THE ALBERTA TEACHERS' ASSOCIATION

MATHEMATICS COUNCIL

NEWSLETTER

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Results of the 1970 Junior Mathematics Contest

A.S.B. Holland
Assistant Dean of Arts and Science
Associate Professor of Mathematics

Once again, students in Alberta performed particularly well in the Junior Mathematics Contest held April 8, 1970. A total number of 18,700 competitors, representing 870 schools from Ontario to British Columbia, took part in the contest. The top score in Canada was obtained by Robin Jackson of Gloucester High School, Ottawa, with a score of 100 out of a possible 120. The top school in Canada is St. George's School in Vancouver with a score of 229.75. The second in Canada was a joint placing of Parkside High School at Dundas, Ontario, with a score of 220.75 (all members of the Parkside team are girls) and *HARRY AINLAY COMPOSITE HIGH SCHOOL IN EDMONTON, ALBERTA,* with a score of 220.75. The members of the Harry Ainlay Composite were *Harold Climenhaga, Geoff Tate, and Samson Tu.* The top score in Alberta was obtained by *Clement Yeung, Ross Sheppard High School in Edmonton,* with a score of 87. Mr. Yeung ranked sixth in Canada. The top Grade X student in Canada was Jorgito Tseng, Killarney Secondary School, Vancouver, with a score of 71 (29th in Canada). The top Grade IX student in Canada was Allan Listoe, Rosetown Composite High School in Rosetown, Saskatchewan, who was third in Canada with a score of 92.25. The top girl in Canada is Janice Crago of Parkside High School, Dundas, Ontario, with a score of 88.

Congratulations are extended to all those who participated in the contest.

in this issue

Newsletter Editor: Mary Beaton
Faculty of Education, University of Calgary

Individualized instruction in mathematics

by *R.S. Mortlock*
Department of Secondary Education
Faculty of Education, University of Alberta

Editor's Note: The ideas found in this article are being implemented in a project concerned with individualized instruction in Grade VII mathematics at Hardisty Junior High School in Edmonton.

What is meant by individualized instruction in mathematics?

Individualized instruction in mathematics is instruction with which is attempted to take account of individual differences between students on factors influencing the learning process of mathematics. These factors include

1. mathematical ability,
2. previous achievement, knowledge and skills in mathematics,
3. rate of assimilation of mathematical ideas and skills,
4. extent of practice needed to develop skills in mathematics,
5. depth of assimilation of mathematical ideas.

With this kind of instruction, attempts may also be made to account for individual differences between students on such factors as

6. interests in mathematics,
7. ability to accept responsibility for learning,
8. independence,
9. motivation and attitude toward mathematics,
10. learning style.

The individualized teaching method allows most students to receive instruction individually by working independently from materials specially prepared for independent study. On occasion, students may also work in small groups. The range of individual differences between students tends to make independent study necessary if this range is to be adequately accounted for.

Why individualize instruction in mathematics?

One of the reasons for such instruction is to take account of the individual differences listed earlier. Very able and interested students can move ahead more rapidly at greater depth and in greater breadth than average and less able students. The latter can similarly progress at their own rate on materials geared to their level of performance.

Secondly, this approach will hopefully result in equal or superior mathematics achievement to that obtained in the traditional setting; and it will expose able students to a far greater range of the mathematics spectrum than they meet in the traditional setting.

Thirdly, gains are expected in areas other than mathematics achievement - for example,

- attitude toward mathematics and the learning of mathematics,
- independence, acceptance of responsibility, self-direction, decision-making,
- attitude toward school, teachers, and authority.

Is individualization of instruction, as described here, new?

Many teachers have practised limited forms of individualized instruction when they attempted to cater to individual differences by such means as groups within classes, differentiated assignments, remedial work, enrichment and individual assistance. Formal approaches in the past have included such methods as the Dalton Plan and Winnetka System. The form of individualized instruction to be described is new to the extent that it attempts to more thoroughly cater to individual differences than other methods.

What has accounted for the renewal of interest in individualized instruction?

There are probably several cources which include

- programmed instruction, its use in instruction and as a research tool,
- computer-assisted instruction and the flexibility it offers in catering to individual differences,
- team teaching and efforts at more effective utilization of school staffs,
- non-graded approaches to instruction.

What are some of the particular characteristics of individualized instruction in mathematics as it has developed in recent times?

1. Behavioral objectives. In individualized instruction it is very important that those involved in preparing the materials to be used have very clearly in mind the particular behaviors which are intended to be developed by the students.

This is important in any form of instruction but particularly so in individualized instruction. Students are working independently without the presence of a teacher who will either say or show (through examples, exercises, and test items) what they are expected to learn and do.

These behaviors should be set out in detail and given to the student so that he can clearly see what is expected of him.

2. Pre-tests. A pre-test for a topic is a test with items including all the behaviors which the student is expected to develop during the topic. There is a test item for each behavioral objective.

The reason for pre-testing is that the student does not waste time during the study of a topic on things that he can already do - he may show that he has achieved certain of the objectives for a topic before studying the topic.

Pre-testing is particularly important in today's approach to mathematics teaching, an approach in which ideas are introduced on several occasions, each time at greater depth, in the course of the mathematics curriculum.

By pre-testing, individual differences in previous knowledge are catered to in a way which would be difficult to carry out in a regular classroom. The student is not expected to study parts of a topic which he already knows. A pre-test also reveals items which are included in the topic. Of course, students have to get used to pre-tests since it is normal for many of them to perform quite poorly on the pre-test. They must see it as an aid to learning and not as some grading device.

3. A brief introduction to each topic. The introduction puts the topic in perspective for the student by telling him

(a) what the topic covers in general terms,
(b) why the ideas in the topic are important,
(c) how the topic fits in with other mathematics he has learned and perhaps how it relates to what he will learn later,
(d) how and where the ideas in the topic are used both in the school mathematics course and in applications outside the mathematics course.

The introduction is primarily to give perspective and to provide motivation

4. Independent study materials. Desirably this is material written specifically to help students develop the behaviors included in the objectives for a topic. As such, the material is expected to introduce and discuss the mathematical ideas and skills to be developed and to provide opportunities to practise with these. Sometimes, although not very often, it may be possible to use existing school texts for this independent study material. The material must be tied closely to the objectives, and it must be readily apparent to the student to which objectives it applies.

5. Progress check items. These are test items which assess whether the student is making adequate progress through a topic. They are used to tell when he is ready to move from one part of the topic to the next. The items may be in the form of exercises which, if they can be done correctly by the student, show he is ready to move on to the next part.

6. Summary and review. At the end of each topic, there should be some form of summary and perhaps review exercises which tie together the ideas developed in the topic.

7. Post-test. A post-test for a topic is a parallel form to the pre-test, having an item for every objective. On this test, given at the completion of the topic, the student demonstrates his achievement of the objectives for the topic. Students would normally do only items for objectives they did not achieve on the pre-test.

Note that the tests used in individualized instruction are what might be called "criterion-referenced" tests. There is an item for every objective, *not* just a representative sample of items. The emphasis in testing is on the achievement of an objective by the individual - for evaluation, diagnosis and as an aid in learning the behaviors for the topic. The emphasis is *not* on comparisons between students for the purpose of grading. The tests are *not* "norm-referenced" tests.

8. Follow-up materials. These are for students who do not achieve adequately on the post-test; they provide re-teaching as it is needed. Again, these materials are tied to the objectives. Often 80 percent of objectives achieved is considered the criterion for advancing to the next topic.

9. Records. Individualized instruction usually has very detailed record-keeping procedures associated with it. This is necessary as students are normally working at varying points in the material due to the varying rates of progress. Most recording materials include

(a) the student's name, class, course, and topic;
(b) percentage of objectives achieved on pre- and post-tests or an actual record of when each objective is achieved;

(c) remedial work required and success on this;
(d) comments.

10. Enrichment. Desirably there should be opportunities for more able students who progress more rapidly, to look in greater dept at ideas presented in each topic, and also to be exposed to a wide range of additional materials and topics.

What is involved in developing individualized instructional materials of the type described?

A very substantial task is to develop materials for individualized instruction on a large scale - that is, for a full year course or for a whole curriculum, and to do it so as to thoroughly implement each of the points dealt with earlier. *It is far beyond what can be reasonably expected of teachers who also have a regular teaching load.* The task needs to be tackled by people, preferably teachers, receiving some help and guidance from those who have had experience in developing such materials, who can devote a considerable amount of time to the task. Presumably this sort of thing will be tackled on a large scale at some time in the future.

What can teachers do about individualized instruction here and now?

On a small scale and at the present time, it is feasible for teachers (even with a full teaching load) to develop one or perhaps two topics along the lines indicated in any one year. This would come to about two weeks per month of individualized work for average students. Experienced teachers who are not overburdened with lesson preparations now do a great deal of things in addition to the routine task of teaching. Preparation of individualized instructional materials could be one of these things. Or, even more desirably, a team of teachers in a school, with classes in the same grade or with common interests, could work cooperatively to develop topics. Different people could take responsibility for different aspects after team planning. This could be a very valuable inservice activity.

In junior high school, the mathematics academic elective offers a golden opportunity to develop topics for individualized instruction along the lines discussed.

To become involved in these things *now* will give teachers extremely valuable experience in the use of individualized instruction - experience from which they will greatly benefit as the individualization of instruction becomes widespread.

The following article is of special interest to teachers of elementary and junior high school mathematics.

Approaches to Teaching Percent

Lillian M. Beswick
Spring Coulee

"Percent", which means "per one hundred", does not need to be a difficult topic to teach in the elementary school. Coulson[1] feels that one reason for it being considered a difficult unit in the traditional math courses was because it appeared to be unrelated to previous topics. Pupils had to learn three approaches to the three types of problems. The three types of problems were, and still are today, of the following kinds:

(a) 25% of 64 is ?
(b) 16 is __% of 64.
(c) 16 is 25% of __.

However, as mathematics is taught now in the elementary grades, there are five different ways in which each type of problem can be solved. These will be discussed later.

In a meaningful mathematics program there should be continuity with new concepts developing naturally. Leslie Dwight[2] would have percent taught directly after decimals and developed from the idea of ratio of any two numbers such as $2 for a hat and $5 for a skirt. The ratio of the cost of the hat to the cost of the skirt would be 2:5 or 2/5.

Previous knowledge of working with fractions and converting to equal ratios with hundredths as the denominator are necessary for this. The important fact being brought out with this illustration is that the percent is to be related to something familiar.

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[1]W.F. Coulson, *Modern Concepts in Elementary Mathematics*, Improvement of Instruction Series No. 4, 1963, The Alberta Teachers' Association, Barnett House, Edmonton.

[2]Leslie Dwight, *Modern Mathematics for the Elementary Teacher*. Toronto, Ontario: Holt, Rinehart and Winston of Canada Ltd., 1966. No. 8, Sec. 14.

Jeanne Nelson[3] is of the opinion that percent should be introduced intuitively as early as the Grade IV level, and if done her way, the class must be familiar with the set theory of mathematics. Ratio is the relationship of a set to a set, but pupils can form rational numbers involving a subset to a set as early as Grade IV. If they have seven words right out of ten, they can figure out how many they would have had right out of one hundred. By the time they reach Grade V, they should be ready to compare two separate sets, or disjoint sets. With the previous work in Grade IV pupils should be able now to understand the difference between a subset-to-set relation and a set-to-set relation. Here again, this approach is to set the stage for the growth of percent out of a familiar background.

Grossnickle and his co-authors[4] would put the child on familiar ground by developing the idea from a ratio built around interesting topics such as *"Our team won 7 out of 10 games."*

We verbally say that "We won 7 out of 10" and write it as a fraction: 7/10, which is the decimal (decimals have presumably been taught before this) .7. Then, if 100 games had been played, it would have been 70 out of 100, 70/100, or .70.

The child would then be informed that per hundred could be called per cent and written 70%. Use of the pegboard is suggested along with this introduction. The method of solving could be any one of those which will be mentioned later.

The value of approaching the teaching of percent from a familiar starting point is stressed by Swain[5]. Assuming that the child has a working knowledge of ratios, he can work with ratios and rates to represent comparisons of "a part with a whole", hence, of a smaller number with a larger. Explain that it is convenient to express this with a MULTIPLE of 100 so that he can use the symbol % as an abbreviation for "percent" which is Latin for per centum, or out of a hundred. The familiar landmark here is the ratio which he will convert to a multiple of 100, and this will be his jumping-off spot to tackle per cent.

All these techniques are for use by teachers to ease the child into per cent. The introduction need not necessarily be preceded by decimals.

Some would advise learning percent before decimals and by using pegboards with 100 holes, teaching the pupils to compare fractions with percents. By using an overhead projector, with overlays as illustrated in Figure 1, the child can compare every ratio equivalent to 3:5 through to 60:100.

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[3]Jeanne Nelson, "Percent: A Rational Number or a Ratio", *Arithmetic Teacher*, Vol. 16, No. 2, February, 1969.

[4]Grossnickle, Brueckner, Reckzeh, *Discovering Meanings in Elementary School Mathematics*, published by Holt, Rinehart and Winston of Canada Ltd., Toronto, 5th edition, 1968.

[5]R. Swain and E. Nichols, *Understanding Arithmetic*, Chap. 6.6. Published by Holt, Rinehart and Winston of Canada Ltd., Toronto, 1966.

Figure 1

XXX 00000	XXX 00000	XXX 00000
XXX 00000	XXX 00000	XXX 00000
XXX 00000	XXX 00000	XXX 00000
XXX 00000	XXX 00000	XXX 00000
XXX 00000	XXX 00000	XXX 00000
XXX 00000	XXX 00000	XXX 00000

XXX 00000	XXX 00000

3	6	9	12	15	30	36	48	60
5	10	15	20	25	50	60	80	100

Riedesel[6] stresses the fact that there is no agreed-upon "best" method for solving percent problems, but there are indications that interrelationships between percent problems should be stressed. Having reached the point where solutions of percent problems come next, it is assumed that the children have had experience in discovery, understand the ideas of rate and ratio, and have at least some acquaintance with fractions and possibly decimals. With these facts in mind it is suggested by him that the teacher present problems to the pupil in which percent is used and follow the lead they take (if it is mathematically correct). Such a procedure is in keeping with a discovery approach, allows

[6]C. Alan Riedesel, *Guiding Discovery in Elementary School Mathematics*, Meredith Publishing Co., New York, 1967.

greater opportunity for success in handling individual differences, and makes per cent meaningful. It is presumed that the child will move into a method which is familiar to him depending on what he has learned previously. For each of the three types of problems there are five possible and acceptable ways. The manner in which they can be taught for each is described as follows:

1. THE DECIMAL APPROACH

Type A — 25% of 64 would be .25 x 64 equals ________

Type B — The child wants to know what part 16 is of 64. He decides it is 16/64 or $64\overline{)16.00}$

Type C — He needs to find the total when 16 is 25% of the number. In this situation he divides the number by the decimal or $.25\overline{)16.00}$

2. THE RATIO APPROACH

Type A — 25% of 64 is ______, is written as 25/100 : n/64
He can solve this by the equivalent ratio or finding the cross product (ratio test)

Type B — 16 is ______% of 64, is written n/100: 16/24 and solved as he would any ratio problem.

Type C — 16 is 25% of ______, is written 25/100 : 16/n and solved as above

3. THE UNITARY-ANALYSIS APPROACH

Type A — To find 25% of 64, they first say 1% of 64 is .64, so 25% is 25 x .64

Type B — To find what % 16 is of 64, they reason that if
64 represents 100%,
1 is (100/64)%, therefore, 16 is 16 x 100/64 or 25%

Type C — 25% is 16,
1% is 16/25, therefore, 100% is 100 x 16/25

4. THE FORMULA APPROACH

With this approach, each part is labelled.

Example: What is 10% of 80. Answer 8.

STUDENT MATHEMATICS
– a new Canadian publication

W. W. Sawyer

The first issue of this publication has gone to the printer and should be ready by the beginning of April. Its aim is to report and encourage student initiative in mathematics, pure and applied, at all levels.

Some of the articles could be read at Grade IV or V level - for example, a study of goal scoring in hockey by R. de Peiza, a Type A student at C.E.U.T. Suitable for a somewhat more advanced level is a problem in number theory, sent to us from Geelong, Australia, for which David Vaskevitch of Northview Heights C.I. devised a computer program. An article by Dan Gautreau (U.T.S.) describes the independent reading he did which enabled him to start university work in second-year mathematics. A first-year university student offers advice and experiences to those still in high school. D. Burbulla of York Mills presents his original ideas on series for π. Professor Duff has an article on optimization and rocketry. There are a number of puzzles, including an original one composed by Simone Chan while at Oakwood C.I., and an investigation proposed by George Lount of La Salle. The publication also contains a number of brief items, including a report on Ken Tucker in Grade XI four-year program at R.H. King, who has been reading about relativity in his spare time.

This bulletin is offered to students at the price of 10¢ per copy. It consists of eight pages; size - 8.5 x 11 inches. We are trying to find ways of making its existence known to students in secondary schools, and of distributing it economically. We would be most grateful to any school or education authority who would take a parcel and be responsible for selling copies to students.

In the outlying parts of the country, there may be schools in which only a single student wants a copy. If such a student will send a stamped, self-addressed envelope, at least 9 x 4 inches large and 10¢, we will be glad to mail him a copy. One of the aims of the publication is to assist such isolated students.

All orders for *Student Mathematics* should be sent to:

The Secretary, "Student Mathematics",
Room 373 - College of Education,
371 Bloor Street West,
Toronto 181, Ontario, Canada.

We hope this publication will spread out beyond Toronto and Ontario and get the cooperation of mathematics students throughout Canada. If you have contacts with other provinces, we would welcome your assistance.

PROBLEM PAGE

A.S.B. Holland
Assistant Dean of Arts and Science
Associate Professor of Mathematics

The Mathematics Council, ATA, thought that the addition of a problem page to the *Newsletter* would be another item of interest to you, the reader. Problems for solution should be sent to the Editor, Mathematics Council Newsletter,

Murray R. Falk,
2112 Palisdale Road, S.W.,
Calgary, Alberta.

Do not forget to include a solution for each problem if one is available; and mention your name and school, please. If solutions are typed, they should be double-spaced; if they are handwritten, they should be in clear writing. We would like to receive from our readers many, many problems of a topical nature, covering all branches of mathematics and of not too advanced a nature.

PROBLEMS FOR SOLUTION

1. Prove that if the bisectors of the base angles of a triangle are equal in length, then the triangle is isosceles. (This is an old problem, rather like the one: If the medians of a triangle are equal in length, then it is isosceles.)

2. Given Sin A, Sin B, Sin (A + B): show how to find Cos (A + B) using only rational combinations of the data.

   (Example: $\text{Cos}\ (A + B) = \sqrt{1-\text{Sin}^2\ (A + B)}$ is no good since this is not a rational use of the data). This problem had considerable practical use and was asked by the War Office in Britain in World War II.

3. Prove that the only number which will divide two consecutive numbers of the form $n^2 + 1$ is the number 5.

4. Two positive integers are chosen at random. Find the probability that they are relatively prime.

   (Example: 9 and 20 are relatively prime although each has various factors.)

   The answer is not zero.

5. Prove the Dudeney-Steinhaus theorem: if X,Y,Z divide the sides of a triangle ABC in cyclic order, in the ratio 2:1, then $\Delta LMN = 1/7\ \Delta ABC$ where L,M,N are the intersections of AX, BY and CZ.