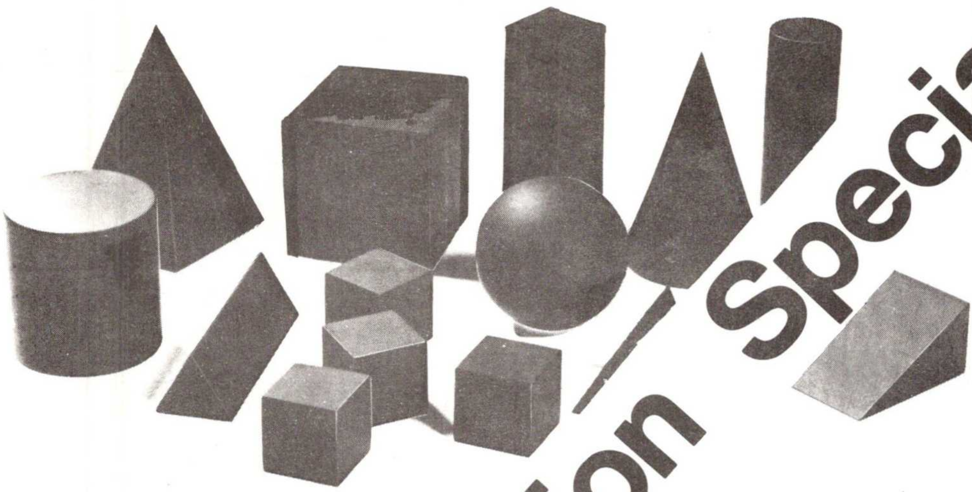
THE ALBERTA TEACHERS' ASSOCIATION

MATHEMATICS COUNCIL

NEWSLETTER

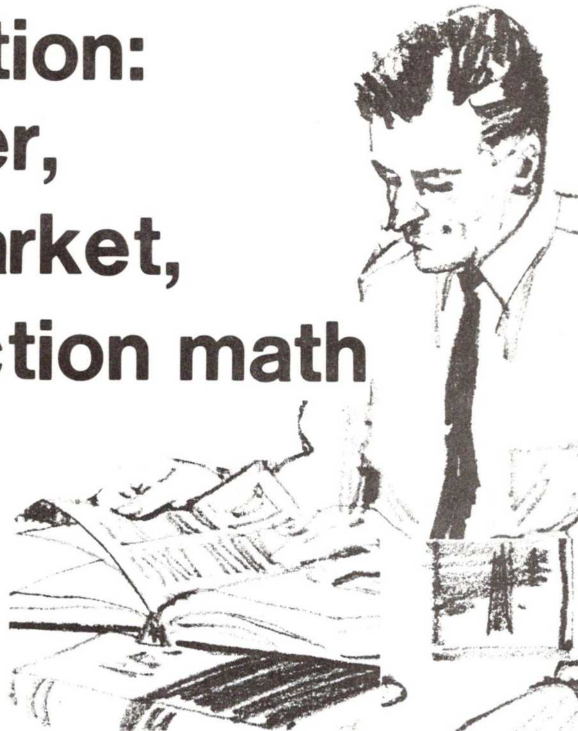
Volume IX, Number 4, June, 1970

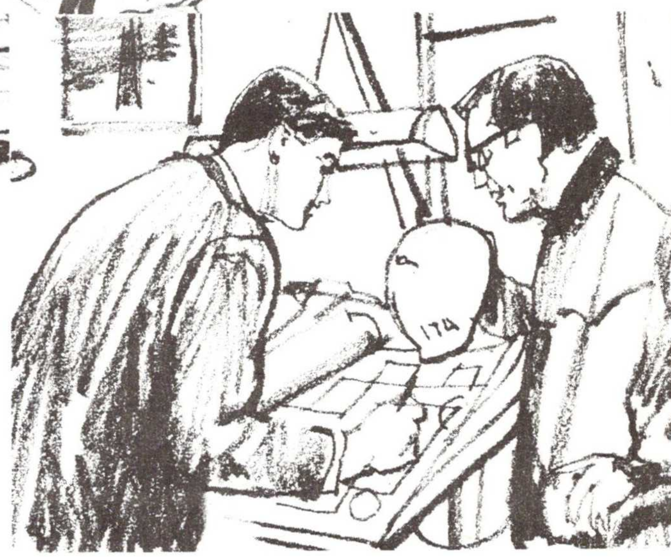


Math Option Special

Newsletter Editor: Mary Beaton
Faculty of Education, University of Calgary

Math option: computer, stockmarket, construction math





Bruce Hedderick

When last year's teaching drew to a close, it was suggested by Marshall Bye, the mathematics supervisor, that a mathematics option be started in Milton Williams Junior High School. Mr. Hamilton, the principal, scheduled choices for student options in mathematics, science, social studies, drama, art, French, language arts, and music. Thirty-six students chose math option out of five Grade VIII classes. The classes were scheduled for three periods per week.

The math option deals with four areas. For the first three areas, the students are divided into groups of twelve students each. Computer math is the first

---

Mr. Hedderick teaches mathematics at Milton Williams Junior High School in Calgary.

area and the intent of this group is to investigate and build simple computers, getting more sophisticated as the year progresses. Stockmarket math is the next area and the intent of this section is to invest $20,000 imaginary money scientifically and mathematically in bonds, and the Calgary and Toronto Stock Exchanges. The construction math, in the third area, deals with the designing of a boat-camper combination that can be towed behind an eight-cylinder car with a maximum price tag of $5,000. If the design looks successful a scale model will be built. The last area involves all the students in the option class in making math puzzles and questions on recipe cards, which allows a variety from the textbook and creativeness on the part of the option class. For the designing of problems a list of story ideas is given including such things as business problems, cost of articles, travel and time problems. Finally a general area guideline is provided, within which students should work on natural numbers, signed numbers, decimals or fractions.

Each individual in the computer company investigates a specific area such as binary language, abacus, Napier's bones, exponent computer, logarithms, slide rule, monographs, digital and analog computers, adding and calculating machines, toy and game computers, and the uses and limits of computers. The stockmarket section deals specifically with bonds, convertible stocks, mutual funds, oil section of common stocks, industrial section of common stocks, annual reports, daily reports in the paper, difference between speculating and investing, and the best method of graphing stocks. The construction company delves into power campers, boat trailers, hull design, interior design, and types of construction materials.

The three groups would like to make field trips to companies dealing with their particular areas. For example, the stockmarket company would like to visit the Calgary Stock Exchange and have a broker speak to the class. The computer company would like to visit a computer installation such as the University Data Centre or International Business Machines, and the boat-camper group would like to visit a company such as Small Craft of Canada Ltd. Some of these field trips have already taken place.

From this point on, it is hoped and expected that the three groups will slowly take over more direction of their own affairs, so that they may learn leadership, responsibility, and cooperation toward a single goal. An excerpt from one of my student's essays on this topic may drive home the point: "In the field of mathematics there is more than just addition, subtraction, multiplication and new forms of applying answers. Things like computers, the stock market, and construction are some of these areas. These topics can be more interesting, more fun, and maybe we can even learn more in terms of this new, complex kind of mathematics."

For teachers wishing to attempt the math option two suggestions might be helpful. First, let us not allow the math option to become just a remedial math period when the province allows us to design our own courses. And second, if you are teaching math option, pick three or four topics with which you feel comfortable; and don't worry too much about the noise level because your students are going to be active.

Reprinted from *The Math Post*, Volume 2, No. 3

Academic math program at Simon Fraser

W. G. Sorge

This year we are attempting a "B" option in each grade level. We have two classes in Grade VII and one each in Grades VIII and IX.

GRADE VII

Mrs. Acton and I are attempting to develop a structured course for our students. We started with a unit on number systems other than base 10. We are now progressing in two different directions so that we can better analyze our programs at year end.

The Grade VII program I have developed has been based on Irving Adler's *Magic House of Numbers,* plus any interesting material I am able to find in my reading. The program to date includes the following areas of study:

(a) quick check method for base 10 calculations (casting out 9's)
(b) number curiosities (patterns, etc.)
(c) triangular numbers
(d) square numbers
(e) cube numbers
(f) perfect numbers
(g) puzzle problems with and without numbers
(h) magic squares
(i) topology (moebius strips)
(j) games
(k) items of interest from research.

GRADES VIII AND IX

Since I am involved only in teaching the Grade VIII and IX programs, I have allowed considerable overlapping in the two and will consider them together. More freedom and less teacher direction is allowed with these levels. Approximately one period in three will involve teacher-directed activities while the other two periods involve pupil interest.

PROGRAM OF BASIC UNITS OF INSTRUCTION

(a) puzzles, commercial games, and library projects
(b) teacher-directed activities:
    pure math (areas not covered in text - for example, vectors)
    general interest (from personal reading, any activity I feel informative and interesting will be attempted), for example, hexahexaflexagons and flexatubes from Scientific American publications or math materials purchased from J. Weston Walch, Portland, Maine.
(c) games (I will have students make their own games after introducing several from the Walch publications).

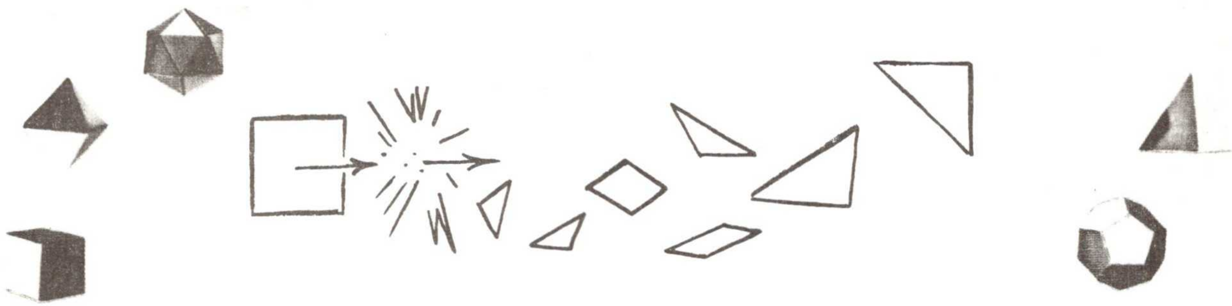
Mr. Sorge is vice-principal at Simon Fraser Junior High School.

EQUIPMENT REQUIRED FOR A SUCCESSFUL MATH PROGRAM

(a) good library, scissors, paper, rulers, glue, pipe cleaners and straws, etc.
(b) money for the purchase of math oriented games, puzzles, and for texts.
(c) adequate space.

EQUIPMENT FOUND USEFUL IN OUR PROGRAMS

(a) *Magic House of Numbers* I. Adler
(b) *Scientific American Book of Mathematical Puzzles*
(c) some cubes (excellent)
(d) Hi Q and Brainbuster
(e) Instant Insanity and number cubes
(f) wooden puzzles: cubes, octagons, boats, pyramids, tower of Hanoi, barrels, spheres
(g) games: Tuf, Wff'n Proof series - Wff'n Proof, On Sets - (games of logic), Equations
(h) Publications from J. Weston Walch: bingo, graphing pictures, Yes - Math Can be Fun (excellent), For the Math Wizard (excellent), Games for Learning Mathematics, Poster Sets on Math (wide variety to choose from).



DIFFICULTIES

(a) lack of time for adequate research
(b) maturity of child involved
(c) evaluation
(d) teacher's math background

The following chart is an attempt to show mathematical concepts which some of our projects have introduced.

PROJECT	MATHEMATICAL CONCEPT
1. bases other than ten	(a) relationship of different numeration systems to our system (b) students learn how our numeration system was formed (c) place value (d) reading numerals (e) addition, subtraction, multiplication and division in different bases

2. check method of casting out nines	(a) drill in mathematical computations with excellent way of checking solutions quickly and accurately (b) we demonstrate how system applies to bases other than ten
3. triangular, square, cubic, rectangular and perfect numbers	(a) our numeration system is made up of a series of different numbers forming different geometrical shapes (b) introduces idea of research in mathematics because no one knows if there is a seventh perfect number
4. word puzzles with and without numbers	(a) demonstrates importance of reading, interpretation, collection of relevant facts, and solution finding
5. wooden puzzles, Soma, Brainbuster, etc.	(a) students become familiar with prisms (b) assembly of puzzles takes considerable thought and patience (c) teaches organization and symmetry
6. flexagons	(a) construction of angles (b) types of triangles (c) formation of different geometrical figures from a strip of connected equilateral triangles, e.g., trapezoid, parallelogram, centennial design, etc. (d) introduction to regular solids, e.g., tetrahedron, octahedron
7. magic squares	(a) pattern of formation (several) (b) sequence of numbers
8. tower of Hanoi	(a) relationship to base 2 (b) formation of formula to determine number of moves for x number of blocks
9. poster problems (Walch Publications)	(a) mathematical calculations relating to everyday business world
10. games	(a) mathematical calculations (b) proper use of parentheses (c) set theory (d) logic

Reprinted from *The Math Post* Volume 2, No. 3

MORE THAN $S_N = \frac{N(N+1)}{2}$:

a class project for math option students

D. W. Annesley

DAY I

The lesson begins with a short *lecture* - for those who are missing this word in their vocabularies, it means to tell students something - about Karl Friedrich Gauss and how he found the sum of the integers from 1 to 100 in just a fraction of the time expected by his teacher. I asked the students in my class how long it would take them to do this formidable job and guesses ranged from five minutes to one hour. My students thought I must be relating a fairy tale as just nobody could do the job as fast as Gauss did! It was at this stage that we developed, together, the formula $s_n = \frac{n(n+1)}{2}$ The calculation procedure was mastered very quickly by having various groups of students sum different sequences. A summary of these sums appears below:

Series	1-10	1-20	1-30	1-40	1-50	1-60	1-70	1-80	1-90	1-100	1-110	1-120
Sums	55	210	465	820	1275	1830	2485	3240	4095	5050	6105	7260

Series	1-130	1-140	1-150	1-160	1-170	1-180	1-190	1-200
Sums	8515	9870	11,325	12,880	14,535	16,290	18,145	20,100

DAY II

The class is begun by asking students, "Could we arrive at these sums without using the formula $s_n = \frac{n(n+1)}{2}$?" It turns out that it can be done in a unique way. Students will very quickly begin noticing rather peculiar number patterns.

Mr. Annesley is mathematics coordinator at Avalon Junior High School, Edmonton.

Sequence of last two digits in alternate sums starting at 55	Sequence of last two digits in alternate sums starting at 210
55	10
65	20
75	30
85	40
95	50
05	60
15	70
25	80
35	90
45	100

What about the other digits?

How do you arrive at these sequences?

4) Note:	12 - 4 = 8
12)	24 - 12 = 12
24)	40 - 24 = 16
40)	61 - 40 = 20 + 1
61)	85 - 61 = 24
85)	113 - 85 = 28
113)	145 - 113 = 32
145)	181 - 145 = 36
181)	

2) Note:	8 - 2 = 6
8)	18 - 8 = 10
18)	32 - 18 = 14
32)	50 - 32 = 18
50)	72 - 50 = 22
72)	
98)	
128)	
162)	
201)	

Differences of pairs of sums

465 - 55 = 410
1275 - 465 = 810
2485 - 1275 = 1210
4095 - 2485 = 1610
and so on.

Powers of 10

Series	Sums
1 - 10	55
1 - 100	5050
1 - 1000	500500
1 - 10000	50005000

Series	Sums
1 - 100	5050
1 - 200	20100
1 - 400	80200
1 - 800	320400

etc.

Once students begin searching for number patterns, all kinds will be found and a genuine interest will be developed. *Students will not preoccupy themselves* thinking, *What good is all this stuff?*

To the readers of this article: Could you determine certain sums of integers using some of the patterns suggested or others you have considered? GOOD LUCK!

Junior high school math option

Harry Topolnitsky

The introduction of an option in junior high mathematics is a matter of some concern to teachers involved in setting up programs in their schools. To aid the teachers, a panel was formed for the annual fall mathematics conference. This panel, which has also been active at two conventions, includes Ted Rempel of Londonderry Junior High, Edmonton, as chairman; Dennis Annesley, Avalon Junior High, Edmonton; Jim Barnes, Montgomery Junior High, Calgary; Dick Daly, D.S. MacKenzie Junior High, Edmonton; and Harry Topolnitsky, Ellerslie Junior High, County of Strathcona. The whole topic of junior high mathematics option was divided into the following sub-topics, to which the members of the panel reacted.

PERSONNEL, RESOURCES AND FACILITIES

It is necessary to keep in mind that each school is individual in nature. Personnel, resources, and physical facilities available or applicable in one situation may be totally unrealistic in another.

It seems logical to assume that the option should be handled by teachers of mathematics. This may be true for a large number of the possible topics, but when you consider the variety of available topics, you begin to realize that some of them may receive better guidance from teachers other than mathematicians. The section dealing with Mathematical Instruments could, perhaps, be best handled by a science teacher who is familiar with the calculator, sextant, micrometer, calipers, etc., and art teachers might prove valuable with mosaics, designs, symmetry and optical illusions. The History of Mathematics or Biographies of Mathematicians may be in the domain of the language arts teacher; Careers in Mathematics may best be handled by the guidance counsellor. Thus we see that the realm of mathematics may be extended to include other teachers because the topics are so varied that no one teacher can be expected to be conversant with all of them. However, the teacher should have initiative and act as a resource person or consultant.

Mr. Topolnitsky, principal of Ellerslie Junior High, County of Strathcona, is a teacher of mathematics.

Resources and facilities vary with the size of the school and with the school boards in their allotment of funds for this purpose. It is difficult to say what is ideal in the way of resources and facilities. Some teachers may prefer to have a room assigned as the math room in which all the projects are done and the displays are stationary. Others may wish to have a mobile class. The science laboratory seems like the logical place to work with mathematical instruments. The social studies room, if such exists, might be used for map work, map coloring, topology, etc., which might be left on display for the social studies students. The library should be available for research work and display of art work done in relation to mathematics and mathematicians.

Many devices can be made for a fraction of the cost of commercial ones. A demonstration slide rule may be made by the industrial arts students instead of purchasing one for $47.50. Individual slide rules may be purchased for as low as 87 cents each. Once the students learn to use a slide rule, they prefer to have one for their personal use.

I fully realize that some of the suggestions offered are impractical in view of the resources and facilities of the individual schools. We may be content with or forced into using one corner of the home room for the option.

CLASSROOM ORGANIZATION

The enrolment in the option may vary from fifteen or fewer to thirty-five per class. Although the election of an option should be on the basis of strength and interest of the student, this is not always the case nor always practical. As a consequence, the class may be composed of students who have interest but not much strength, strength but little interest, both strength and interest, and those who had no alternative.

In some situations it may be advisable to have the whole class work on one project. In this case the teacher would present a list of topics from which the students would democratically choose one. If, for example, the topic of paper folding is selected, it might involve all the students in making airplanes as a basis to paper folding. This might be followed by a lesson on drawing regular polygons. To complete the project, the students might work in groups or independently to construct polyhedrons.

Some teachers prefer to have the class divided into groups of students who have the same interest. Although an ideal group size may be four or five students, it will vary depending on the interest shown.

The interests of the students may be so varied that they would perform best independently. It is also possible to have a combination of groups and individuals. The teacher is in a position to assess which of the above would be suitable to his class.

TIME ALLOTMENT

The *Junior High School Handbook* suggests that the time allotted to the options be between 75 and 175 minutes or the equivalent of 2 to 5 periods per week. Where feasible, blocking of the options on the timetable is perhaps the most desirable, but with the variety of situations in our schools, this is not always possible. A six-day timetable seems to be the most suitable. This allows for the options to be offered every second day. Schools on the semester or trimester system would offer the option to one group of students for only a part of the year, and use a rotation. Another possibility is to offer the core in mathematics for four periods and to use the fifth period for exploring the mathematics option where no regular time can be timetabled for the option.

Setting a time limit for a project or topic is advisable. This should prevent the students' interests from waning. The anticipation of a new project should keep the students interested in completing the project at hand in a minimal time.

EVALUATION OF STUDENT PROGRESS

The evaluation of student progress in the options varies with the requirements of the different school systems. Some of the methods for evaluation are: (a) a five-point scale, (b) three-point scale, (c) written comment, (d) interview, and (e) no indication of progress.

However, where an evaluation is necessary or desirable, some criteria must be used. Following are several points that may be considered.

(a) student participation
(b) quality of records or finished product
(c) growth
(d) use of time
(e) independent study
(f) ability to organize
(g) ability to follow instructions
(h) effort
(i) enthusiasm
(j) completeness of project.

A log may be kept for each student in which would be recorded the title of the project, the date started, the date completed, and any of the above criteria that are applicable.

EVALUATION OF PROGRAM

This may be the most difficult task that we face. However, through participation we can learn about good points or ideas to propagate and pitfalls to avoid. Perhaps an indication of the success of the program might be determined by the students themselves - their desire to re-enrol in the option for another year.

Ordinarily, the proper evaluation of a program requires more time than we have had this year. The success of the program depends to a large extent on the teacher in charge - his interest, enthusiasm, and suitability will be reflected in the final results.

The program has a lot of merit and gives the teacher an opportunity to use his initiative. It provides increased flexibility in the area of content and methodology. The option should cultivate interest, develop special abilities, and provide for research.

To assist teachers and administrators in planning the mathematics option, the mathematics coordinators of the Edmonton Public School System have spent considerable time and effort in the preparation of the topics suggested below:

Business Mathematics

(a) Stock Market - stocks, bonds, corporations, trading and quotations
(b) Banking - saving accounts, checking accounts
(c) Instalment Buying - kinds, interest rates
(d) Consumer Buying - discounts, comparative shopping
(e) Mortgages - house, second mortgage
(f) Insurance - home, car, life
(g) Taxes - kinds, assessment, rates
(h) Car Operating Expenses - fees, operating costs, credit cards, depreciation costs, instalment paying, insurance
(i) Bookkeeping
(j) Operation of a School Business (e.g., a store) - organization, shares, operation of the business, reports.

Recreational Mathematics

Magic Squares, Cross - Number Puzzles, Paper Folding, Number Sequences, Numerology, Math Challenges or Puzzles, Codes, Mathematics Cartoons, Math Games - Krypto, Radix, Equations, Wff'n Proof, etc., Aestheometry (three-dimensional string constructions).

Number Theory

(a) The Natural Numbers - sums of consecutive numbers, consecutive odd and consecutive even numbers; triangular numbers; Fibonacci numbers.
(b) Prisms and Composites - distribution of primes, relative primes, Sieve of Eratosthenes.
(c) Divisors of a Number - prime numbers or factors, GCF, divisors, perfect numbers
(d) Facts of Number Theory - Fermat's two-square theorem, Lagrange's theorem.
(e) Conjectures in Number Theory - Goldbach conjecture, Fermat's last theorem.

Topology - "Rubber Space Geometry"

(a) Two-Dimensional Topology - simple closed, transformations
(b) Networks
(c) The Moebius Strip
(d) Map Coloring Problems
(e) Mazes
(f) Three-Dimensional Topology - Kline bottle

Set Theory and Logic

(a) Truth Tables
(b) Implication
(c) Simple Proof
(d) Relating Truth Tables to Set Theory
(e) Using Intersection and Union to Teach GCF and LCM

Probability and Statistics

1. Probability: (a) mutually exclusive events
   (b) independent events
   (c) experiments (coins, die)
   (d) odds and simple games
   (e) fair games (math expectations)
2. Statistics: (a) histograms
   (b) measures of central tendency (mean, mode, medium)
   (c) surveys to illustrate the above concepts

Math Instruments

Slide rule, transit, sextant, abacus, calculator, micrometer, calipers.

Measurement

1. Direct Measure: (a) metric and British
   (b) greatest possible and relative error
   (c) scientific and standard notation
   (d) significant digits
2. Indirect Measure: (a) scale drawing
   (b) similar triangles
   (c) numerical trigonometry
   (d) vectors

Finite Math Systems

1. Group Theory: (a) modular arithmetic (clock)
   (b) permutations
   (c) plane transformations
   (d) 2 x 2 matrices under addition
2. Fields: (a) real numbers
   (b) prime modular systems

Geometry

(a) Plane Constructions
(b) Geo-Board Applications
(c) Optical Illusions
(d) Mosaics and Designs
(e) Finite Geometrics
(f) Tangrams
(g) Paper Folding
(h) Symmetry
(i) 3 - D Constructions - prisms, pyramids.

Computer Science

(a) Numeration Systems - base 10, 5, 2
(b) Functional Relationships Between Parts of a Computer
(c) Flow Charts
(d) Programming in APL
(e) Concentrated Area in Mathematics Using Flow Charts
(f) Field Trip to a Computing Center

Graphs

(a) Pictographs
(b) Bar Graphs - horizontal, vertical
(c) Circle
(d) Rectangular
(e) Line Graphs

History

1. Ideas:
   (a) Need for Counting - early methods, importance of zero, calendars
   (b) Need for Calculating - trade
   (c) Need for Measuring - surveying, building, navigation
2. People: -

Thales	Hipparchus	Pascal	
Pythagoras	Ptolemy	Newton	
Plato	Copernicus	Leibniz	
Euclid	Galileo	Boole	and others
Archimedes	Napier	Einstein	
Eratosthenes	Kepler	Euler	
Apollonius	Descartes	Gauss	

BOOK REVIEW

Gerald Worger

Mr. Worger is completing the M.Ed. degree in Curriculum and Instruction in Mathematics at the University of Calgary.

THE HOUGHTON MIFFLIN MATHEMATICS ENRICHMENT SERIES

Last month, three books in the Houghton Mifflin Mathematics Enrichment Series were reviewed:

1. *Stereograms* - Stover
2. *Mosaics* - Stover
3. *Legislative Apportionment* - Meder

This month the last four booklets in the series are reviewed.

The 52-page *Topics from Inversive Geometry* by Albert E. Meder, Jr. deals with transformations of a fixed circle, in a plane. A transformation is defined as a pairing of points, according to some law or rule. The problem of transformations is dealt with in three ways: intuitively, algebraically, and geometrically. The book, therefore, lends itself to various levels of treatment by a teacher.

Most of the work on transformations is done using the Cartesian coordinate system in two dimensions. However, polar coordinates and three-dimensional transformations are discussed. Though the topics are presented in a simple manner in the first part of the book, later treatments require some knowledge of trigonometry, general equations for lines and circles, locus parabolas and other conics. The book should be useful at any secondary level.

Sequences is an 84-page book by Katherine E. O'Brien. It is designed for students having about two years of algebra and geometry. Topics covered include: Euler's ϕ Function; recursive definitions of sequences; π and e; and harmonic series. The treatment of the sequences brings in such concepts as function relationships, successive differences between members of a sequence, generalized algebraic terms of a sequence, bounds and limits.

Many theorems, relating to sequences, are presented, but many are not proven. It is suggested that students apply the theorems to different sequences to see that they work. The book would likely be useful to high school teachers who wish to provide further experiences for their students.

Another useful book for the high school mathematics teacher is *Induction in Mathematics*, by Louise Johnson Rosenbaum. The pamphlet is quite short (30 pages) but it comprehensively covers the topic of inductive proof. The rationale behind the method is adequately presented, and there are many excellent examples discussed and problems given. The presentation in the book might be followed by a teacher, rather than that of most texts.

Fibonacci and Lucas Numbers, by Verner E. Hoggatt Jr., is the last of the seven books in the series. The concept of Fibonacci numbers is developed in context with a problem of finding out the number of rabbits produced, given the breeding rate of rabbits. Various forms of Fibonacci numbers are discussed - those arising from work with the "Golden Section" of a line, as well as those arising from geometrical work with triangles and rectangles.

The different methods of generating Fibonacci and Lucas numbers are discussed, along with proofs of various theorems dealing with the numbers. The last part of the book relates Fibonacci and Lucas numbers to other mathematical areas such as Pascal's triangle; 2 x 2 matrices; completeness property of sequences; identities, periodicity and divisibility.

The series of books would be an interesting and useful addition to a teacher's library or a school library. Many students would enjoy working through the books on their own, and most teachers would find the different topics and their approaches worthwhile incorporating into their own lessons.