The geoboard:
a versatile instructional aid

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Manipulative materials play a useful role at most levels of mathematics instruction. Some can be used to predict or check answers when solving problems which involve the four operations; others can lead to the discovery of patterns and relationships; and still others, such as games, can aid the development of problem-solving strategies. It is the purpose of this article to illustrate, with specific examples, how the geoboard could be used in each of the settings described above.

Commercial geoboards come in many shapes and sizes. However, they are relatively simple to construct from a piece of plywood (approximately 6" by 6") and 25 finishing nails. Observations seem to indicate that a 5-nail by 5-nail geoboard is more than adequate and that it is advantageous not to paint or draw lines onto the piece of plywood (between the nails) since they seem to distract from the constructions created by the children. If the spacing of the nails is a little less or a little more than one inch, even a small rubber band can be used to create various ingenious designs. Larger spacings (1 1/2" or more) seem to be unsuitable for shorter bands, tend to restrict constructions to only one area of the board, and frequently result in broken rubber bands when they are stretched across the whole geoboard. A spacing of exactly one inch seems to lead to the adoption of that unit whenever discussions arise and tends to make the students less flexible in adopting an arbitrary unit and/or name which at times could be advantageous.

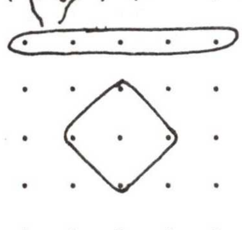
SUGGESTED ACTIVITIES

The suggestions which follow are meant to achieve two puposes:

1. Illustrate the versatility of the geoboard by describing some of its uses in a variety of topics and settings, and
2. Present some sample instructions and questions which could be used in developing activity sheets for students related to these topics.

Number - numerals

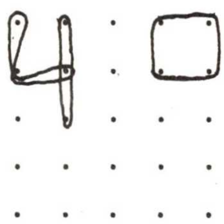
1. On your geoboard, in how many different ways can you put one rubber band around 5 () (or 6, or 7, etc.) nails? Record and display your results.
   Example:



   How are the arrangements different and how are they the same?

2. Put a rubber band around some nails and from these cards
$\boxed{2}$, $\boxed{3}$, $\boxed{4}$, $\boxed{5}$, $\boxed{6}$ ... choose the correct name for the number of nails enclosed.

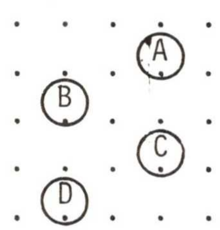
3. How many different numerals can you make on the geoboard with your rubber bands? Record and display.
Example:



Match the numerals with the appropriate number of nails.

Counting

1. If we call the nail labelled A, number 7, where we have started to count?



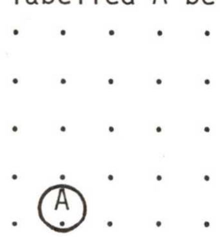
Using the same starting point, what number would you assign to nail B?, C? and D?

2. If the number 14 is assigned to the nail labelled A above, how did we count? Where did we begin to count? What number would you now assign to nails B, C, and D?

3. Assign 18, 21, and so on to nail A and repeat the questions in part 2.

Ordinal number

The first task is to invent or agree upon a notation which would assign every nail on the geoboard a distinct number name. A variety of possibilities exist: (Row 2, Nail 4) or (R-2, N-4); (Up 2, Across 4) or (U-2, A-4) or (↑2, →4); (Down 2, Across 4) or (D-2, A-4) or (↓ 2, → 4); or an agreement is reached on the starting point and a simple number pair is used, for example, (2, 1) for the nail labelled A below.



After a notation has been agreed upon:

1. Give a name to all the nails on your geoboard.

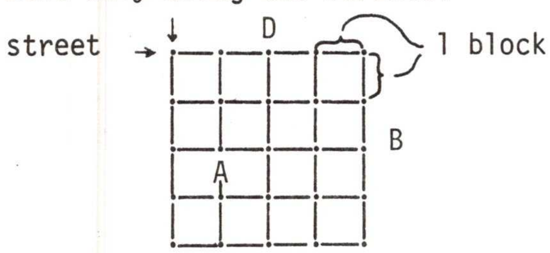
2. Use a piece of colored construction paper or wooden beads to mark the following:

   a. Both parts of the number pair names (first and last number for each nail) are even.
   b. Both parts of the number pair names are odd.
   c. The sum of the first and second number for each nail is 6.
   d. Both parts of the number pair names are the same.
   e. The second part of the number pair name is twice as large as the first part.
   and so on.

3. Use pieces of construction paper or beads to make a pattern on your geoboard. Write the number pair names for your pattern. What did you notice?

4. Pretend there are streets along the rows and columns on your geoboard.

The distance between any 2 nails (corners) is "one block". You are allowed to walk only along the streets.



a. How many blocks is it for the shortest walk along the streets from corner A to corner B? A to D? A to C? C to B?
b. How many different ways can you find to walk the shortest distance from corner A to corner B?
c. Write the address (number pair) for corner A. Write the address for B. Is there any way of finding the answer for (a) from the addresses? How? Does it work for the others? Does it work for any two corners on your geoboard? Try it.

Addition/subtraction

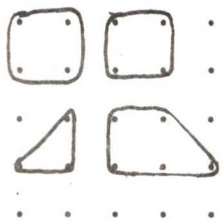
Addition and subtraction problems involving simple basic facts could be simulated on the geoboard.

1. Put a rubber band around 8 nails. Put a rubber band around 7 *different* nails. How many nails are in both rubber bands? How do you know? Can you find another way of showing the sum? (regrouping by fives, or by tens and ones)

2. Put a rubber band around 13 nails. Put another rubber band around 6 of the 13 nails. Of the 13, how many nails have only one rubber band around them?

3. Using 2 rubber bands, how many different ways can you find to show the number 8? How are they different and how are they the same?

   Example:



   Use the same technique for 9, 12, 6, and so on.

4. Repeat part 3 using 3 rubber bands.

ultiplication/division

1. On your geoboard, show 7 groups of 3. What is the total?

2. On your geoboard, show 5 groups of 2. Now show 2 groups of 5. How are they different? How are they the same?

3. Put rubber bands across 3 rows on your geoboard. Take one rubber band and stretch it across one column. Count the number of nails which have 2 rubber bands around them, or the intersections, and record your results in the table.

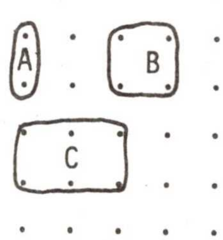
No. of Bands Across or Horizontal	No. of Bands Up and Down or Vertical	No. of Nails inside two Bands or Intersections
3	1	3
3	2	
3	0	
3	4	

   Reorganize the data entered into the table (column 2 from smallest to largest). Do you see a pattern?

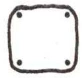
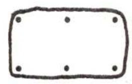
4. Put a rubber band around 16 nails. If you use 2 other bands to make 2 groups of the same size, how many would there be in each group? What if you were to make 4 groups of the same size, how many would there be in each group? How about 8 groups?

Patterns

1. Use this pattern.



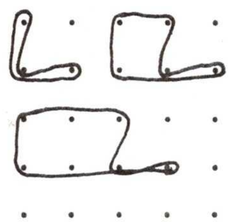
a. Can you tell what the next 3 (or 4) would look like? Construct them on your geoboard and record your results in the table. Describe the pattern.

Sketch	Number of Nails
A 	2
B 	
C 	
D	
E	

b. If you add any 2 numbers from the table above, can you make the sum look like the sketches? Try it on your geoboard.

c. If you were to multiply any 2 numbers from the table, will the product look like the entries in the table? Try it.

2. Use this pattern.



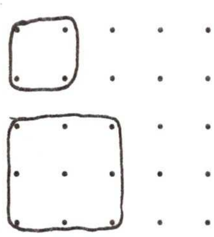
a. Find the next 3 (or 4). Construct a table.

b. Add any 2. Construct your result. What can you say about the appearance of the sum?

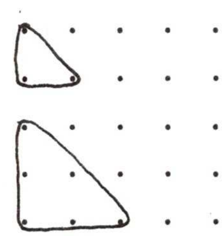
c. Multiply any 2. Construct the result. What can you say about the product?

3. Follow the same 3 steps for these patterns.

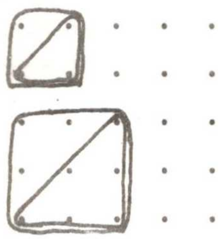
a.

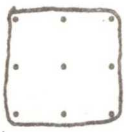
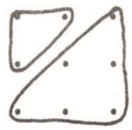


b.



4. Combining the 2 previous patterns results in the following pattern.



Sketch	Number	Triangles	Sum
	4		1 + 3
	9		3 + 6

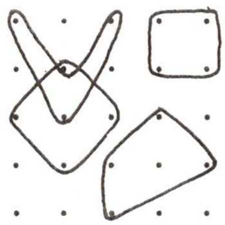
Describe the pattern. Does the pattern work for other "square" numbers? Use your geoboard to find out.

5. a. Take a number from the table in part 1 and add it to a number from the table in part 2. Display the sum on your geoboard. Can you make the sum look like an entry in the table of part 1 or part 2? Try other sums. What can you say about them?
   b. Take a number from the table in part 1 and multiply it by a number from the table in part 2. Display the product on your geoboard. Can you make the product look like an entry in the table of part 1 or part 2? Try other products. What can you say about them?

Figures

1. Familiar figures
   a. On your geoboard, make figures that look like something in your classroom; a kitchen; a store, and other places. Record, label, and display.
   b. Look at a figure someone else has constructed and try to guess what it could be after he has told you where it might be found.
   c. Does your figure look the same when you turn the geoboard? How many corners (sides) does your figure have? Can you make a figure with more (fewer) corners (sides)?
   d. Make a figure. Now use another rubber band and try to construct a figure which looks the same but is larger or smaller; longer or shorter; narrower or wider.
   e. Construct a figure which has 4 (or 3) sides which is long; short; long and wide; long and narrow; short and wide; short and narrow. Record and display your results.

2. Construct all the different 4-sided figures you can think of. Record, display and compare your results.

Examples: 

3. Do the same as in part 2, but with 3-sided or 5-sided figures.

4. Construct different figures on your geoboard and record your results in the table.

	Number of	
Sketch	Sides	Corners
	3	3
⋮	⋮	⋮

What is the pattern? Does it always work?

Segments

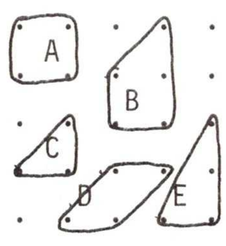
1. Try to make segments that are short; long; straight; "crooked". Record, display and compare your results.

2. Construct segments that touch; do not touch; will never touch; cross each other. Record, label, display and compare.

3. Construct various segments leading to 2 (or more) nails; which are equal in length; which are not equal in length.

Area

Call the size (area) of square A, one "unit".



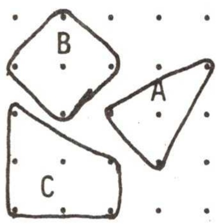
1. If figure A has an area of one "unit", use rubber bands to find the area of figures B, C, D, and E.

2. Construct "gardens" with an area of 2, 2 1/2, 3, 6, 7 1/2, and so on units. Label and display your results.

3. Construct as many gardens as you can which have an area of 8 (or 12, and so on) units. Sketch, label, and compare your results. How are the gardens different? How are they the same?

4. Construct more gardens but always leave one tree (nail) in the interior.

   Example:



   Find the size of the gardens you have constructed and record your results in the table.

Garden	No. of Posts (nails) used in the fence of garden	Size of garden
A	3	
B		
C		

   Build gardens with 6, 7, and so on fence posts. Don't forget the tree in the middle. Record your results in the table. Do you see a pattern? How could you predict the size of the garden from knowing the number of fence posts which were used? Does this work for bigger gardens? Try it.

5. Construct gardens which have no trees in the interior. Count the number of fence posts, calculate the area, and record your results in a table. Find a pattern for these gardens.

6. Construct gardens with 2 (3) trees in the interior. Count the number of posts used in the fence, calculate the area, and record your results in a table. Find a pattern for these gardens.

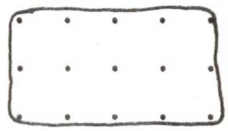
Perimeter

Call the distance between 2 adjacent nails in a row or column, one "unit".

1 unit 

1. Use rubber bands to construct figures that have a fence which is 4, 6, 8, 10, and so forth units long. Record, label, display and compare your results. How are they different or how are they the same?

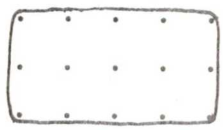
2. Construct as many figures as you can which have a fence that is 12 (or 8, or 16) units long. Record your results in a table and display.

Sketch	Length of fence
	12 ⋮

How are the figures different? How are they the same?

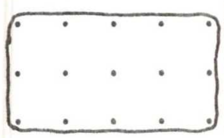
Area and perimeter

1. Construct a number of different rectangular figures with an area of 8 "units". Record your results in a table.

Sketch	Area	Length of fence or Perimeter
	8 ⋮	

What can you predict about the area and perimeter of these figures?

2. Construct a number of different rectangular figures with a perimeter of 12 units. Record your results in a table.

Sketch	Dimensions	Perimeter	Area
		12 ⋮	

What can you say about the dimensions and the area of these figures?

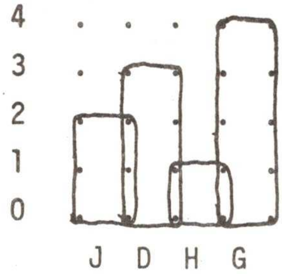
Graphs

1. John, Dick, Harry and George played hockey for the school team. The table shows how many goals each one of them scored in the last 2 games.

Name	No. of Goals
John	2
Dick	3
Harry	1
George	4

Use 4 rubber bands and your geoboard to show these results. Label your graph and display it.

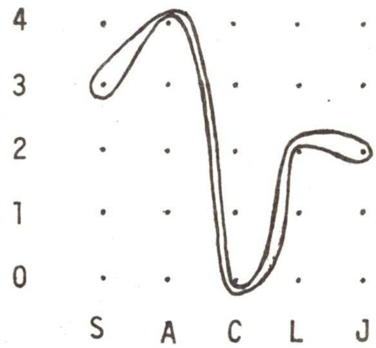
Example:



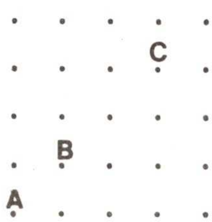
2. Collect other information (such as hair or eye color, days away from school), construct similar graphs and display your results.
Summarize the results shown in your graph by writing a story. Display the story. Which do you prefer, the graph or the story? Why?

3. Line graph. Use one long rubber band and your geoboard to show the results from the table. Label your graph and display it. Collect other information, construct similar graphs, label and display them.

Name	No. of Pets
Suzie	3
Ann	4
Caren	0
Linda	2
Joan	2



Notation



Consider the following directions ↑↓ ↗↙ →← ↖↘ and use the nail labelled A above as a starting point. Various directions can be taken to get from A to B.

Examples: A ↗
A → ↑
A ↑ →
A → ↖ →

Can you think of others? Try them.

1. Put a bead or piece of construction paper onto the nails which would be the end point of the following instructions:
   a. A →
   b. A → → →
   c. A → ↑ ↑
   d. A → → → ↑ ↑
   e. A → → ↑ ↑ ↑

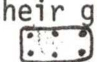
   What does this construction look like to you? Make up similar directions for a construction of your own. Write number pair names for a to e above.

2. Write as many different directions as you can for a route from nail A to nail C. Compare your answer with a friend. Which one is the shortest? Which one is the longest route?

3. Using the nail labelled A as a starting point, could you find a nail for A ← ↖ ? Why or why not? Could you find it if you were to use another geoboard? Make up some directional expressions which take you from A to a nail on the second geoboard.

4. Using the nail A as a starting point, could you find nails for A ↓ ↓ → or A ↓ ↓ ← ↙ ? Why or why not? How could you solve these problems?

Hide and seek (battleship)

Two contestants, or two teams, face each other in a game of "Hide and Seek". They construct on their geoboard, hidden from each other's view, an agreed-upon figure (ship),  , in any position they like. The task consists of each person trying to find the opponent's figure first, by taking turns and asking questions in terms of number pairs, for example, "Does your figure touch (1, 2)?" Positive responses could be marked with beads or pieces of construction paper. The person who has determined the exact location of the figure first is then declared the winner. (After each positive response try to consider all the possible locations for the opponent's figure. Try to determine which "shot" or position could give you the most information when it is your turn to ask the next question.) Try to find a champion for this game in your group or room.

Tic-tac-toe; four in a row:

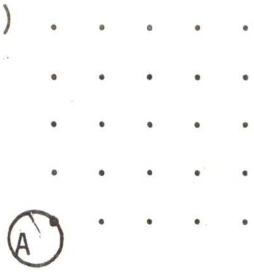
Two players take turns in placing different colored beads or pieces of construction paper onto the nails of the geoboard. The first one to get four in a row, column or diagonal is the winner. Are there any good (bad) moves? Why? Who is the champion in your group or room?

Letter - messages (codes)

1. How many different letters of the alphabet can you make on your geoboard? Are there any that cannot be made? Try as many as you can, sketch, display and compare your results.

2. Use rubber bands and follow these directions:

call A, (1,1)



a. Join (1,1) to 1, 3)
and (1, 3) to (2, 1)
and (2, 1) to (2, 3).

b. Join (3, 1) to (3, 3)
and (3, 3) to (4, 3)
and (4, 3) to (4, 1)
and (4, 1) to (3, 1).

What does a and b say? ______________________________

3. Make up directions for a "secret" message of your own. Let a friend try to figure it out.

Treasure hunt

1. a. Join (2, 2) to (4, 2).
   b. Join (3, 1) to (3, 3)
   and (3, 3) to (5, 3)
   and (5, 3) to (5, 1)
   and (5, 1) to (3, 1).

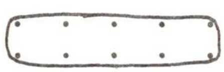
   The treasure is hidden where a and b intersect. What is its location?

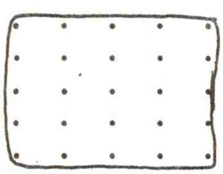
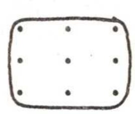
2. Make up directions for a treasure hunt of your own and let your friend try to find it.

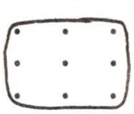
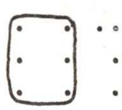
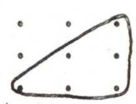
Fractions

By assigning the value of "one whole" to either a row of nails, a row of squares or to the whole geoboard, various activities can be designed to illustrate:
1. fractional notation or the meaning of fractions,
2. equivalent fractions,
3. addition of fractions.

Examples:  = Unit, then  = ? etc.

 = Unit, then  = ? etc.

 = Unit, then  = ? etc.

 = Unit, then  = ?,  = ? etc.

CONCLUSION

Some suggestions for activities have been made to illustrate that the geoboard can be used at any grade level for a great variety of topics. The aid has various other advantages: it withstands wear and can be constructed by the students, at least at the upper elementary level; concepts can be classified or reinforced through demonstrations, or the discovery method can be employed to teach these concepts; various settings can be created which could develop a student's appreciation of mathematics, develop his interest, enhance his initiative, arouse his imagination and provide recreation and enjoyment.

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