# *Simple computing devices for children to build and use*

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"... the conclusion is inescapable that children can study mathematics more satisfactorily when each child has abundant opportunity to manipulate suitable physical objects." (*Goals for school mathematics*, 1963, p.35)

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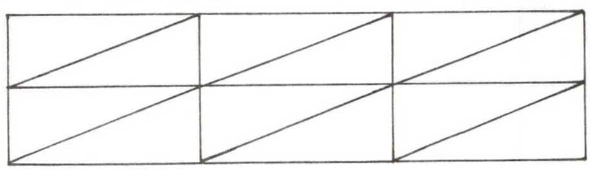
The use of manipulative devices in teaching mathematics is becoming a widely accepted procedure and is supported by some research by learning theorists, and by practitioners - teachers themselves. Many teachers have long advocated that children need to manipulate a device or variety of devices before being required to abstract a concept or a model. Thus, there is available commercially an abundance of instructional materials designed to provide the concrete experiences necessary for the development of a child's concepts of number and space. One of the areas in which a large variety of aids is not available is that of computation. Yet, we live in a highly technological society in which computing devices or calculators play a major role; in addition, Man made use of computing devices even before numeration systems were fully developed. It is the purpose of this paper to outline a number of simple computing devices that elementary school children can build and use. One caution must be made: devices themselves will teach very little mathematics. It is their use under the guidance of a wise teacher that determines their effectiveness in learning.

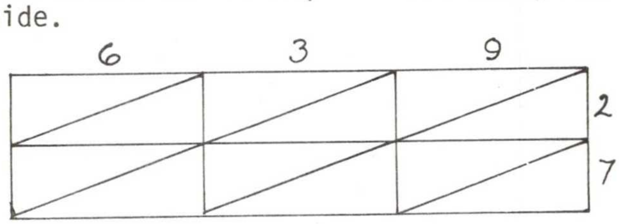
## *GELOSIA (LATTICE) MULTIPLICATION*

The gelosia or lattice method is one of the very first methods Man used to release himself from the tedious work of multiplication. The device consists of equal sized cells, each divided into two parts by diagonals drawn from upper right to lower left. The number of cells depends on the number of digits in each factor. For example, to multiply a three digit number by a two digit number, six cells are needed:

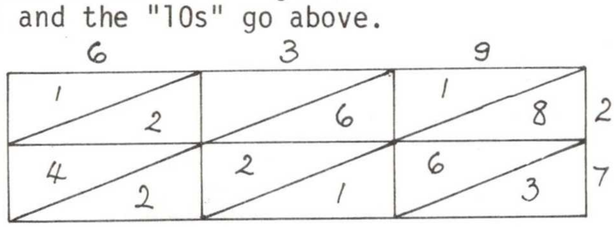


To multiply 639 by 27, follow these steps:

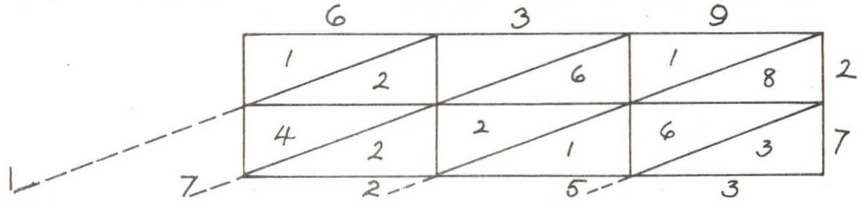
1. Place the digits of one factor at the top of each cell, and the digits of the other factor at the side.




2. Multiply each digit at the top by each digit at the side. The product is written in the cell corresponding to each pair of factors. If the product has one digit, it is written below the diagonal. If it has two digits, the "ones" go below the diagonal and the "10s" go above.



3. To find the product, add the numbers in each of the diagonals.



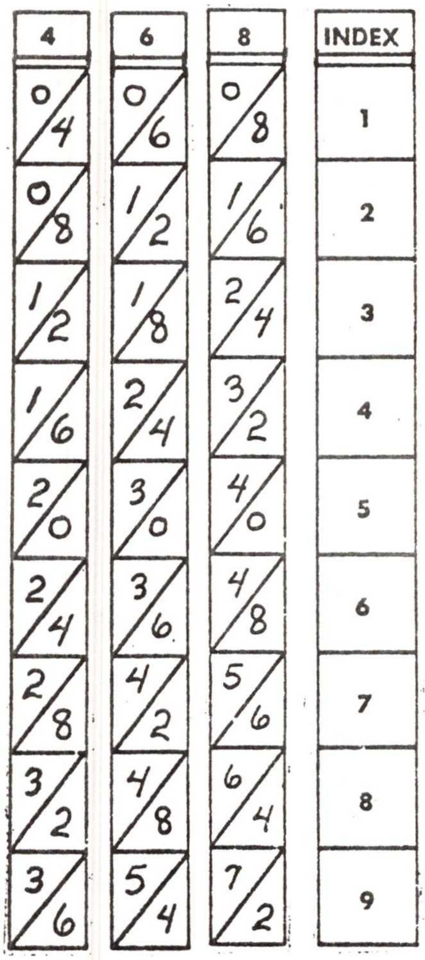

Pupils may also work on multiplying numbers having more digits, for example 368 x 472; 4562 x 835. A discussion of "why it works" should point out that the diagonals separate the numerals according to place value.

## NAPIER'S BONES

In 1617, a Scottish mathematician, John Napier, developed a mechanical device that simplified the monotonous work of long multiplication. His method made use of a set of numerating rods called Napier's "bones", which were based on the gelosia or lattice method of multiplication.

To make a set of Napier's bones, use 11 strips of heavy cardboard. On 10 of them write the multiples of the numbers 0 to 9. The eleventh strip is used as an index rod and lists the digits 1 through 9. A completed set of bones is shown below. Notice that each one is actually a kind of multiplication table.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | INDEX |
|---|---|---|---|---|---|---|---|---|---|---|
| 0/0 | 0/1 | 0/2 | 0/3 | 0/4 | 0/5 | 0/6 | 0/7 | 0/8 | 0/9 | 1 |
| 0/0 | 0/2 | 0/4 | 0/6 | 0/8 | 1/0 | 1/2 | 1/4 | 1/6 | 1/8 | 2 |
| 0/0 | 0/3 | 0/6 | 0/9 | 1/2 | 1/5 | 1/8 | 2/1 | 2/4 | 2/7 | 3 |
| 0/0 | 0/4 | 0/8 | 1/2 | 1/6 | 2/0 | 2/4 | 2/8 | 3/2 | 3/6 | 4 |
| 0/0 | 0/5 | 1/0 | 1/5 | 2/0 | 2/5 | 3/0 | 3/5 | 4/0 | 4/5 | 5 |
| 0/0 | 0/6 | 1/2 | 1/8 | 2/4 | 3/0 | 3/6 | 4/2 | 4/8 | 5/4 | 6 |
| 0/0 | 0/7 | 1/4 | 2/1 | 2/8 | 3/5 | 4/2 | 4/9 | 5/6 | 6/3 | 7 |
| 0/0 | 0/8 | 1/6 | 2/4 | 3/2 | 4/0 | 4/8 | 5/6 | 6/4 | 7/2 | 8 |
| 0/0 | 0/9 | 1/8 | 2/7 | 3/6 | 4/5 | 5/4 | 6/3 | 7/2 | 8/1 | 9 |




To multiply 468 by 7, take the 4, 6, and 8 bones and the index and place them as shown at the left.

7 x 468 is shown in the seventh row. Add the numbers in the diagonals, as in the gelosia method.

7 x 468 = 3,276.

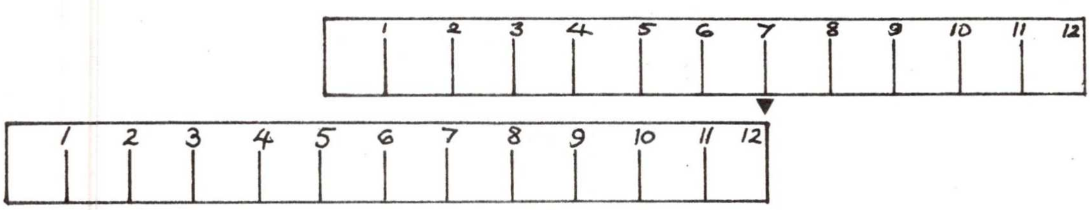
Pupils can discover how to use the bones to multiply by two and three digit numbers. The bones can then be used to check multiplication examples done the "ordinary" way.

## SLIDE RULES

*Addition and subtraction slide rule*

Two rulers may be used as a very simple form of slide rule for addition and subtraction. Children can make their own by using two strips of heavy cardboard and marking a scale on each one, possibly using graph paper to assist them in making the scale.

To add 7 and 5, for example, place one strip above the other and "slide" the top strip to the right, until its left or 0 end is above the 5 on the bottom strip. Find the 7 on the top strip and look directly below it to the answer 12.

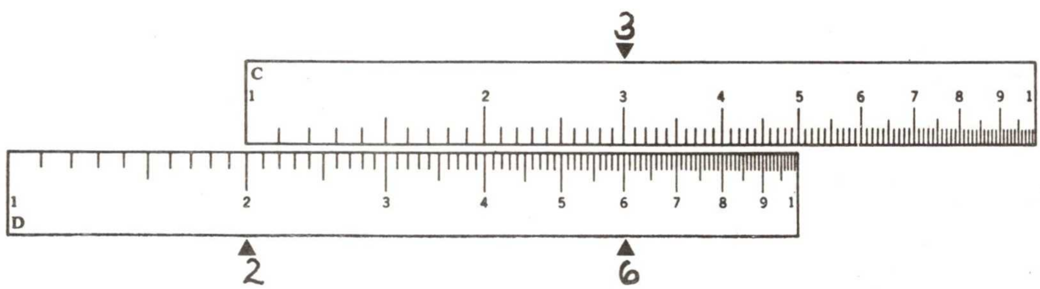


The strips can be used for subtraction by simply reversing the above procedure. Any scale may be used on these strips as long as the same one is used on both strips. For example, a scale going to 20 would allow practice on all the addition and subtraction facts through 18; a scale going to 50 would provide examples of addition and subtraction of two digit numbers through 50. Addition and subtraction of fractions and decimals may also be shown.

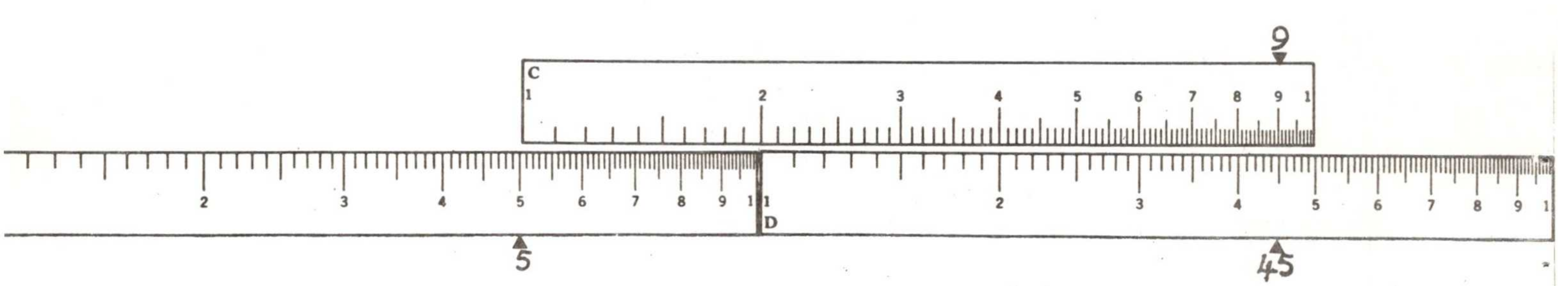
*Multiplication and division slide rule*

The basic mathematical idea behind this slide rule is the logarithm - an idea for which John Napier was also responsible. (A logarithm is an exponent, and exponential numbers are multiplied by adding the exponents. For example, $4^2 \times 4^3 = 4^{2+3} = 4^5$. On a slide rule, the exponents are represented by distances, and two numbers are multiplied by adding distances.)

The following diagram illustrates 2 x 3. Note that the "index" is now 1, the multiplicative identity, rather than 0, the additive identity.



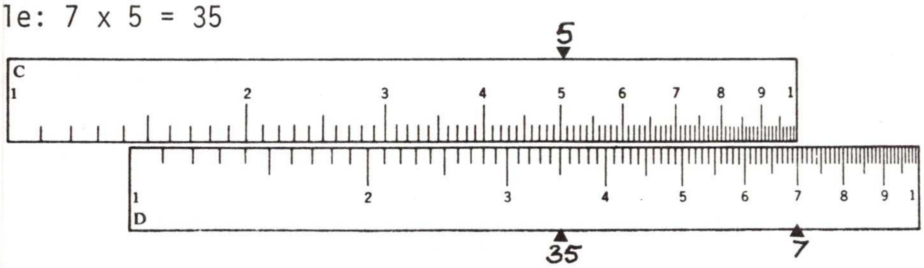

The strips show the scale for a multiplication-division slide rule. The scale goes from 1 to 10; to make a slide rule with a scale from 1 to 100, simply put two of the D strips end to end, as shown in the following example, illustrating, 5 x 9 = 45.



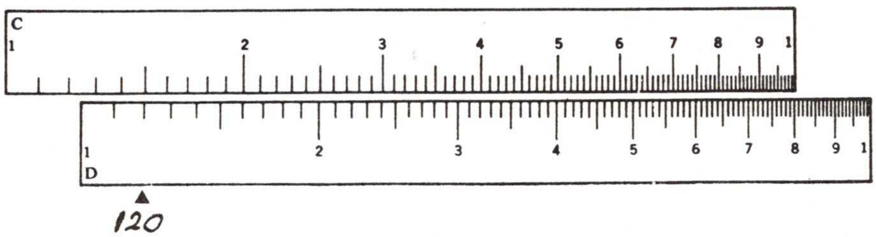
The second "D" scale can be thought of as representing the numbers from 10 to 100. To use this slide rule to provide basic fact practice, the products through 81 could be marked on the D scale.

The process of connecting the scales end-to-end to allow multiplication of any two numbers could go on forever, but it is not too practical, nor is it necessary. By sliding the C scale to the left rather than to the right, that is using the 10 on the right of the C scale as the index, all the products through 100 may be found:

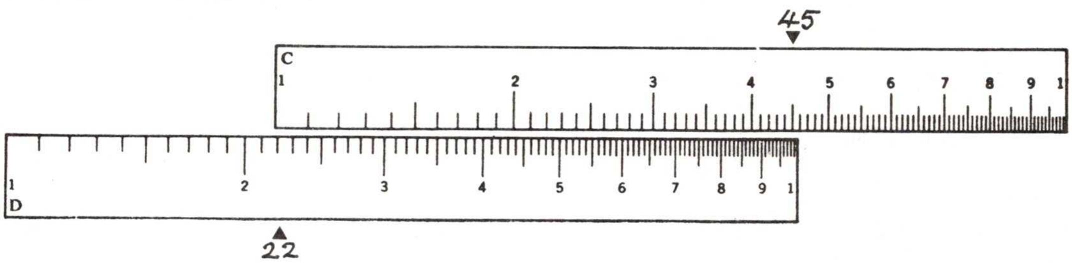
Example: 7 x 5 = 35



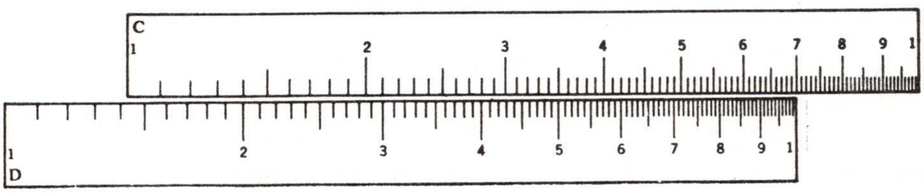

Multiplication involving a 2-digit number may be done in the same manner as above. For example, to multiply 8 x 15:




Finally, 2 2-digit numbers such as 22 x 45 may be shown. Note that the 99 is considered as 99 tens or 990.



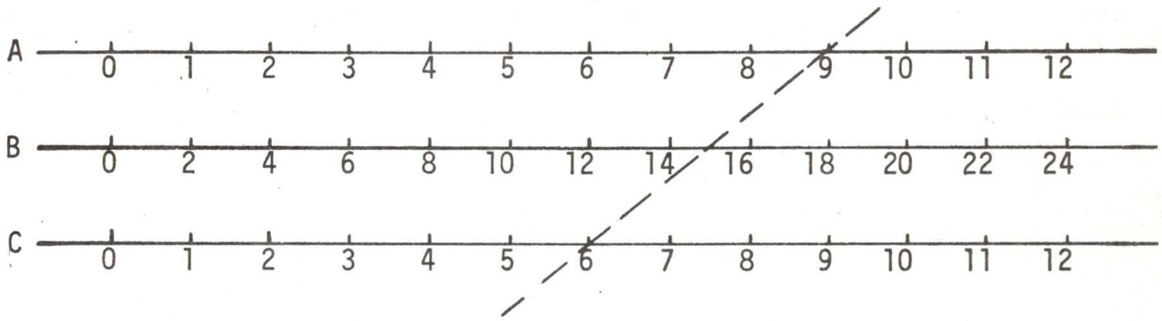

Division with this slide rule is just the inverse of multiplication. For example, to show 35 ÷ 5, put the 5 under the 35 and look above the right index to find the quotient, 7.



## NOMOGRAPHS

*Addition-subtraction*

Devices called nomographs have been designed to allow a person to do rapid computing by just reading numbers from scales drawn on graph paper, or plain paper. One type of nomograph is shown below:




To add any two numbers, locate one on the top line and the other on the bottom line. Place a straightedge at these two points; the straightedge will cross the middle line at the sum. The diagram above illustrates 9 + 6 = 15. For subtraction, use the top and middle lines and read the difference from the bottom line. The diagram above also illustrates 15 - 9 = 6.

To make such a nomograph, start with three equally spaced, horizontal or vertical, parallel lines, A, B, and C. Mark off equal spaces on each line. On lines A and C, number corresponding marks with the same numerals. Mark each point on line B with a numerical value twice that of the corresponding marks on lines A and C. These lines are related to one another by the formula, Top + Bottom = Middle. Using this formula, you can construct a nomograph for any set of numbers you wish - naturals, integers, fractions, decimals.

*Multiplication-division*

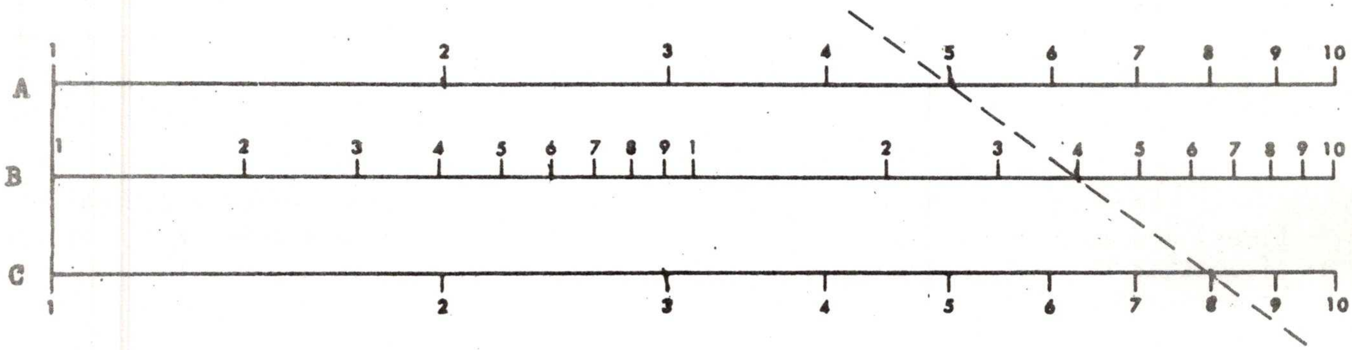
A nomograph that can be used for multiplication and division works in much the same manner as the addition-subtraction one. The diagram shows a nomograph for multiplication and division. Again there are three equally spaced, horizontal or vertical, parallel lines, A, B, and C. Lines A and C are marked off exactly like the scale of a multiplication slide rule. Line B has half the scale of the other two, thus has two slide rule scales in the same length that lines A and C have one.

To multiply any two numbers, locate one on line A and the other on line C. Join with a straightedge, which will cross line B at the product of the two numbers. These lines are related to one another by the formula, Top X Bottom = Middle. The following diagram illustrates 5 x 8 = 40. (Remember, from the slide rule, that the 4 means 4 groups of 10 or 40.)

Division on the nomograph is, of course, just the inverse. Locate the dividend on the B scale, the divisor on the A scale, connect the points with a straightedge which will cross the C scale at the quotient, the answer. Thus, the nomograph below shows 40 ÷ 5 = 8.

Note that you can find the square of any number on the A and C scale by looking at the corresponding point on the B scale, and you can find the square root of a number on the B scale by looking at the corresponding numbers on the A and C scales.

To make a multiplication-division nomograph, use the scale below or the scale from one of your slide rules. To use for basic fact practice, mark the products through 81 on the B scale.




## *POCKET CHARTS*

Pocket charts are useful not only for computing, but also for developing place value and grouping by 10 ideas.

To make one chart with 4 pockets, use a 24" x 18" sheet of construction paper and fold as follows: measure down from the top 6 1/4", then fold UP. Then measure down from the fold, 1 1/4" and fold DOWN. Measure 3 1/4" down from the second fold, and fold UP. Continue measuring 1 1/4" and fold down, then 3 1/4" and fold up, until you have four "pockets". Staple or glue the folded construction paper to a heavy cardboard backing, 13" by 18", and trim the edges with black tape.

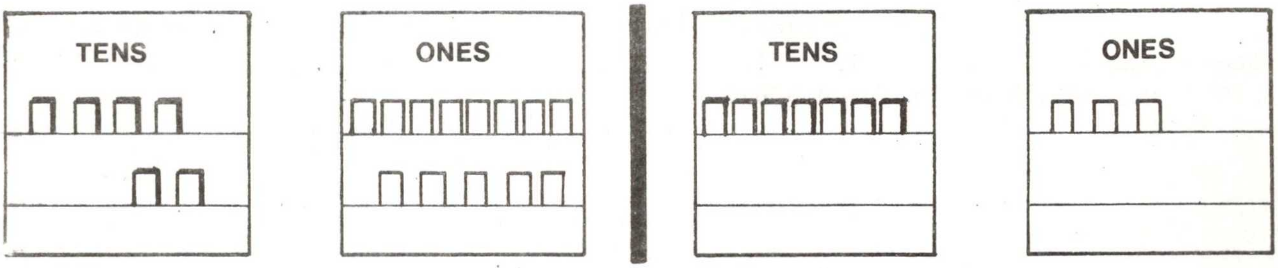
Three charts allow one each for ones, tens, hundreds; or for tenths, hundredths, thousandths if working with decimals; or for three places if working with other bases. Cards containing these titles could be made, and clipped onto the top of the chart with a paper clip.

Cut a supply of 1 1/2" x 3" cards for use as markers in the charts.

Small individual charts for each pupil are more easily made by folding a sheet of paper into thirds and labeling appropriately. Pupils can use tooth-picks, popsicle sticks, or cardboard strips for markers.

The following figure illustrates an addition example and shows how the pocket charts help pupils see the meaning of regrouping. The example, 48 + 25, is shown with 4 bundles of 10 in the 10s place, and 8 ones in the ones place. 25 is shown with 2 bundles of 10 in the 10s place, and 5 in the ones place. The pupil puts all the ones together, then takes 10 of them and makes a bundle of 10 which he puts in the 10s place. Thus he sees 73 as the sum.

Borrowing or regrouping for subtraction can be similarly shown, as can multiplication and division.

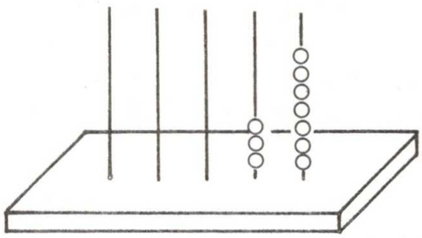



*ABACUS*

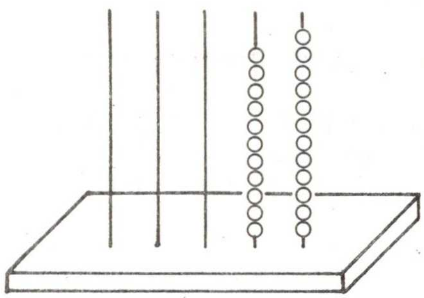
Some form of the abacus has been used as a computing device since earliest times, and is still in use in some countries today. Although there are many types of abaci available, perhaps the easiest for children to make and use is the spike or open-end abacus.

A 1" by 3" by 5" board or piece of styrofoam may be used for a base. Pieces of coat hanger wire can be pushed into the styrofoam, or put into holes drilled in the board. The wires should be uniformly spaced; 3 wires are sufficient for primary grades, and 5 or 6 for intermediate grades. Wooden or plastic beads are used to put on the wires.

To do an addition example such as 37 + 85, 7 beads are put on the ones wire and 3 on the 10s wire.



Then 5 beads are put on the ones wire, and 8 on the 10s wire.



Ten beads are then removed from the ones wire, and exchanged for one bead which is put on the 10s wire. Similarly, 10 beads are removed from the 10s wire and exchanged for one bead which is put on the 100s wire and exchanged for one bead which is put on the 100s wire. Pupils can see the result - 122.

Subtraction may be shown by reversing the steps for addition.

The wires can also represent decimal places, and illustrate computation with decimals in the same manner as above. If upper elementary students are working with bases other than base 10, the abacus can again be used to show place value, regrouping, and computation.

Some pupils may be interested in looking into the history of the abacus, and becoming familiar with the types of abaci still in use today in countries like Japan.

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