data, thinking about it, trying to draw conclusions from it and, then, thinking about probabilities. The Commission on Mathematics realized that probability and statistics were important. Many of you will remember, with pleasure, the little grey book that they prepared. The topic is so important that it cannot, and should not, be done in one semester course or a year course. First, an accumulation of experience is needed, and

second, a certain amount of maturity is required before one can begin to organize logically and systematically.

The other big and very new thing is the computer. It is an essential part of civilization today. Probably more than 50 percent of the students sitting in front of you every day have a computer in their future, one way or another. Where do you teach about computers? When you need to use them? At the age of 20 or 25? No! Here, again, we must start early and keep going.

I wanted to talk a little about what this would mean in the grades and in the high school. I will just consider a couple of things about trends and directions we should be thinking about. I think we should consciously start to de-emphasize some of the things (such as sets) that we have overemphasized. The basic ideas are important, but let's not overdo them. In the same way, we need de-emphasis on a hangover, namely algorithms and computations. I am *not* advocating not to teach computation, but I think understandings come first.

Let me conclude with a possible outline of what a new textbook for Grade VII would include:

- (1) Finite Number Systems
- (2) Sets and Operation
- (3) Mappings
- (4) The Integers
- (5) Probability and Statistics
- (6) The Integers (again)
- (7) Lattice Points in the Plane
- (8) Sets and Relations
- (9) Transformations of a Plane (this means shoving things around and folding them over)
- (10) Segments, Angles, Isometrics
- (11) Elementary Number Theory
- (12) The Rational Numbers

Some of these topics may sound frightening, but so did the idea of sets and functions when they were first introduced. They will become parts of an ever-improving and ever-expanding curriculum of the future.





## A Mathematics Laboratory in GRADES VII AND VIII

James H. Vance

Mr. Vance, assistant professor in the Faculty of Education at the University of Victoria, is a doctoral candidate in the Department of Secondary Education at the University of Alberta.

In the June issue of this *Newsletter* Bernice Andersen suggested developing a mathematics laboratory program for the academic option. Despite recent enthusiastic reports concerning laboratory methods in mathematics, Alberta teachers may wonder how they could set up labs in their schools and what the results of such a venture would be. A recent study conducted in an Edmonton junior high school not only confirmed that students like studying mathematics in a laboratory situation, but indicated that they are able to effectively learn new concepts and acquire desirable attitudes toward learning mathematics through experiences in this kind of instructional setting.

A mathematics laboratory program consisting of 10 activity lessons and designed to function as an adjunct to the regular courses for Grades VII and VIII was developed for the study. Each activity was designed to lead to the discovery of a new concept or relationship through the manipulation of some type of concrete material. The physical materials accompanying each lesson were to stimulate interest, to provide a real setting for the problem to be investigated, to provide a means of gathering data leading to the solution of the problem, and to provide a way for students to check hypotheses and answers independently of the teacher or textbook. The problems and ideas encountered in the lessons were new to the students but had been chosen to support and supplement topics studied in the regular program as well as to enrich the curriculum. Topics investigated in the laboratory program included probability, numeration, the circumference and area of a circle, the number of subsets of a set containing elements, a finite mathematical group, and Euler's formula for polyhedra. Other activities were centered about a balance arm, a set of logic blocks, and a geoboard. The main idea was to create learning situations in which the students could be active and free to investigate mathematical ideas in their own way and at their own rate.

These laboratory activities were held on a once-a-week basis in place of regular mathematics classes. In the lab the students worked in pairs with the various physical materials, taking directions from prepared sets of written instructions. The teacher's role was that of a guide or advisor, giving assistance to groups of students who needed and requested help but allowing the learners to discover mathematical ideas through their own experiences.

Several measures were used to compare the lab students with students who had taken the experimental lessons in a teacher-directed class setting and also with students who had not been exposed to the experimental materials but who had continued to study the regular program (*Seeing Through Mathematics*), the full time allotted for instruction in mathematics. It was found that the use of 25 percent of class time in mathematics for informal exploration of new mathematical ideas did not adversely affect achievement in the regular program over a three-month period. In addition, tests of learning, retention, transfer, and divergent thinking indicated that students in both experimental groups had benefitted mathematically from participating in the program. Although test scores were slightly higher for the students who had studied the lessons under a teacher in a class situation, the reaction of the lab group to their instructional setting was more favorable. The lab students also rated higher than students in the other two groups (a) in feeling that learning mathematics is fun and enjoyable, and (b) in the view that mathematics is a subject which can be investigated and developed experimentally by using real objects rather than restricted to a textbook subject in which symbols are manipulated.

The most popular feature of the laboratory method as identified by the students was the opportunity which it provided for working independently of the teacher. The following are comments made by students in the laboratory group:

I liked the privilege of working at your own speed and without a teacher always telling you what to do. It was fun and helped me a great deal. I think it is better than teaching from the book and is a lot more interesting.

I liked where you could find out and prove things yourself so you would know for a fact that something is true.

## **Teaching Mathematics**

The following letter from R.A. Staal, president of the Ontario Mathematics Commission, was published in the "Letters to the Editor" column of the *Globe and Mail* on April 21, 1968.

The Ontario Mathematics Commission, an independent, voluntary association founded in 1959, representing all aspects and levels of mathematics education in Ontario, is currently studying recent trends in education and their likely effect on mathematics teaching. In particular, it is concerned with possible outcomes of the Hall-Dennis Commission Report, *Living and Learning*. While the OMC supports the broad objectives of *Living and Learning*, it is of the opinion that some of the trends attributed to this report may have serious and undesirable consequences in the teaching of mathematics unless treated very cautiously.

One theme of *Living and Learning* is an approach to curriculum development which is based mainly on the current needs and interests of the child rather than on a subject-matter-oriented structuring. Our concern is that a carelessly "unstructured" approach to mathematics teaching may fail to account for the fact that mathematics is, by its nature, a strongly structured subject. Curriculum structuring may have been too rigid, but the tendency to regard structure as inherently undesirable is not acceptable.

Another theme of *Living and Learning* is a major reduction in the role of examinations. The report qualifies its remarks about examinations, but most public comment overlooks such reservations - the school of the future is already being pictured as being without structure and without examinations. Formal examinations may have been over-emphasized, but to abolish examinations is to fail to face the real issue of evaluation.

In mathematics it is useless for the student to try to proceed unless he both understands and can actually do things with the material which has gone before. Proper evaluation requires more than piecemeal testing of short units. We feel that the time has come in this public debate for the problems of structure and examinations to receive more serious consideration. Realistic comparative emphasis upon creativity and workmanship is urgently needed in public discussion of the foundations of educational policy.

Committees of the OMC and other bodies have given, for some time, serious thought to such matters as ungrading and a more varied and humane use of testing procedures. Groundwork has been laid in such reports as *Intermediate Mathematics Methodology*, published by the Ontario Institute for Studies in Education and written in cooperation with the OMC. In our view, the real value of the report will emerge with continuing scrutiny of it by such bodies as the OMC, endeavoring to extract what is both realistic and feasible.

> From Ontario Teachers' Federation Reporter May, 1969

# CUSO Seeks 200 Math and Science Teachers

This year, Canadian University Service Overseas has received requests for some 200 mathematics and science teachers to work on two-year assignments in 40 developing nations of Africa, Asia, the Caribbean and Latin America.

An independent, non-profit Canadian organization, CUSO recruits, selects, trains and sends professionally and technically qualified personnel abroad in response to specific requests from governments and agencies overseas. It is a technical assistance manpower program - not a relief, religious or emergency aid scheme.

Teachers, assigned to established positions in secondary school systems, are paid by the host country employer at approximately counterpart, not Canadian, salaries. CUSO provides health and life insurance and can defer Canada student loans by paying interest during term of service overseas.

Applicants must be Canadian citizens or landed immigrants of at least two years' residence in Canada. Married couples are considered if both husband and wife are qualified for available assignments. Couples with one child can be considered in special cases, provided the child is under school age at the time of posting and will remain so during the assignment. There is no upper age limit, but good health is essential.

All outgoing personnel attend a six- to eight-week orientation and training course in July and August, which covers language and area studies, be-fore flying out to assignment in September.

Those interested should apply to their nearest CUSO committee located at 80 universities, colleges, institutes of technology, and community colleges across Canada. CUSO Information, 151 Slater, Ottawa 4, Ontario, will send committee addresses on request.

Alberta teachers may contact any of the following:

Dr. L.D. Cordes, Department of Geography, University of Calgary.

CUSO, Box 400, University of Alberta, Edmonton.

Santokh Anant, Psychology Department, University of Lethbridge.



#### CANADIAN UNIVERSITY SERVICE OVERSEAS

## "I find the work very stimulating and rewarding. I know I would not have had the same opportunity in Canada."

Born in Calgary, 25-year-old Stephen Gibbons grew up in Saskatchewan and graduated from the University of Saskatchewan with degrees in Arts and Education, both with a major in chemistry. Stephen is now teaching chemistry and physics at Union College, Sierra Leone.

"Union College, Bunumbu, is a teacher training college. We are concerned mainly with teachers for the primary school system of Sierra Leone. Future plans call for expansion into secondary school teacher training.

"I am teaching physics and chemistry from the General Science syllabus as well as two classes of mathematics. Since we are training teachers, our instruction involves methodology as well as subject matter. Some of our students have completed Form 5 and thus are fairly well advanced in the field of science, while others have only completed Form 3 which unbalances the class somewhat. All my teaching involves lab work as I use it to illustrate the scientific phenomena I wish to explain.

"I am doing research of a sort in the local primary school, a practice teaching school associated with the college. My students and I are trying to introduce primary school science units developed at Njala University College. We are also experimenting with some units of our own. The results of this work are being sent to the London Institute of Education as part of a competition in science teaching. The main aim of our participating in the competition is not to win but to provide a continuing hard look at the work we are doing here so that we may constantly re-evaluate ourselves.

"There will be a conference of science teacher trainers at Njala University College sometime in March where our college will present the results of some of the work we have been doing. It will also give us the opportunity to hear and discuss the work being done at other colleges.

"The help and support given to me by my principal is excellent. He is willing to throw the resources of the college behind all staff and he helps lecturers with any teaching problems they may have. Working at Union College means being treated as a professional. You are left alone in your classroom and allowed to get on with your job. This gives one great opportunity to develop his teaching technique.

"The satisfactions here are the same as those experienced by teachers around the world. You watch students develop their potential under your guidance. When you go into the practice teaching school and watch a student teach a lesson well, you feel so good you could extend your assignment for 10 years! You're satisfied.

"The frustrations are similar to those encountered in Canada. The teaching profession rates comparatively low on the social ladder and teachers are grossly underpaid.

"All in all I find the work very stimulating and rewarding and I wouldn't have missed it for the world. I know I would not have had the same opportunity to work in a teachers college back in Canada." Stephen Gibbons

## REFERENCES

#### for MATHEMATICS 15 and 25

Marshall Bye

Dr. J.S.T. Hrabi, director of curriculum in the Province of Alberta, has requested that this bibliography by Marshall Bye, supervisor of secondary mathematics, Calgary School Board, be included in this issue of the *Newsletter*. Both Dr. Hrabi and Mr. Bye have been members of the MCATA Executive Committee.

Subsequent to the publication of the Curriculum Guide for Mathematics 15 and 25, the following teacher references have been reviewed. These references are highly recommended; they contain otherwise unavailable suggestions and approaches to topics suitable to the Mathematics 15 and 25 program.

S.M.S.G. (School Mathematics Study Group), Mathematics and Living Things, revised edition. McGill University Press, Montreal, 1965; 221 pp. Contains many laboratory exercises in mathematics relating to the "Life Sciences" Especially appropriate for use with students interested in

Sciences". Especially appropriate for use with students interested in Biology.

A.J. Moakes and others, Pattern and Power of Mathematics, Bks. 1,2,3,4. Macmillan Co. of Canada Ltd., Toronto, 1967.

A set of books of about 220 pages, each printed in England. Contains many interesting diversions relating to basic mathematics.

, School Mathematics Project, Bks. 1,2,3. Macmillan of Canada, Toronto, 1965.

Part of a set of books published in England with a distinctly different approach co mathematics. Contains many excellent activities for the mathematics class.

Available from the Queen's Printer, Ottawa is *Consumer Calculator* for 50¢. This calculator might motivate a series of classes relating to purchasing and to the use of fractions especially with a class of girls.

## **Curriculum News**

At the third meeting of the Secondary School Curriculum Board, held on June 5 and 6, 1969, experimental mathematics classes using the materials indicated were approved as follows:

1. For Grades VII-VIII

Developing Mathematical Ideas, Books 1 and 2, by Sobel et al (Ginn and Co.)

Modern School Mathematics Structure and Method, Grades VII and VIII, by Dolciani et al (Thomas Nelson & Sons)

- For Grade VII Mathematics, Concepts, and Applications, by Van Engen et al (W.J. Gage & Co.)
- 3. For Grade VIII The second item above if available.
- For Mathematics 10 and 20 (Geometry) Geometry - A Modern Approach, by Wilcox (Addison-Wesley)

Modern Geometry, by Nichols et al (Holt, Rinehart and Winston)

5. For Mathematics 10 and 20 (Algebra) Modern Intermediate Algebra, by Nichols et al (Holt, Rinehart and Winston)

Modern Algebra and Trigonometry, by Dolciani et al (Thomas Nelson & Sons)

6. For Mathematics 30 Functions, Relations, Transformations, by Elliot et al (Holt, Rinehart and Winston)

Algebraic Structures and Probability, by Elliot et al (Holt, Rinehart and Winston)

Mathematics 10 was revised effective September, 1969, by deleting Reviews A and B and by adding Chapter VI (Relations) and Chapter VII (Systems of Linear Equations).

Mathematics 20 was revised effective September, 1969, by adding Chapter XII (Sections 12.1 to 12.12 only).



# **Book Review**

Modern Trigonometry, by Eugene D. Nichols and F. Henry Garland. Holt, Rinehart and Winston of Canada Limited. 833 Oxford Street, Toronto 18, Ontario, 1968. viii and 328 pages. \$6.15 (Canadian)

The book contains 10 chapters, three appendices, tables and an index. The first chapter introduces the inevitable notation of sets, inequalities and functions. A rather unsatisfactory attempt at defining irrational numbers is made, and it would have been better not to introduce these numbers at all. As with all other chapters, diagrams are very neat, colored and numerous. In some places there are perhaps too many, as in Chapters 3 and 6.

All the standard work is covered, but not well organized. For example, in Chapter 3, the Cosine is defined: Given W, for which W(U)=P(x,y), the cosine function is (0,x): and one has to build up a theory of a Wrapping function in Chapter 2. Pedagogically, confusion will result in teaching from this book, since it is inclined to fall over backwards to be modern. Later chapters on trigonometric equations and identities are done much better; however, the use of "o" for degrees in all expressions:  $\cos u^{\circ} + \sin u^{\circ}$  etc. is probably necessary. In order to see if a trigonometry text is any "good", I just turn to the chapter that deals with the proof that  $\cos (A-B)= \cos A \cos B + \sin A \sin B$ . This book does the proof admirably - see page 99. Again, the diagram, although neat, is difficult to understand.

In my opinion, Chapter 9 on Complex Numbers is the best in the book. The topic of vectors is well covered in Chapter 10. A large number of good exercises may be found throughout the book. The section which was covered the least satisfactorily was Chapter 8 on Inverse of a Function. Inverse functions can be a little tricky to teach to newcomers, and definition should be kept to a minimum. The issue is clouded in this book by the introduction of a new function called "Cap Sin", which is then related to Arcsin. Great confusion will result unless the functions are related to their graphs. The writers of the book attempt to do this on pages 218 and 219 but do not really succeed.

In conclusion, I would say that *Modern Trigonometry* is quite well written, the printing is excellent, exercises are first class, answers are adequate, definitions are too numerous, and some chapters are confusing. The book is well worth having in the library, but considerable experimentation is required before it can be adopted confidently for classroom use.

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