PROBLEM PAGE

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The Mathematics Council, ATA, thought that the addition of a problem page to the *Newsletter* would be another item of interest to you, the reader. Problems for solution should be sent to the Editor, Mathematics Council Newsletter,

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Do not forget to include a solution for each problem if one is available; and mention your name and school, please. If solutions are typed, they should be double-spaced; if they are handwritten, they should be in clear writing. We would like to receive from our readers many, many problems of a topical nature, covering all branches of mathematics and of not too advanced a nature.

PROBLEMS FOR SOLUTION

1. Prove that if the bisectors of the base angles of a triangle are equal in length, then the triangle is isosceles. (This is an old problem, rather like the one: If the medians of a triangle are equal in length, then it is isosceles.)

2. Given Sin A, Sin B, Sin (A + B): show how to find Cos (A + B) using only rational combinations of the data.

   (Example: $\text{Cos } (A + B) = \sqrt{1 - \text{Sin}^2 (A + B)}$ is no good since this is not a rational use of the data). This problem had considerable practical use and was asked by the War Office in Britain in World War II.

3. Prove that the only number which will divide two consecutive numbers of the form $n^2 + 1$ is the number 5.

4. Two positive integers are chosen at random. Find the probability that they are relatively prime.

   (Example: 9 and 20 are relatively prime although each has various factors.)

   The answer is not zero.

5. Prove the Dudeney-Steinhaus theorem: if X,Y,Z divide the sides of a triangle ABC in cyclic order, in the ratio 2:1, then $\Delta LMN = 1/7\ \Delta ABC$ where L,M,N are the intersections of AX, BY and CZ.