# MATMEMATICSCOUNCII newsLetter 

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# New Mathematics Curriculum at the University of Calgary 

Reported by A.S.B. Holland University of Calgary<br>Assistant Dean of Arts and Science<br>Associate Professor of Mathematics

Since the Faculty of Arts and Science at The University of Calgary is to establish four-year Bachelor of Arts and Bachelor of Science degrees to commence September, 1970, the Department of Mathematics has revised its first-year courses accordingly. First-year mathematics courses now listed in the calendar of The University of Calgary will be withdrawn at the end of the Summer Session, 1970. The following courses have been deleted.

## in this issue

NEW MATHEMATICS CURRICULUM
NEW MATHEMATICS CURRICULUM
AT THE UNIVERSITY OF CALGARY
1

- A.S.B. Holland

EUCLID MUST GO:

- Marshall P. Bye

MATHEMATICAL PREPARATION OF ALBERTA MATH TEACHERS

- Donald O. Nelson

ONTARIO MATHEMATICS CONTEST April 8, 1970

- A.S.B. Holland

ETV CONFERENCE IN MATHEMATICS
MATHEMATICS CURRICULUM NEWS
MCATA SPEAKERS' LIST
BOOK REVIEWS
EXECUTIVE COMMITTEE, 1969-70 22

Newsletter Editor: Mary Beaton

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Computing Science 252 (1/2), 253 (1/2), 254 (1/2), 255 (1/2)
Mathematics 111(1/2), 212 (1/2,) 211, 213(1/2), 219, 239, 285, 287
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New first-year courses are the following:
PMAT 101 Selected Topics in Mathematics I (3-0; 0-0)
PMAT 103 Selected Topics in Mathematics II ( $0-0 ; 3-0$ )
PMAT 201 Calculus (3-IT); and ( $0-0$; 3IT)
PMAT 203 Algebra and Geometry (3-IT; 0-0) and ( $0-0 ; 3-I T$ )
PMAT 211 Sets and Mappings (3-IT; 0-0)
PMAT 231 Living Mathematics (0-0; 3-0)
PMAT 211 Vector Methods (3-IT; 0-0) and ( $0-0 ; 3-\mathrm{IT}$ )
AMAT 203 Mathematics and the Real World (3-IT; 0-0) and ( $0-0 ; 3-I T$ )
STAT 201 Elements of Finite Probability (3-IT; 0-0) and (0-0; 3-IT)
STAT 211 Facts from Figures (3-3; 0-0) and (0-0; 3-3)
CPSC 201 Introduction to Computer Programming ( $3-2 ; 0-0$ ) and ( $0-0 ; 3-2$ )
CPSC 203 Elementary Programming for Business ( $3-2 ; 0-0$ )
CPSC 205 Elementary Programming for the Social Sciences ( $0-0 ; 3-2$ )
CPSC 207 Elementary Programming for the Natural Sciences (3-2; 0-0)
CPSC 209 Elementary Programming for the Humanities ( $0-0 ; 3-2$ )
CPSC 221 Elements of Computing Equipment (3-2; 0-0) and ( $0-0 ; 3-2$ )
CPSC 223 Introduction to Data Processing Equipment ( $0-0 ; 3-2$ )
Any enquiries concerning the nature or the details of the new first-year mathematics curriculum should be directed to the Department of Mathematics The University of Calgary. Questions in particular should be directed to Dr. Schaer (telephone 284-5202). Further changes which will be promulgated from year to year will be advertised in the subsequent calendars of The University of Calgary.

## EUCLID MUST GO!

Marshall P. Bye
Mr. Bye is Supervisor of Mathematics for the Calgary public secondary schools

Euclid must go! Surely anyone who utters such sentiments must be sacrilegious. Yet these are the words of the outstanding mathematician Professor J. Dieudonné in his address to the Organization of European Economic Council in France in 1959. ${ }^{1}$ Why did he make the statement? Perhaps I can bring some light to this.

We read so much today about what should and should not be included in the school curriculum that, I am sure, we all wonder just what mathematics will become in another decade. One such indicator is the Report of the Commission on Mathematics of the College Entrance Examination Board², published in 1959. The very bold programs set forth in that document (very bold for its day) are being realized in varying degrees around the world today - just 10 years later. A number of topics and concepts listed have yet to be included in the Alberta curriculum, but we are surprisingly close to the programs outlined.

The report of the Cambridge Conference on School Mathematics - Goals for School Mathematics ${ }^{3}$, published in 1963, listed a still more startling set of objectives for mathematics from $K$ through 12. This document might well be the preview of the next decade.

Why do I mention these two reports? I do because in both reports strong reference is made to transformations. In the Commission Report ${ }^{4}$, some time is spent on the three primary weaknesses of the so-called Euclidean geometry as it has been presented for so many years. I shall be concerned primarily with only one of the weaknesses. Professor Dieudonné had these weaknesses in mind when he made the statement "Euclid must go!"

There has been a pronounced trend away from "traditional" geometry in countries outside of North America. A number of the British programs - to name three: the School Mathematics Project, the Nuffield Project, the Scottish Mathematics Group - emphasize transformation.

Belgium, perhaps as a result of Papy's work, leans heavily on transformations from elementary school up. Here in Canada, Professor Dienes of Sherbrooke, Quebec, has made transformations an integral part of his program. De1 Grande and Egsgard of Toronto have come out with high school texts integrating transformations into the program. ${ }^{5}$ The Secondary School Mathematics Curriculum Improvement Study (SSMCIS) ${ }^{6}$, produced by Teachers' College, Columbia University - two of the authors are Dr. Julius Hlavaty and Professor Ray Cleveland - has utilized transformations in algebra and geometry. The NCTM publication Geometry in the Secondary School (1967) ${ }^{7}$ devotes nearly half of its space to discussions about transformations of one type or another and hardly mentions traditional Euclidean Geometry as it has been taught for years.

## TRANSFORMATIONS

My objective will be to show quickly and easily how transformations may be used in high school geometry and, at the same time, not get involved with "motion" of a geometric figure or set of points. (At times, for clarification only, I shall call upon your intuition as to the motion of a figure.) I shall not be rigorous in such a brief presentation. I shall, also, make statements which, in a more formal presentation, would need more firm and rigorous attention.

## DEFINITION OF TRANSFORMATION

A transformation is a one-to-one mapping. Since we will be talking about plane geometry, I will say that a transformation is a one-to-one mapping in which the domain and range are the set of points of a plane.

Let us now look at a particular set of transformations - the set known as isometries.

## DEFINITION OF ISOMETRY

An isometry is a distance preserving function. Any figure transformed under an isometry is said to be invariant; that is, a figure is its own image under an isometry. Another way to say this, and perhaps crucial to this discussion, is: a figure transformed under an isometry is congruent to its image.

I shall sum up the properties of isometries after we have looked at three such transformations.

REFLECTION (in a line)
Consider a triangle reflected in a mirror.

This is the intuitive concept of a reflection. Now let me draw to your attention some of the pertinent details.

1. Every point of the figure $A B C$ is associated with one - and only one point in its image figure $A^{\prime} B^{\prime} C^{\prime}$.
2. Points: $A \rightarrow A^{\prime} ; B \rightarrow B^{\prime} ; C \rightarrow C^{\prime}$;

$$
P \rightarrow P ; \quad Q \rightarrow Q ; \quad R \rightarrow R .
$$

3. Notice that the points in the mirror line are invariant: each maps on to itself.
4. Segments: $\overline{\mathrm{AB}} \rightarrow \overline{\mathrm{A}^{\prime} \mathrm{B}^{\prime}} ; \quad \overline{\mathrm{AC}} \rightarrow \overline{\mathrm{A}^{\prime} \mathrm{C}^{\prime}} ; \quad \overline{\mathrm{BC}} \rightarrow \overline{\mathrm{B}^{\prime} \mathrm{C}^{\prime}}$;

$$
\overline{\mathrm{AP}} \rightarrow \overline{\mathrm{PA}} ; \quad \overline{\mathrm{BQ}} \rightarrow \overline{\mathrm{QB}}{ }^{\prime} ; \quad \overline{\mathrm{CR}} \rightarrow \overline{\mathrm{RC}}
$$

5. Notice that the mirror line is invariant. $\quad \overleftrightarrow{P R} \rightarrow \overleftrightarrow{P R}$
6. Consider the angles formed by $\overleftrightarrow{A B}$ and $\overleftrightarrow{A^{\prime} B^{\prime}}$ with $\overleftrightarrow{P R}$. The angles are congruent.

7. The perpendicular distance between a point and the mirror is congruent to the perpendicular distance between the image point and the mirror, or stated
differently, the axis of reflection is the perpendicular bisector of the segment joining a point and its image:
8. $\triangle A B C \cong \triangle A^{\prime} B^{\prime} C^{\prime}$
9. The sense of $\triangle A B C$ is opposite to that of $\triangle A^{\prime} B^{\prime} C^{\prime}$. The order of vertices of the object triangle listed clockwise is $A-B-C$, whereas the order of vertices of the image triangle, clockwise, is $A^{\prime}-C^{\prime}-B^{\prime}$.

Let us now look at a double reflection: a reflection of a reflection. In the first, the two axes of reflection are parallel (Figure 3), whereas in the second (Figure 4) the two axes are not parallel. Notice that we have one transformation followed by another. This is called "composition of transformations".


These two illustrations may lead us intuitively to accept the statement that given any two congruent triangles in a plane, there is a series of reflections such that one triangle is mapped onto the other. It is an interesting exercise to determine the maximum number of axes of reflection necessary to transform any given triangle into a specific congruent triangle and where those axes of reflections are.

At this time I wish to emphasize that we are not moving triangles or lines or points. When you look in a mirror and see your eyes, you do not, for a moment, have the notion that your eyes have moved behind the mirror. As one author states, in terms of a bowling lane, "We are setting up pins in another alley." As for motion in a plane to explain congruency, there is no motion that would permit you to move $\triangle A B C$ to coincide with $\triangle A^{\prime} B^{\prime} C^{\prime}$ (Figure 1). The motion would have to come out of the plane.

To make my point clear, let me digress for a moment to a transformation that is not an isometry. Consider the inversive transformation. For this transformation, consider a circle in a plane with center 0 and fixed radius $r$. Any point $M$ is mapped into $M^{\prime}$ such that $m(\overline{0 M}) \cdot m\left(\overline{0 M^{\top}}\right)=r^{2}$. Refer to Figure 5 .

--.--------- - - - L'

Figure 5

This transformation results in some strange things:

1. Every point in the interior of the circle is mapped into a point in the exterior of the circle and, conversely, every point in the exterior is mapped into the interior.
2. Every point on the circle remains fixed (mapped into itself).
3. Every circle in the interior of the circle that passes through the center is mapped into a straight line.
4. Every line that contains the center 0 of the circle is mapped into itself.
5. Every line that does not contain the center 0 of the circle is mapped into a circle.
6. Every circle not containing the center 0 of the circle is mapped into another circle.

Clearly, we have not "moved" figures - we have not preserved shape or size.

However, let us return to isometries. While we can use reflections to establish our mappings of one figure into congruent figures, other transformations may be used as well. We shall only spend time with two others in this paper.

Refer to Figure 6. Notice that we can think of $A \rightarrow A^{\prime \prime}, B \rightarrow B^{\prime \prime}$ and $C \rightarrow C^{\prime \prime}$. If we place this figure on the coordinate plane, it is easy to think of this transformation as mapping any point $M(x, y)$ into $M^{\prime}(x+1, y+h)$.


Intuitively, a translation can be thought of as the transformation of the set of points, taken in order, through a certain fixed distance in some direction.

Review some of the properties of this invariant transformation:

1. Corresponding sides are parallel and congruent.
2. Corresponding angles are congruent.
3. The sense of the figure is preserved.

## ROTATION

The third and last transformation discussed by me in this paper is illustrated in Figure 7. I have reproduced it here to show the mapping of $\triangle A B C$ onto $\triangle A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$. You can visualize the mapping: $A \rightarrow A^{\prime \prime}, B \rightarrow B^{\prime \prime}$, and $C \rightarrow C^{\prime \prime}$. As you know, this is a rotation.


In the next figure, we clearly see the mapping on the Cartesian plane.


The point of rotation is the origin. The angle of rotation is the measure of $\angle A O A^{\prime}$.

Points to observe in this isometry:
$-\angle A O A^{\prime} \cong \angle B O B^{\prime} \cong \angle C O C^{\prime}$

- The perpendicular bisector of the segment determined by two corresponding points contains the point of rotation 0 . ( $\ell$ bisects $\overline{C C}{ }^{\top}$ )
- The said perpendicular bisector of the segment $\overline{C C^{\prime}}$ bisects $\not \angle C^{\prime}$.
- Sense is preserved.
- The point of rotation is the only point in the plane that is invariant.
- The image is congruent to the object.

Another isometry is the glide-reflection. It is a combination of the translation followed by a reflection. Some books use the glide-rotation. These are simply compositions of other isometries.

## PROPERTIES OF AN ISOMETRY

At this point we will sum up briefly and state our understandings in the form in which we will be using them.

1. If there is an isometry or isometries which transform one geometric figure into another, the two figures are congruent.
2. Suppose in polygons $A B C D$ and $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ the mapping is an isometry and suppose $A \rightarrow A^{\prime}, B \rightarrow B^{\prime}, C \rightarrow C^{\prime}, D \rightarrow D^{\prime}$.
(a) Distance is preserved: $m(A B)=m\left(A^{\prime} B^{\prime}\right)$ $m(B C)=m\left(B^{\prime} C^{\prime}\right)$ $m(A C)=m\left(A^{\prime} C^{\prime}\right)$ etc.
(b) Measure of each angle is preserved:

$$
m \angle A B C=m \angle A^{\prime} B^{\prime} C^{\prime}, \text { etc. }
$$


(c) Straightness is preserved - that is, lines map into lines.
(d) Paralle1 lines map into parallel lines.

If, $A B \| C D$, then $A^{\prime} B^{\prime} \| C^{\prime} D^{\prime} \quad$ (Hence, perpendicularity is preserved)
Now we arrive at the main point of the discussion. I have gone neither into any detail on the method of presentation nor into interesting side trips. I have only laid the foundation for that which I want to present at this time.

## GEOMETRIC PROOFS USING ISOMETRIES

Definition: Two figures are said to be congruent if there is an isometry (or a composition of isometries) that maps one of the figures onto the other. Let us look at specific instances.

## Example 1

Figure XYZVW
Segments XW and $\overline{V Y}$ intersect such
that $\overline{X Y}$ is parallel to $\overline{\mathrm{VW}}$ and $\overline{Y Z} \cong \overline{\mathrm{VZ}}$.
Prove: $\overline{X Z} \cong \overline{W Z}$
Proof: Consider the $180^{\circ}$ rotation of $\triangle X Y Z$ about $Z$.
Thus $Z \rightarrow Z$


Since $\overline{Y Z} \cong \overline{V Z}, Y \rightarrow V$
Let X $\rightarrow$ X'
Since $X$ lies on $\overparen{Z W}, X^{\prime}$ lies on $\overleftrightarrow{Z W}$
Now in a rotation of $180^{\circ}$ a line not through $Z$ maps onto a parallel line (property of rotations).
$\because \overline{X Y} \rightarrow \overline{V X^{\prime}}$
$\therefore \overline{X Y} \| \overline{V X^{\prime}}$ and $\overline{X Y} \simeq \overline{X^{\prime}}$
But, $\quad \therefore \overline{X Y}\left\|\frac{\overline{V W}}{V X^{\prime}}\right\| \frac{V W}{}$
Two parallel segments with one common point must lie in the same line. (Euclid we have not banished him completely. Saccheri does not dethrone Euclid here!)
$X^{\prime}$ lies in $\widehat{V} W$
But $X^{\prime}$ lies on line $\grave{Z} W$
$W$ is the only point common to the two lines $\overleftrightarrow{Z W}$ and $\overleftrightarrow{V W}$
$\therefore X^{\prime}=W$
$\therefore \overline{x Z} \rightarrow \overline{W Z}$
$\therefore \overline{X Z} \cong \overline{W Z}$
Let us look at another example.

Example 2
Consider the square $A B C D$
$P$ and $Q$ are midpoints of $\overline{A B}$ and $\overline{B C}$ respectively.
Prove: $\overline{P D} \perp \overline{A Q}$


We will not set up the detailed proof, but I will work through the general approach. First, by translation, we transform $\triangle D A P$ along the $\bar{D} A$ the distance equal to the measure of $\overline{D A}$. Thus we get the $\triangle A A^{\prime} P^{\prime}$. Now we rotate $\triangle A A^{\prime} P^{\prime}$ about point $A$, through an angle of rotation of $\pi / 2$. We can then show that $\triangle A A^{\prime} P^{\prime}$ has been mapped onto $\triangle A B Q$. Hence, $\overline{A D}$ (which is parallel to $\overline{D P}$ ) maps onto $\overline{A Q}$ by a rotation of $\pi / 2$. Hence $\overline{A Q} \perp \overline{D P}$.

A final example to illustrate the use of a reflection:

## Example 3

Given: In the figure, $\overline{\mathrm{AB}}$ and $\overline{\mathrm{PQ}}$ are parallel chords of two circles with a common center 0 .

Prove: (a) $\angle A O P \cong \angle B O Q$
(b) $\overline{\mathrm{AP}} \cong \overline{\mathrm{BQ}}$

Proof: Consider a reflection of $\triangle P A O$ in a line $M$ through 0 perpendicular to $\overline{\mathrm{PQ}}$. From previous work we know line $M$ bisects $\overline{\mathrm{PQ}}$. (Can prove, but will accept now.)
$0 \rightarrow 0$
$S \rightarrow S$
$R \rightarrow R$

$$
\begin{aligned}
& \because \overline{P R} \rightarrow \overline{Q R} \text { and } \overline{P R} \perp \overline{O R} \text { (previously proven) } \\
& \therefore P \rightarrow Q
\end{aligned}
$$

Similarly, we can show $A \rightarrow B$.

$$
\begin{aligned}
& \therefore \angle A O P \rightarrow \angle B O Q \quad \therefore \angle A O P \\
& \cong \angle B O Q \\
& \therefore \overline{P A} \rightarrow \overline{Q B}: \quad \therefore \overline{P A} \cong \overline{Q B} .
\end{aligned}
$$

I have illustrated the use of transformations in proofs from plane geometry. Transformations also may be used to let students discover construction techniques and in turn make the work on constructions far more meaningful and a far more powerful unit in geometry. Time will not permit a discussion of this area.

## A FEW CONCLUDING REMARKS

I have restricted my discussion to plane geometry. There is no need for this restriction; transformations allow an easy transition into 3-dimension or even $n$-dimension. This is an advantage of transformations.

Transformations can also be used to solve quadratic equations of the form $a x^{2}+b y^{2}+2 y x+2 f y+c=0$ by simply transforming them to the form $a x^{2}+b y^{2}=C$. This is another advantage of transformations.

Finally, transformations provide ample opportunity to show that Euclidean geometry is one particular element of the set of geometries in which a certain set of properties are invariant. Whenever we change the set of invariant properties we have a new geometry. Within the scope of transformations lies a host of geometries of such a simple nature that students at an early age can develop, if given an opportunity, an intuitive understanding of them.

Professor Dieudonné viewed the broader field of mathematics which is possible through the new freedom provided by a break from Euclid. He does not advocate throwing out all of Euclid, but rather stresses that for young students there is a richness in geometry possible when parts of Euclid are set aside.

## FOOTNOTES

${ }^{1}$ New Thinking in School Mathematics. Organization For European Economic Cooperation, 1961, p. 35.
${ }^{2}$ Report of the Commission on Mathematics. College Entrance Examination
Board. Princeton, New Jersey. 1959.
${ }^{3}$ Goals for School Mathematics. Report of the Cambridge Conference on School Mathematics. Boston: Houghton Mifflin Company. 1963.
${ }^{4}$ Report of the Commission on Mathematics, op.cit., pp.109-110.
${ }^{5}$ J.J. Del Grande, J.C. Egsgard, Mathematics 11. Toronto: W.J. Gage Ltd.
${ }^{6}$ Unified Modern Mathematics. Secondary School Mathematics Curriculum Improvement Study. 1968. Board of Trustees of Teachers College, Columbia University.
${ }^{7}$ Geometry in the Secondary School, National Council of Teachers of Mathematics, Washington, D.C., 1967.

# Mathematical Preparation of Alberta Math Teachers 

Donald O. Nelson, M.Ed.<br>Mr. Nelson teaches mathematics in<br>Claresholm, Alberta

It may seem obvious that a teacher must know the subject matter which he or she is teaching, particularly in a subject such as "modern" mathematics. The Mathematical Association of America considered the problem of the background in college mathematics which would be needed by teachers of mathematics. To do this, the Committee on the Undergraduate Program in Mathematics (CUPM) was formed; it recommended the following courses in university mathematics:

4 semester courses for elementary school teachers
7 semester courses for teachers of beginning Algebra (Grades VII and VIII)
11 semester courses for teachers in Grades IX through XII mathematics.
They stated that these recommendations were minimal, and a comparison with recommendations of other committees and groups would show much higher standards being proposed.

Since the CUPM organization is a well-recognized mathematical body, it seemed interesting to compare Alberta's teachers of mathematics to this standard. The study became part of an M.Ed. thesis at the University of Calgary. In May of 1968, questionnaires were mailed to a random selection of 209 schools in Alberta. Teachers of any mathematics were asked to complete the questionnaire which dealt with their mathematical training and other selected topics. Results were tabulated from the 910 questionnaires returned.

Only 10 percent of Alberta elementary school mathematics teachers and 20 percent of Alberta junior and senior high mathematics teachers had sufficient mathematics background to satisfy the CUPM recommendations. Sixty-seven percent of Alberta elementary mathematics teachers reported no training in university mathematics. The CUPM felt that the elementary teacher in the self-contained classroom who must teach every subject should have taken four semester courses in mathematics plus whatever was required in the other subject areas. They expressed doubt as to whether or not this could be accomplished in only four years of a teacher preparation program. However, about 60 percent of Alberta teachers have less than four years of training. ${ }^{1}$

[^0]At the junior high school level, 23 percent of the responding mathematics teachers reported no university mathematics courses. Fourteen percent of the senior high school mathematics teachers responded in a similar way. Only 62 percent of the junior high mathematics teachers and 51 percent of the senior high mathematics teachers stated that mathematics was their main field of interest. Since subject matter specialization is practised in these grades in many schools, it is assumed that anyone teaching mathematics would prefer the subject and would have some background beyond his own high school training. Has this large proportion of Alberta mathematics teachers been "mis"-assigned in their teaching positions? Is it not considered important that the teacher instructs in the subject for which he has been prepared? Or is it simply that there are not enough qualified mathematics teachers?

Ten percent of elementary teachers and 33 percent of secondary mathematics teachers expressed a desire to take further mathematics courses. It was found that the intention to take further mathematics courses was greater for the teachers who had previously taken one or more university mathematics courses than for those who had not. This may be an obvious observation, as most of those who have taken mathematics courses have taken them because they were interested in the subject and consequently would be interested in taking more courses. At the secondary school level, teachers' desire to take further mathematics courses was greater for younger teachers than for older teachers. This may be because the older teachers have completed their training to their satisfaction and, therefore, are not interested in taking further courses in any area.

If it is important that the modern teacher have a broad background in the subject he teaches, something must be done to persuade every mathematics teacher to further his own training to the point where he at least reaches a standard as set by the CUPM, which regards its own standards as minimal.

Inservice projects in mathematics may be an answer to the problem of insufficient background in mathematics, but it was found that approximately one-third of the respondents had not participated in such projects. Many inservice projects are of such short duration that they are relatively ineffective, but 26 percent and 37 percent of the elementary, junior high, and senior high mathematics teachers, respectively, reported over 15 hours of inservice activities in modern mathematics. However, the problem of reaching the teacher of mathematics who is not primarily interested in mathematics exists, as approximately 65 percent of the respondents who had no university background in mathematics had taken part in less than 15 hours of inservice modern mathematics activities, and thus the effect of these activities would be minimal.

Formal courses or inservice activities may not be the only answer. The teacher's initiative in self-study may give a good knowledge of the subject not measurable in the number of formal courses taken. The studying of topics in mathematics which appear in grades higher than the one taught by the teacher is a good project. Films on modern mathematics topics are available from our Mathematics Council, and could be valuable.

The training in mathematics of Alberta mathematics teachers may be lower than what is considered advisable in terms of formal courses; however, rather than bemoan the situation, we must be constructive. Have we as members of the

Mathematics Council prepared ourselves at least to the level advocated by the CUPM? What can we do in promoting more interest and enthusiasm in activities which foster further mathematical knowledge in our own school and area? How can we draw people who are not primarily interested in mathematics into these activities? The answers to these questions are not clear, but they must come from our clear thinking on the issue. Each and every one of us will have to work on finding a solution to existing problems in this important matter.

# ONTARIO MATHEMATICS CONTEST <br> (formerly Ontario Junior Mathematics Contest) 

April 8, 1970

A.S.B. Holland

Once again the Ontario Mathematics Contest, a national contest designed by the Mathematics Department of the University of Waterloo, is made available to junior and senior high schools with students principally in Grades IX, X and XI. The main aim of the contest is to stimulate students who are interested in mathematics and to provide an opportunity for schools and students to test their mathematical prowess against others.

This year's contest will be the third in which Alberta schools compete. The contest is sponsored by the Department of Mathematics at The University of Calgary. Registration forms have been sent out to nearly all schools in Alberta that teach students in Grades IX, X and XI. Included with the registration forms were order forms which should be returned to the University of Waterloo for those schools wishing to order copies of previous contests. The basic fee for the contest is $\$ 5$; this amount entitles any school to enroll students in the age bracket up to and including 20. For every student in excess of age 20 a further charge of $25 \phi$ is made. A maximum number of 99 students from any one school is permitted.

The contest will be held from 9 to 10 a.m. on Wednesday, April 8, 1970 (not Wednesday, April 1, 1970, as we previously advertised). It will consist of 32 questions in three categories. Category $A$ is fairly simple and valued at three points each. Questions in Category B are more challenging and valued at four points each. Those in Category $C$ are quite stimulating and valued at five points each. The contest is multi-choice with five answers supplied for each question. The contestant is required to indicate which one, in his opinion, is correct. Points are deducted for incorrect or multiple answers. The contest is on an individual and school basis. The school team score consists of the sum of the three highest scores obtained by individual contestants. Each school receives a listing of the scores of all students in their own school, one of the top students in their district, one of the top students in their province and one of the top students in Canada; as well as listing of all schools in their
district, one of the top schools in their province and one of the top schools in Canada. The school with the highest team score will receive the University of Waterloo shield for one year. This award was won in 1969 by Sir Winston Churchill Secondary School, Vancouver.

The Mathematics Council of The Alberta Teachers' Association will offer a prize of $\$ 30$ and award a plaque to the best team of three in the province. The Department of Mathematics of The University of Calgary will also offer two prizes of $\$ 30$ each for the next two teams of three in the province. In addition, the Mathematics Department of The University of Alberta in Edmonton will offer two prizes of $\$ 30$ to be disbursed among the top teams in the province.

Registration forms should be mailed, by the end of this month, to Mrs. Scott Niven, Department of Mathematics, The University of Calgary, Calgary 44, Alberta. All the forms for copies of previous contests should be mailed to the Ontario Mathematics Contest, Faculty of Mathematics, University of Waterloo, Waterloo, Ontario.

It is hoped that schools in the Province of Alberta will enter teams once again in this contest. On a national level the teams from Alberta did particularly well in the previous year's contest, and - who knows - an increased participation in the 1970 competition may be forthcoming.

## ETV CONFERENCE IN MATHEMATICS

## TORONTO, APRIL 17-18, 1970

The Canadian Association of Mathematics Teachers is holding an ETV mathematics conference on Friday and Saturday, April 17 and 18, at the Skyline Hotel in Toronto. At this conference representatives from the provinces will explain what is being done throughout Canada with ETV in the mathematics classroom at the present time and what will be done in the future. Each province will show selections from tapes produced by its ETV department. Teachers from across Canada who are using TV or TV recorders in a novel way will be given the opportunity to describe their work. Commerical TV equipment will be on display.

Anyone interested in attending this conference or any teacher who would like to describe his or her novel endeavors in ETV is asked to contact John C. Egsgard, 1515 Bathurst Street, Toronto 349, Ontario.

## Mathematics Curriculum News

- The following series are being tried on an experimental basis in various schools at the junior high school level:


## Developing Mathematical Ideas, Books 1 and 2, by Sobel et al

 (Ginn and Company)Modern School Mathematics, Books 1 and 2, by Dolciani et al (Thomas Nelson and Sons)

Mathematics, Concepts and Applications, Books 1 and 2, by Van Engen et al (Gage and Company)

- The new pattern of Mathematics 13, 23 and 33 which is designed to replace Mathematics 12, 22 and 32 has the following objectives:

1. to assist the student in the learning process by developing mathematical concepts through an inductive approach;
2. to use applications from various areas such as mensuration, science, and the real world, for the purpose of reinforcing concepts;
3. to develop powers of analyzing problems and presenting solutions in a clear manner;
4. to develop and maintain an understanding of the operations and concepts of mathematics by using an essential core supplemented by exploratory topics;
5. to develop and maintain skill in mathematical operations by these means.
[^1]- The Board approved a plan for phasing out the following courses and texts:

Mathematics 12: Mathematics in Practice (Brown, Bridge and Morrison, revised edition) at the end of the 1970-71 school year

Mathematics 22: Senior Technical Mathematics (Heywood) at the end of the 1971-72 school year

Mathematics 11: Canadian Business Mathematics, Book 1, at the end of the 1969-70 school year

Mathematics 21: Canadian Business Mathematics, Book 2, at the end of the 1970-71 school year.

## MCATA Speakers' List

Revised for 1969-70 by M.R. Falk and Professor A.S.B. Holland

The purpose of this list is to provide a source of available speakers classified according to general and specific topics as well as a recommended type of audience. Interested groups should contact the speaker directly, and all details of arrangements should be planned to the mutual satisfaction of the speaker and the group.

Many outstanding talents backed up by years of study and experience are represented on this list. We hope that it will be put to good and frequent use.

Aggarwala, B.D., Department of Mathematics, The University of Calgary, Calgary. Topics in applied mathematics - for example "What is Applied Mathematics?" Senior high school students and/or teachers.

Anderson, Ruby, teacher and consultant in mathematics, Vincent Massey Junior High School, Calgary.
Topics in junior high school mathematics. Junior high school teachers.
Amour, Carol, Senior High Mathematics Curriculum Committee, Ernest Manning High School, Calgary. Vice-President of MCATA, 1969-70.
Topics in senior high school mathematics curriculum and instruction. Senior high school teachers.

Atkinson, Tom, Faculty of Education, University of Alberta, Edmonton. "Number Systems", "Problem Solving". Secondary school teachers.

Baecker, Harry D., Department of Mathematics, The University of Calgary. Computer programming, compilers, data banks, computers and high school curricula.

Beaton, Mary, Faculty of Education, The University of Calgary, Calgary. Topics in mathematics curriculum and instruction. Elementary school teachers.

Blumell, R.E., Department of Education, 9th flr. Bowlen Bldg., Calgary. Topics in mathematics education, especially pertaining to high schools.

Bruns, A., superintendent, Lacombe County schools, Lacombe. Topics in elementary mathematics curriculum. Elementary teachers.

Bye, Marshall P., junior and senior school mathematics consultant, Calgary School Board, Calgary.
Topics in secondary school mathematics curriculum and instruction. Secondary school teachers.

Cleveland, Ray, Department of Ed.C.I., The University of Calgary, Calgary. Topics in mathematics education, especially "active learning" (He is a co-author of the STM series used in Alberta)

Falk, M.R., Department of Ed.C.I., Faculty of Education, The University of Calgary, Calgary.
Topics in mathematics instruction as arranged.
Farajat, H.K., Head, Department of Mathematics, The University of Calgary. Topics in mathematics, mathematics instruction and mathematics organizations.

Freedman, H.I., Department of Mathematics, University of Alberta, Edmonton. Topics in mathematics - for example, "Large Numbers", "What are the Odds?", "How Many Equations Are There?"

Gibb, A.A., Associate Dean, Faculty of Education, The University of Calgary. Topics in mathematics education. Any interested group.

Harrison, D.B., Faculty of Education, The University of Calgary, Calgary. Contemporary learning theories and mathematics (secondary) learning (Piaget, Bruner, Dienes, Skemp). Reflective intelligence and mathematics learning. Secondary teachers.

Holland, A.S.B., Department of Mathematics, The University of Calgary, Calgary. Topics in geometry and calculus. "Geometry in the High School System", "Geometry in the University". Teachers of mathematics.

Jagoe, Olive, Head, Mathematics Department, Henry Wise Wood High School, Calgary. Topics in secondary school mathematics curriculum and instruction. Secondary school teachers.

Kean, Jim, Lord Beaverbrook High School, Calgary.
Topics in mathematics instruction. Mr. Kean is President of the MCATA for 196970 and wishes to sepak to groups of mathematics teachers who are interested in organizing Regional Mathematics Councils. He may be able to speak entirely at MCATA expense.

Kieren, T.E., Faculty of Education, University of Alberta, Edmonton.
"The Computer in Mathematics Learning and Teaching", "Creative Problems in Mathematics", "Very Elementary Functions", or "Functions - How and Why?"

Lancaster, Peter, Department of Mathematics, The University of Calgary, Calgary. Topics in mathematics - for example, "Can it be calculated?" Secondary school students and/or teachers.

Lindstedt, S.A., Head, Education and Curriculum and Instruction, Faculty of Education, The University of Calgary, Calgary.
Topics in mathematics education. Any interested group.
Macki, J.W., Department of Mathematics, University of Alberta, Edmonton. Topics in mathematics as arranged.

Milner, E.C., Department of Mathematics, The University of Calgary, Calgary. Topics in set theory or graph theory - for example, "Arithmetic of Infinite Numbers". Grade XII students or high school teachers.

Naimpally, S.A., Department of Mathematics, University of Alberta, Edmonton. Topics in mathematics - for example "Light-hearted Approach to Topology". Secondary school mathematics students.

Nelson, L.D., Faculty of Education, University of Alberta, Edmonton. Topics in mathematics education - for example, "Concept Development in Primary Mathematics". Elementary school teachers.

Neubeld, K.A., Façulty of Education, University of Alberta, Edmonton. Topics in mathematics education - for example, "Structured Mathematics Teaching Bruner and Bloom". Secondary school mathematics teachers.

Radomsky, Ron, Ernest Manning High School, Calgary.
Administrative problems related to secondary school mathematics. Mathematics for the low achiever. Secondary teachers and/or administrators.

Rogers, T., Department of Mathematics, University of Alberta, Edmonton. Topics in mathematics - for example, "Inequalities", "Group Theory", "Basic Calculus", "Elementary Game Theory", "Elementary Optimization Problems". Grade XII mathematics students and/or mathematics teachers.

Sahney, B.N., Department of Mathematics, The University of Calgary, Calgary. Topics in convergence of series - for example, "Series Expansion and Their Convergence". Grade XII mathematics students and/or mathematics teachers.

Sawada, D., Faculty of Education, University of Alberta, Edmonton. "Computers in Elementary School Mathematics". Teachers (and/or administrators).

Sigurdson, S.E., Department of Secondary Education, Faculty of Education, University of Alberta, Edmonton
"Secondary School Mathematics Curriculum". "Discovery Teaching". Mathematics teachers.

Strickland, Roy, elementary consultant, Calgary Public School Board, Calgary. Topics in teaching elementary mathematics. Elementary teachers.

## BOOK REVIEW

MATHEMATICS AND SOCIETY - The COSRIMS Report


Teachers of mathematics and mathematicians are regularly called upon to answer questions such as "What is mathematics? What role and importance does it have in life? What mathematics should we teach in the schools and to whom?" And teachers are vitally concerned with yet another question: "How do we do it?" Any responsible group considering the problem of the mathematics curriculum has had to address itself to these questions. They are dealt with again in a report that is attracting widespread interest.

Several years ago the Division of Mathematical Sciences of the National Academy of Sciences, becoming concerned about the decreasing federal support for research in mathematics, appointed the very prestigious Committee on Support of Research in the Mathematical Sciences (COSRIMS). The report of this Committee has been published in three volumes: A Report (vol.1), Undergraduate Education (vol.2), and A Collection of Essays (vol.3).

While a report on research ordinarily has limited relevance to the world of the man in the street or to the daily work of the teacher in his classroom, this report is a signal exception to that rule. Especially the first half of the first volume and all of the third volume will be of prime interest to both the general reader and the classroom teacher.

Beginning with an articulate, eloquent, detailed, and up-to-date account of the role of mathematics in modern life, the first volume traces the mathematicization of our culture from physics through the other physical sciences and now the life sciences. Also discussed is the current penetration of mathematics into the social and behavioral sciences and into traditionally humanistic areas, not to mention the worlds of government, industry, and business. Applications in economics, anthropology, sociology, and even linquistics are touched upon.

With the vastly increased need for people who can understand and use mathematics, teachers should seek, at all times, illustrations of the nature and uses of mathematics in order to motivate more students to pursue mathematical studies. The Mathematical Sciences: A Report is clearly a gold mine of such motivational illustrations.

The brilliant third volume, A Collection of Essays, was written by stars of the first magnitude in the galaxy of mathematicians. It provides nontechnical summaries of the current state of several branches of core mathematics such as complex analysis, functional analysis, differential topolegy, combinatorial analysis, point-set topology, and the continuum hypothesis. Exciting illustrations are given of the coordination of mathematics and other disciplines: the use of mathematics in the social sciences, the part played by analytic functions in research on the elementary particles, the role of mathematics in economics, the development of mathematical linguistics, and the use of mathematics in the biomedical sciences.

Certainly these volumes should be on the shelves of all teachers of mathematics. They will be useful to students as well - and, yes, to interested parents. The first two volumes, The Mathematical sciences: A Report (\$6.00) and The Mathematical Sciences: Undergraduate Education (\$4.25), are available from the Printing and Publishing Office, National Academy of Sciences - National Research Council, 2101 Constitution Avenue, Washington, D.C. 20418. The third volume, The Mathematical Sciences: A Collection of Essays ( $\$ 8.95$ ) is available in the paper edition ( $\$ 3.95$ ) from The M.I.T. Press, Massachusetts Institute of Technology, Cambridge, Mass. 02142.

Julius H. Hlavaty
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## EXECUTIVE COMMITTEE, 1969-70

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[^0]:    ${ }^{1}$ J.E. Wicks and T.F. Rieger, The Alberta Teaching Force, September, 1966. Research Monograph No. 12 (Edmonton, Alberta: The Alberta Teachers' Association, 1967) p. 13.

[^1]:    - The Secondary School Curriculum Board approved the offering of Mathematics 13 and Mathematics 23, effective September, 1970, and recommended the following texts:

    Mathematics 13, by Dean and Graham (Holt, Rinehart and Co. 1969)

    Mathematics 23, by Dean and Graham
    (Holt, Rinehart and Co. 1969)

    - A proposed change in the Mathematics 10 and 20 courses with an indication of multiple authorization of new texts was tabled until the spring meeting of the Board.

