MORE THAN $S_N = \frac{N(N+1)}{2}$:

a class project for math option students

D. W. Annesley

DAY I

The lesson begins with a short *lecture* - for those who are missing this word in their vocabularies, it means to tell students something - about Karl Friedrich Gauss and how he found the sum of the integers from 1 to 100 in just a fraction of the time expected by his teacher. I asked the students in my class how long it would take them to do this formidable job and guesses ranged from five minutes to one hour. My students thought I must be relating a fairy tale as just nobody could do the job as fast as Gauss did! It was at this stage that we developed, together, the formula $s_n = \frac{n(n+1)}{2}$ The calculation procedure was mastered very quickly by having various groups of students sum different sequences. A summary of these sums appears below:

Series	1-10	1-20	1-30	1-40	1-50	1-60	1-70	1-80	1-90	1-100	1-110	1-120
Sums	55	210	465	820	1275	1830	2485	3240	4095	5050	6105	7260

Series	1-130	1-140	1-150	1-160	1-170	1-180	1-190	1-200
Sums	8515	9870	11,325	12,880	14,535	16,290	18,145	20,100

DAY II

The class is begun by asking students, "Could we arrive at these sums without using the formula $s_n = \frac{n(n+1)}{2}$?" It turns out that it can be done in a unique way. Students will very quickly begin noticing rather peculiar number patterns.

Mr. Annesley is mathematics coordinator at Avalon Junior High School, Edmonton.

Sequence of last two digits in alternate sums starting at 55	Sequence of last two digits in alternate sums starting at 210
55	10
65	20
75	30
85	40
95	50
05	60
15	70
25	80
35	90
45	100

What about the other digits?

How do you arrive at these sequences?

4) Note:	12 - 4 = 8
12)	24 - 12 = 12
24)	40 - 24 = 16
40)	61 - 40 = 20 + 1
61)	85 - 61 = 24
85)	113 - 85 = 28
113)	145 - 113 = 32
145)	181 - 145 = 36
181)	

2) Note:	8 - 2 = 6
8)	18 - 8 = 10
18)	32 - 18 = 14
32)	50 - 32 = 18
50)	72 - 50 = 22
72)	
98)	
128)	
162)	
201)	

Differences of pairs of sums

465 - 55 = 410
1275 - 465 = 810
2485 - 1275 = 1210
4095 - 2485 = 1610
and so on.

Powers of 10

Series	Sums
1 - 10	55
1 - 100	5050
1 - 1000	500500
1 - 10000	50005000

Series	Sums
1 - 100	5050
1 - 200	20100
1 - 400	80200
1 - 800	320400

etc.

Once students begin searching for number patterns, all kinds will be found and a genuine interest will be developed. *Students will not preoccupy themselves* thinking, *What good is all this stuff?*

To the readers of this article: Could you determine certain sums of integers using some of the patterns suggested or others you have considered? GOOD LUCK!